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Multi-objective optimization of water distribution networks using particle swarm optimization

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ABSTRACT

Most applied optimization problems involve several objectives, which must be solved simultaneously within a set of constraints. When conflicting objectives exist, there is no single solution to be chosen as the best one, but a set of optimal solutions to the problem. When the decision variables are discrete, the complexity of the problem increases even further. In the present work, a multi-objective model is presented to solve the problem of water distribution network optimization with discrete variables. The problem has a mixed discrete nonlinear programming formulation. A new algorithm based on particle swarm optimization is proposed in order to solve the model. Hydraulic simulator EPANET v2.1 is used to calculate the pressure at each node and flow velocity of water in each pipe. Two problems from the literature are studied, having as objectives the minimization of pipeline installation costs and the minimization of pumping energy costs for the system. Through the weighted sum method, the problems of WDN were solved. The proposed algorithm is verified to be efficient, with equal or better results than those found in the literature.

Keywords: Mixed discrete nonlinear programming; Multi-objective optimization; Water distribution networks; Particle swarm optimization; EPANET

1. Introduction

In practice, there are factors that influence decision making when setting up a water distribution network (WDN), which may be technical, economic, political, etc. When these different objectives are conflicting, the final result is not, necessarily, the best one. For most WDN, the global cost comprises the costs of pipeline installation and of the pumping system, pumping energy, and infrastructure work. The infrastructure work (water intake structure, water treatment plant, raw water pumping system, etc.) are defined in advance. The pipeline installing cost and the pumping energy cost are variables to be optimized within the particular constraints of the network.

Problems that present a set of objectives to be minimized (or maximized) within a set of constraints are named multi-objective optimization problems (MOOP). These objectives are usually conflicting, that is, a function cannot have its value improved without worsening the value of another function. Therefore, there is a set of solutions, which have advantages in certain objectives but are worse in others [1].

In the present work, a problem with a multi-objective formulation is used in WDN dimensioning, involving two

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conflicting functions: the minimization of fixed costs C_P (pipeline installation costs) and the minimization of operational costs C_E (pumping energy costs).

The reduction of the pipes diameters results in greater energy consumption. One of the challenges in WDN design is the optimization of operational efficiency, which includes the pumping system, the volumes of variable-level reservoirs, the pipe diameters, etc. Operational sustainability is a rising issue [2].

WDNs should be designed in an efficient manner to ensure water supply with adequate flow rates and pressures, avoiding excessive installation and operational network costs. The optimization of a WDN comprises searching for the best combination of pipe diameters that results in a minimal installation cost while meeting specific constraints, such as minimum pressures in consumption nodes or minimum and maximum velocities of the fluid in the pipes.

In the steady-state operation, the algebraic sum of the flow rates in each node must equal zero and the algebraic sum of the head losses (starting from and arriving at the same node) in a closed circuit inside the system (loops) must equal zero.

According to Wu and Simpson [3], traditional optimization methods, including linear, nonlinear, and dynamic programming, have provided efficient computational methods to achieve a lower-cost solution. They present, however, some disadvantages, such as: inefficiency in the search for a global optimum due to the zero-gradient optimality criterion, lack of flexibility at handling discrete variables, and complexity of implementation in practical engineering designs.

Therefore, the search is also carried out outside the neighborhood, thus increasing the chances of finding a global optimal solution. Some heuristic methods with applications in WDN optimization stand out, as shown in Table 1.

Stochastic methods use only information on the function to be optimized, which can be nonlinear, nondifferentiable, and multimodal. These methods search for the optimal solution using probability rules, generating possible solution candidates according to a certain pattern.

The particle swarm optimization (PSO) algorithm is a simple and efficient technique that can be naturally extended in order to deal with multi-objective optimization problems (MOOP). Kumar and Minz [27] proposed a solution named multi-objective particle swarm optimization (MOPSO), where the PSO can be modified in two ways: first, each objective function is treated separately, and second, all objective

Table 1
Some heuristic methods applied in WDN optimization

Solving algorithm	Year	Researchers	WDN
ACO – Ant Colony Optimizations	2003	Maier et al. [4]	Two Reservoirs and NYC water Supply System
GA – Genetics Algorithms	1987	Goldberg and Kuo [5]	Serial Liquid Pipeline
	1993	Simpson and Murphy [6]	Two Reservoirs
	1997	Savic and Walters [7]	Two Loop, Hanoi and NYC water Supply System
	2015	Bi et al. [8]	Hanoi, Extended Hanoi, Fosspoly1, ZJ, Balerma, Rural Network, Klmod Network
HBMO – Honey Bee Mating Optimization	2008	Jahanshahi and Haddad [9]	Nyc Water Supply System
	2010	Mohan and Babu [10]	Two Loop and Hanoi
HS – Harmony Search	2002	Geem et al. [11]	Two Loop
ILS – Iterated Local Search	2016	De Corte and Sörensen [12]	Two Loop, Hanoi and HydroGen
PSO – Particle Swarm Optimization	2006	Suribabu and Neelakantan [13]	Two Loop and Hanoi
	2008	Montalvo et al. [14]	Hanoi and NYC Water Supply System
	2014	Ezzeldin et al. [15]	Two Loop and Two-Source
	2017	Surco et al. [16]	Two Loop, Hanoi, R-9, Balerma
	2018	Surco et al. [17]	Two Reservoirs, NYC Water Supply System, Network 1, Esperança Nova
SA – Simulated Annealing	1995	Loganathan et al. [18]	Two Loop and NYC water Supply System
	1999	Cunha and Sousa [19]	Two Loop and Hanoi
SCE – Shuffled Complex Evolution	2004	Liong and Atiquzzaman [20]	Two Loop and Hanoi
SFLA – Shuffled Frog Leaping Algorithm	2003	Eusuff and Lansey [21]	Two Loop, Hanoi, NYC water Supply System
STA – State Transition Algorithm	2016	Zhou et al. [22]	Two Loop, Hanoi, NYC water Supply System
TS – Tabu Search	1999	Fanni et al. [23]	Two Reservoirs and Extended Two Reservoirs Network
	2004	Cunha and Ribeiro [24]	Two Loop, Hanoi, NYC water Supply System
WCA – Water Cycle Algorithm	2014	Sadollah et al. [25]	Balerma
Multi-Swarm Optimizer	2019	Surco et al. [26]	Two-Source, Esperança Nova

functions are evaluated for each particle. A nondominated solution (best position) is chosen as the leader of the group.

Siew et al. [28] presented an algorithm named Penalty-Free Multi-Objective Evolutionary Algorithm (PF-MOEA), based on Nondominated Sorting Genetic Algorithm (NSGA II) to solve a network with multiple pumps and variable-level reservoirs.

Zheng and Zecchin [29] developed an algorithm named decomposition and Decomposition and Dual-Stage Multi-Objective Optimization (DDMO) to minimize installation costs for network pipes and to maximize network reliability, described by Farmani et al. [30].

In the present work, an optimization model was developed for WDN design, considering two conflicting objectives: the minimization of pipeline installation costs and the minimization of pumping energy costs. An algorithm based on PSO was developed to solve the problem. The hydraulic variables were calculated using EPANET, which is a public domain hydraulic analysis package for water supply networks and was developed by the U.S. Environmental Protection Agency in 1993. In July 2016, it was provided the toolkit for EPANET version 2.1, which is used in the present work.

2. Multi-objective function optimization

MOOPs may be defined as follows: a set of objective functions to be optimized, subject to a set of constraints, as shown in Eq. (1), where $\boldsymbol{x} = [x_1,x_2,\ldots,x_{n_4}]^T$ is the vector of n_4 decision variables, also known as the solution vector:

$$\left.\begin{array}{lll} \text{Minimize/Maximize} & f_{i_1}(\boldsymbol{x}), & i_1 = 1,2,\ldots,n_1 \\ \text{Subjectt o} & g_{i_2}(\boldsymbol{x}) \geq 0, & i_2 = 1,2,\ldots,n_2 \\ & h_{i_3}(\boldsymbol{x}) = 0, & i_3 = 1,2,\ldots,n_3 \\ & x_{i_4}^{(L)} \leq x_{i_4} \leq x_{i_4}^{(U)} & i_4 = 1,2,\ldots,n_4 \end{array}\right\} \quad (1)$$

Decision variable space *'D* is limited by $x_{i_4}^{(L)}$ and $x_{i_4}^{(U)}$, which represent the lower and the upper limits of the variable x_{i_4}. The n_2 inequalities defined by $g_{i_2}(\boldsymbol{x})$ and the n_3 equalities defined by $h_{i_3}(\boldsymbol{x})$ are the constraints of the model.

The vector comprising the n_1 objective functions, $f(\boldsymbol{x}) = \left[f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),\ldots, f_{n_1}(\boldsymbol{x})\right]$, forms a multidimensional space named (objective space). Functions $f_{i_1}(\boldsymbol{x})$ are conflicting. For a single-objective optimization problem, $f(\boldsymbol{x}): \mathbb{R}^{n_4} \to \mathbb{R}$, while for a multi-objective one, $f(\boldsymbol{x}): \mathbb{R}^{n_4} \to \mathbb{R}^{n_1}$.

2.1. MOOP solution methods

All Pareto-optimal solutions are, in principle, equally important. According to Deb [1], there are two goals in multi-objective optimization:

- Finding a set of solutions as close as possible to the Pareto-optimal front;
- Finding a set of solutions with the greatest possible diversity.

The first goal is common in optimization works. The second one however has to be carried out including the whole Pareto-optimal front. With a diversified set of solutions, comes a good set of options among objectives. The aforementioned goals should be met with utmost computational efficiency.

Two different approaches may be taken when optimizing multi-objective functions: multiple criteria decision making (MCDM) and evolutionary multi-objective optimization (EMO). These approaches solve the same problem using different focuses [31]. MCDM aims to support the decision maker in identifying the preferred solution, with the ability to choose said preference before, during, or after the search for the ideal solution. In this method, a MOOP can be converted, via some techniques, into a single-objective optimization problem. On the other hand, the EMO method is based on an evolutionary algorithm that tries to find a set of nonconditioned solutions near the Pareto-optimal front.

There are some classic methods for the solution of MOOPs:

- Weighted sum method
- ε-constraint method
- Goal-programming method

Due to the results required in the present work, the weighted sum method will be focused on. Its formulation is presented by Eq. (2), where $w_{i_1}\left(w_{i_1} > 0, \forall i = 1,\ldots,n_1\right)$ are the weight of each objective function; Weight vector *W* is given by $W = \left(w_1, w_2,\ldots,w_{n_1}\right)$. Usually, weights that suit equation $\sum_{i_1}^{n_1} w_{i_1} = 1$ can be chosen. *F*($\boldsymbol{x}$) is the global objective function (GOF):

$$\left.\begin{array}{lll} \text{Minimize } F(\boldsymbol{x}) = \sum_{i_1}^{n_1} w_{i_1} f_{i_1} & & \\ \text{Subject to} & g_{i_2}(\boldsymbol{x}) \geq 0, & i_2 = 1,2,\ldots,n_2 \\ & h_{i_3}(\boldsymbol{x}) = 0, & i_3 = 1,2,\ldots,n_3 \\ & x_{i_4}^{(L)} \leq x_{i_4} \leq x_{i_4}^{(U)} & i_4 = 1,2,\ldots,n_4 \end{array}\right\} \quad (2)$$

The advantages of this method are its simplicity and ease of use. Furthermore, in situations of convex problems, solutions in the Pareto-optimal set are guaranteed to be found.

3. Optimization of WDNs with pumping in the network head

If a network presents *M* pipes and *K* nodes, its optimization consists in finding the diameters of the pipes that compose the network with minimal cost. In the context of WDN design, there is a finite set of available diameters for a given network, which will be named $D_{SET} = \{D_1, D_2, \ldots, D_{nd}\}$, and their respective costs, Cost = $\{Cost_1, Cost_2, \ldots, Cost_{nd}\}$, so that:

$D_1 < D_2 < \ldots < D_{nd}$, where $D_1 = D_{min}$ and $D_{nd} = D_{max}$

3.1. Objective function for pipe installation costs

The objective function can be formulated as shown in Eq. (3), where C_P is the objective function to minimize the total pipe installation costs, L_j is the length of pipe j, and Cost(D_j) is the installation cost per unit length for the pipe with diameter D_j. D_j is the decision variable, $j = 1, \ldots, M$:

$$\text{Min}C_P = \sum_{j=1}^{M} L_j \cdot \text{Cost}\left(D_j\right) \tag{3}$$

3.2. Objective function for pumping energy costs

The objective function to minimize operational expenses of the pumping station (C_E) can be expressed as shown in Eq. (4), where E_h represents the updated cost of water pressurization per meter of elevation, provided by Eq. (5), and H is a decision variable representing the pumping head (m):

$$\text{Min}C_E = E_h \cdot H \tag{4}$$

$$E_h = \frac{9.81 Q_T}{\eta} E_C \cdot n \cdot N_{\text{hy}} \cdot \text{PWF} \tag{5}$$

In the aforementioned equations, E_h is in \$/m, Q_T is the total flow rate of the system (m^3/s), E_C is the electrical energy cost per kWh, η is the efficiency of the pump-motor unit, N_{hy} is the number of pumping hours per year, n is the lifespan of the installation in years, PWF is the present worth factor used in engineering economic for geometric gradient series [32], shown in Eq. (6), for this model e_1 is the annual interest or discount rate and e_2 is the annual rate of increase in the unit energy cost:

$$PWF = \begin{cases} \left[\left[\dfrac{1-\left(1+e_2\right)^n\left(1+e_1\right)^{-n}}{e_1 - e_2}\right]\right] & \text{if} \quad e_1 \neq e_2 \\ \left[\dfrac{n}{1+e_1}\right] & \text{if} \quad e_1 = e_2 \end{cases} \tag{6}$$

The PWF for a projection of n years also named present value factor, converts a series of annual costs into a present value, subject to an interest rate e_1 and to a rate of increase in the unit energy cost e_2.

3.3. Constraints and limitations

- Law of conservation of mass

$$\sum q(k) = d(k), \quad k = 1, \ldots, K(\text{nodes}) \tag{7}$$

This equation shows that the net flow rate in each node must equal to demand d at node k.

- Law of conservation of energy

$$\sum h_f(j) = \text{Ep}, \quad j \in \text{loop set} \tag{8}$$

The sum of head losses h_f in the pipes (starting and ending at the same node) must equal to the energy delivered by a pump belonging to the loop.

- Diameters are discrete, real variables

Diameter D_j must belong to the set of available diameters for the network, D_{SET}:

$$D_j \in D_{\text{SET}} = \left\{D_1, D_2, \ldots, D_{\text{nd}}\right\} \tag{9}$$

- Minimum pressure requirements in the nodes

The pressure in demand node k must be greater than the minimum requirement for the said node ($\forall k = 1,\ldots,K$):

$$\text{pr}(k) \geq \text{pr}_{\min}(k) \tag{10}$$

- Minimum and maximum velocities of the fluid in the pipes

Water velocity in pipe j (v_j) must be between the limits showed in Eq. (11):

$$v_{\min} \leq \left|v_j\right| \leq v_{\max} \tag{11}$$

- Minimum and maximum reservoir elevations

The minimum reservoir elevation must be enough for the water supply by gravity to be viable and the maximum elevation must be defined for each case study:

$$H_{\min} \leq H \leq H_{\max} \tag{12}$$

The head loss, h_f, is calculated, in the international system of units (SI), using the Hazen–Williams Eq. (13), where C_j is the Hazen–Williams roughness coefficient (dimensionless), q_j is the flow rate (m^3/s), D_j is the diameter (m), and L_j is the length (m) for pipe j:

$$h_f(j) = \frac{10.674 q_j^{1.852} L_j}{C_j^{1.852} D_j^{4.871}} \tag{13}$$

3.4. PSO algorithm with continuous variables for the minimization of pumping energy costs

The PSO algorithm initialization for variable H is as follows:

- Particle velocity of variable H may be initialized at zero ($v_H = 0$) or randomly between limits $V_{H\min}$ and $V_{H\max}$.

- Eq. (14) is used to start the H_i particles, $i = 1,2,\ldots,N$, where r is a uniformly distributed random number in the interval [0, 1].

$$H_i = H_{\min} + r \cdot \left(H_{\max} - H_{\min}\right) \tag{14}$$

- Eq. (15) is used to update v_H, where w_{PSO} is the inertia weight, c_1 and c_2 are, respectively, the cognitive and social acceleration coefficients, and r_1 and r_2 are uniformly distributed random numbers in the interval [0,1]. The values of variables p_{H_i} and g_H must be known beforehand for Eq. (15) to be used:

$$v_{H_i}\left(t+1\right) = v_{H_i}\left(t\right) \cdot w_{\text{PSO}} + c_1 \cdot r_1 \left(p_{H_i} - H_i\right) + c_2 \cdot r_2 \left(g_H - H_i\right) \tag{15}$$

Some researchers recommend the inertia weight, w_{PSO}, to be dynamic, that is, decreasing as a function of iterations. Several formulations for w_{PSO} as a function of iterations t are present in the literature [33].

- The velocity of variable H in the PSO algorithm must be limited by defined values of $V_{H\min}$ and $V_{H\max}$:

$$V_{H\min} \le v_{H_i} \le V_{H\max} \tag{16}$$

- The new position of particle i in iteration $t + 1$ is given by:

$$H_i\left(t+1\right) = H_i\left(t\right) + v_{H_i}\left(t+1\right) \tag{17}$$

If the network pump belongs to a set of available pumps it is necessary to convert the continuous variable H into a discrete one. The pump heads are based on the specific curves for each pump:

$$H_{nP}\left(Q_{nP}\right) = a \cdot Q_{nP}^2 + b \cdot Q_{nP} + c \tag{18}$$

The coefficients a, b, and c are presented in Table 2.

H_1 is fixed as the $H_{\min}$ and $H_{n\text{PT}}$ is fixed as the $H_{\max}$. H_i belongs to the set $\{H_1,H_2,\ldots,H_{n\text{PT}}\}$. Fig. 1 illustrates the discretization procedure [16].

According to Fig. 1, variable H_i will assume the value of either H_L or H_U:

$$H_i = \begin{cases} H_L & \text{if } d_1 \le d_2 \\ H_U & \text{if } d_1 > d_2 \end{cases} \tag{19}$$

Table 2
Pump curves coefficients

Pump number	a	b	c	Pump heads (m)
$nP = 1$	a_1	b_1	c_1	H_1
$nP = 2$	a_1	b_1	c_1	H_2
⋮	⋮	⋮	⋮	⋮
$nP = n\text{PT}$	$a_{n\text{PT}}$	$b_{n\text{PT}}$	$C_{n\text{PT}}$	$H_{n\text{PT}}$

This involves cost differences in the construction and installation of the reservoirs. It is not being considered in the optimization, which is a limitation of the work.

3.5. Discrete-variable PSO algorithm for the minimization of pipeline installation costs

Particles are initialized randomly, with continuous values between limits $D_{\min}$ and $D_{\max}$, according to Eq. (20), where r is a uniformly distributed random number in the interval [0,1].

$$x_{i,j} = D_{\min} + r \cdot \left(D_{\max} - D_{\min}\right) \tag{20}$$

The response of Eq. (20), $x_{i,j}$ (diameter of pipe j belonging to particle i), is a continuous number and must be discretized, as shown in the approach of Fig. 1 and Eq. (19). Therefore, $x_{i,j} \in \{D_1,D_2,\ldots,D_{\text{nd}}\}$.

Iteration velocity, $v_{Pi,j}$, is a continuous value. It is initialized at zero or randomly between $-V_{\text{PSOmin}}$ and V_{PSOmax}. The new position of particle $x_{i,j}$ in iteration $t + 1$ is given by Eq. (21), whose result is continuous and must be discretized again, according to Fig. 1 and Eq. (19). Particle velocity $v_{Pi,j}$ is given by Eq. (22):

$$x_{i,j}\left(t+1\right) = x_{i,j}\left(t\right) + v_{Pi,j}\left(t+1\right) \tag{21}$$

$$v_{Pi,j}\left(t+1\right) = v_{Pi,j}\left(t\right) \cdot w_{\text{PSO}} + c_1 \cdot r_1 \left(p_{i,j} - x_{i,j}\right) + c_2 \cdot r_2 \left(g_j - x_{i,j}\right) \tag{22}$$

Iteration velocity values can be controlled by imposing limits, shown in Eq. (23), and V_{PSOmax} itself can be limited by Eq. (24), where $\delta \in (0,1]$.

$$-V_{\text{PSOmax}} \le v_{Pi,j} \le V_{\text{PSOmax}} \tag{23}$$

$$V_{\text{PSOmax}} = \delta \cdot \left(D_{\text{máx}} - D_{\text{mín}}\right) \tag{24}$$

These limits must be tested for an efficient convergence in the search for an optimal solution.

In the case of single-objective optimization problems concerning WDN design, the particle is a vector with dimension M, which corresponds to the diameters of the M pipes, considered to be a possible solution for the network, given by vector $X_i = (x_{i,1},\ldots,x_{i,j},\ldots,x_{i,M})$ where $x_{i,j}$ is the diameter of pipe j belonging to particle i. Vector X_i is named the current position of particle i. Analogously, there are the velocity vector (V_i) and the best position vector (P_i) for particle i, which have the following formats: $V_i = (v_{Pi,1},\ldots,v_{Pi,j},\ldots,v_{Pi,M})$ and $P_i = (p_{i,1},\ldots,p_{i,j},\ldots,p_{i,M})$. The vector of the best position reached by particle i is named local best and vector $G = (g_1,\ldots,g_j,\ldots, g_M)$ is the leader particle, which has the best evaluation in iteration t, named global best.

4. Developed algorithm

For an optimization problem with two objective functions, there are M variables relative to the pipes of the WDN and one variable relative to the pumping system, totaling $M + 1$ decision variables.

Particle i is a vector of dimension $M + 1$ and has the following format:

- $X_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,M}, H_{i,M+1})$ named position vector;
- $V_i = \left(v_{Pi,1}, \ldots, v_{Pi,j}, \ldots, v_{Pi,M}, v_{H_{i,M+1}}\right)$, named velocity vector;
- $P_i = \left(p_{i,1}, p_{i,2}, \ldots, p_{i,M}, p_{H_{i,M+1}}\right)$ is the vector of the best position achieved by particle i, named local best;
- $G = (g_1, g_2, \ldots, g_M, H_g)$ is the vector of the leader particle in iteration t, which has the best bi-objective evaluation, named global best.

At a given iteration t, the position of the N particles from the group forms matrix X of order $N \times (M + 1)$, as shown in Eq. (25). Each particle i from the group moves in the decision space with a certain speed, V_i, in search of a new position. Eq. (26) represents matrix V of order $N \times (M + 1)$. For each movement (iteration) of a particle, its position is updated and evaluated according to the global objective function (GOF), shown in Eq. (27), where L_j is the length of pipe j, $\text{Cost}(x_{i,j})$ is the cost of pipe j with diameter $x_{i,j}$, E_h is a constant calculated by Eq. (5) and H_i is the elevation for particle i:

$$X = \left[\begin{array}{l|l} x_{1,1}\ x_{1,2}\ x_{1,3} \ldots\ldots\ldots x_{1,M} & H_{1,M+1} \\ x_{2,1}\ x_{2,2}\ x_{2,3} \ldots\ldots\ldots x_{2,M} & H_{2,M+1} \\ \quad\quad . & . \\ x_{i,1}\ x_{i,2}\ x_{i,3} \ldots\ldots\ldots x_{i,M} & H_{i,M+1} \\ \quad\quad . & . \\ \quad\quad . & . \\ x_{N,1}\ x_{N,2}\ x_{N,3} \ldots\ldots\ldots x_{N,M} & H_{N,M+1} \end{array}\right] \tag{25}$$

$$V = \left[\begin{array}{l|l} v_{P1,1}\ v_{P1,2}\ v_{P1,3} \ldots\ldots\ldots v_{P1,M} & v_{H_{1,M+1}} \\ v_{P2,1}\ v_{P2,2}\ v_{P2,3} \ldots\ldots\ldots v_{P2,M} & v_{H_{2,M+1}} \\ \quad\quad . & . \\ v_{Pi,1}\ v_{Pi,2}\ v_{Pi,3} \ldots\ldots\ldots v_{Pi,M} & v_{H_{i,M+1}} \\ \quad\quad . & . \\ \quad\quad . & . \\ v_{PN,1}\ v_{PN,2}\ v_{PN,3} \ldots\ldots\ldots v_{PN,M} & v_{H_{N,M+1}} \end{array}\right] \tag{26}$$

$$C_{TGi} = \sum_{j=1}^{M} w_1 \cdot L_j \cdot \text{Cost}\left(x_{i,j}\right) + w_2 \cdot E_h \cdot H_i \tag{27}$$

Eq. (28) shows the global objective function with the combination of weights and the penalization. Penalization happens when the particle (vector X_i) violates the hydraulic constraints of pressure or velocity. The developed algorithm, through variable NV_i, counts the number of nonviable nodes (node pressure lower than $\text{pr}_{\min}$) and the number of nonviable pipes (velocity in the pipe outside the limits of $v_{\min}$ or $v_{\max}$) for particle i, which is then penalized with a value, W_{penal}, for each violation of the hydraulic constraints. If there are NV_i violations, the total penalization will be $W_{\text{penal}} \times NV_i$. This penalization is then added to the value of the global objective.

This function in this process, hydraulic simulator EPANET, is used for the evaluation of node pressures and velocities in the pipes for particle i. This penalized global objective function will be used in the optimization process:

$$C_{\text{TGP}i} = \sum_{j=1}^{M} w_1 \cdot L_j \cdot \text{Cost}\left(x_{i,j}\right) + w_2 \cdot E_h \cdot H_i + W_{\text{penal}} \cdot NV_i \tag{28}$$

Each favorable position (minimal F_i) of particle X_i is named local best evaluation. The position is stored in vector P_i. Matrix P of order $N \times (M + 1)$, shown in Eq. (29), is formed by the vectors P_i. The value of the penalized global objective function of each particle P_i is stored in column vector F_L, shown in Eq. (30), being simultaneously updated with matrix P:

$$P = \left[\begin{array}{l|l} p_{1,1}\ p_{1,2}\ p_{1,3} \ldots\ldots\ldots p_{1,M} & p_{H_{1,M+1}} \\ p_{2,1}\ p_{2,2}\ p_{2,3} \ldots\ldots\ldots p_{2,M} & p_{H_{2,M+1}} \\ \quad\quad . & . \\ p_{i,1}\ p_{i,2}\ p_{i,3} \ldots\ldots\ldots p_{i,M} & p_{H_{i,M+1}} \\ \quad\quad . & . \\ \quad\quad . & . \\ p_{N,1}\ p_{N,2}\ p_{N,3} \ldots\ldots\ldots p_{N,M} & p_{H_{N,M+1}} \end{array}\right] \tag{29}$$

$$F_L = \begin{bmatrix} F_{P1} = w_1 \cdot C_{T1} + w_2 \cdot C_{E1} + W_{\text{penal}} \cdot NV_1 \\ F_{P2} = w_1 \cdot C_{T2} + w_2 \cdot C_{E2} + W_{\text{penal}} \cdot NV_2 \\ . \\ F_{Pi} = w_1 \cdot C_{Ti} + w_2 \cdot C_{Ei} + W_{\text{penal}} \cdot NV_i \\ . \\ . \\ F_{PN} = w_1 \cdot C_{TN} + w_2 \cdot C_{EN} + W_{\text{penal}} \cdot NV_N \end{bmatrix} \tag{30}$$

The vector whose elements equal $D_{\max}$ is named $G_{\max} = (D_{\max}, D_{\max}, \ldots, D_{\max})$. If particle $G_{\max}$ is viable, that is, $\text{pr}(k) \geq \text{pr}_{\min}(k)$ for each node k, the WDN problem is an optimization problem with a single objective, which means the feeding source is sufficiently high to supply every node in the system by gravity. If solution $G_{\max}$ shows pressures in demand nodes lower than the required minimum, the water has to be pumped to a greater elevation. Therefore, the vector becomes $G_{\max} = (D_{\max}, \ldots D_{\max}, H_{\max})$ and the corresponding value of the global objective function is given by C_{TGmax}.

For each new position (iteration), the best position of P is also evaluated, generating vector G (global best), $G = (g_1, g_2, \ldots, g_M, H_g)$ whose evaluation with the global objective function, shown in Eq. (28), is a scalar named global total cost, C_{TGG}, which is, for now, the best result found in

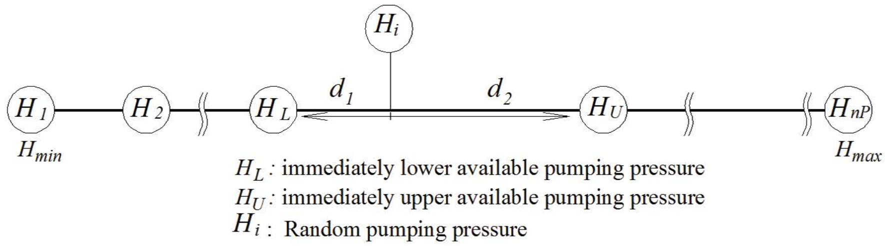

Fig. 1. Discretization of variable H_i.

the optimization process. The movement of the particles (iterations) ends when C_{TGG} shows no variation in the results obtained in subsequent iterations or when the iteration limit, t_{max}, is reached.

For each particle $i = 1,…,N$, the algorithm initializes vectors $P_i = (D_{max},…,D_{max},H_{min})$, $V_i = (0,…,0,0)$, and $F_{Pi} = C_{TGmax}$. Vector global best is initialized as $G = (D_{min},D_{min},…,D_{min}, H_{max})$ and its corresponding value of the global objective function is C_{TGmax}, which corresponds to a reference value in the search for the minimal values of the global objective function. The results to be presented are: $G = (g_1, g_2,…,g_M,H_g)$, the global total cost (C_{TGG}), the pressure (pR) for each node, and the velocity (vR) for each of the M pipes.

The algorithm, which is described below, works with a certain number of attempts (Att_max). Each attempt yields a viable minimal solution to the global function, according to the maximum number of iterations (t_{max}). After carrying out the Att_max attempts, the algorithm will select the best result, showing the values of the decision variables as well as those of the hydraulic variables.

Algorithm description:

- Initialize X_i, V_i, P_i, $\forall\ i \in \{1,…,N\}$: $x_{i,j}$ according to Eqs. (14) and (20) for $j = M + 1$. The particle starts at rest, that is, $v_{Pi,j} = 0$. Vector P_i is given by $P_i = (D_{max},…, D_{max},H_{min})$ and its respective initial performance is $F_{Pi} = C_{TGmax}$. Vector G is given by $G = (D_{min},D_{min},…, D_{min},H_{max})$ and its respective performance is $C_{TGG} = C_{TGmax}$.
- For each particle i ($i = 1,…,N$):
  - Calculate the pressures and the velocities using the hydraulic simulator EPANET, then calculate the number of violations (NV_i) and the value of the penalized global objective function, C_{TGPi}, according to Eq. (28).
  - Compare performance C_{TGPi} with performance F_{Pi} of vector P_i. If it is better, update the performance of vector P_i ($F_{Pi} \leftarrow C_{TGPi}$) as well as its components ($P_i \leftarrow X_i$).
  - Compare performance C_{TGPi} with performance C_{TGG} of vector G_{best}. If it is better and no constraints are violated ($NV_i = 0$), update the performance of G_{best} ($C_{TGG} \leftarrow C_{TGPi}$), the components of vector G ($G \leftarrow X_i$), the vector of pressure results, pR, and the vector of velocity results, vR.
- If the number of iterations is greater than t_{max} and the number of attempts, A_t, is lower than Att_max, return to step 1. Else, end the process and output the results (G, C_{TGG}, pR, vR).

Fig. 2 presents the block diagram of the developed algorithm.

5. Case studies

Two case studies were considered in order to test the applicability of the developed model.

5.1. Grande Setor WDN

Grande Setor WDN corresponds to the water supply system belonging to the city of João Pessoa, in the state of Paraíba – Brazil. The studied WDN was taken from the work by Gomes et al. [34]. The network comprises seven nodes (including the reservoir) and eight pipes. All pipes and the elevation of the reservoir are to be dimensioned with the lowest possible cost, as shown in Fig. 3. The minimal pressure imposed on the network is 25 m for each node and the maximum and minimum velocities allowed in the pipes are 3.0 and 0.2 m/s, respectively.

Network data is shown in Table 3. Water is collected at an elevation of 30 m, coinciding with the altimetric elevation of the terrain.

The materials used for the pipes are rigid PVC and ductile iron. Ductile iron pipes can withstand greater pressure than rigid PVC ones. Therefore, pipes with a diameter greater than 350 mm are ductile iron ones. There are 10 diameters available for use in this network. Table 4 shows the properties and the installation costs of the pipes.

Data on pumping energy costs and pumping conditions is presented in Table 5.

The bi-objective optimization function can be written as shown in Eq. (31):

$$F_P\left(X_i\right) = \sum_{j=1}^{8} w_1 \cdot L_j \cdot \text{Cost}\left(x_{i,j}\right) + w_2 \cdot \left(44{,}657.39\right)\left(H_i\right) + W_{\text{penal}} \cdot NV_i \tag{31}$$

The parameters used for the PSO algorithm were: $w_{PSO} = 0.9$, $c_1 = c_2 = 2$, penalty W_{penal} = US$ 100,000, population $N = 30$, V_{PSOmax} = 100 mm, $V_{HPSOmax}$ = 0.50 m, H_{max} = 14.353 m, H_{min} = 9.287 m, iteration number t_{max} = 120. Considering every new pipe to have a diameter of 619.6 mm, the maximum installation cost, C_P, equals US$ 3,793,777.50, and the maximum pumping system cost, C_E, equals US$ 640,967.50 (H_{max} = 14.353 m), with a global maximum total cost, C_{TGmax}, of US$ 4,434,745.00. Table 6 shows the results for different combinations of weights. The lowest global total cost, C_{TG}, was US$ 2,272,331.75. Elapsed time was 1.0 s for each combination of weight at an Intel Core i5 1.6 GHz CPU.

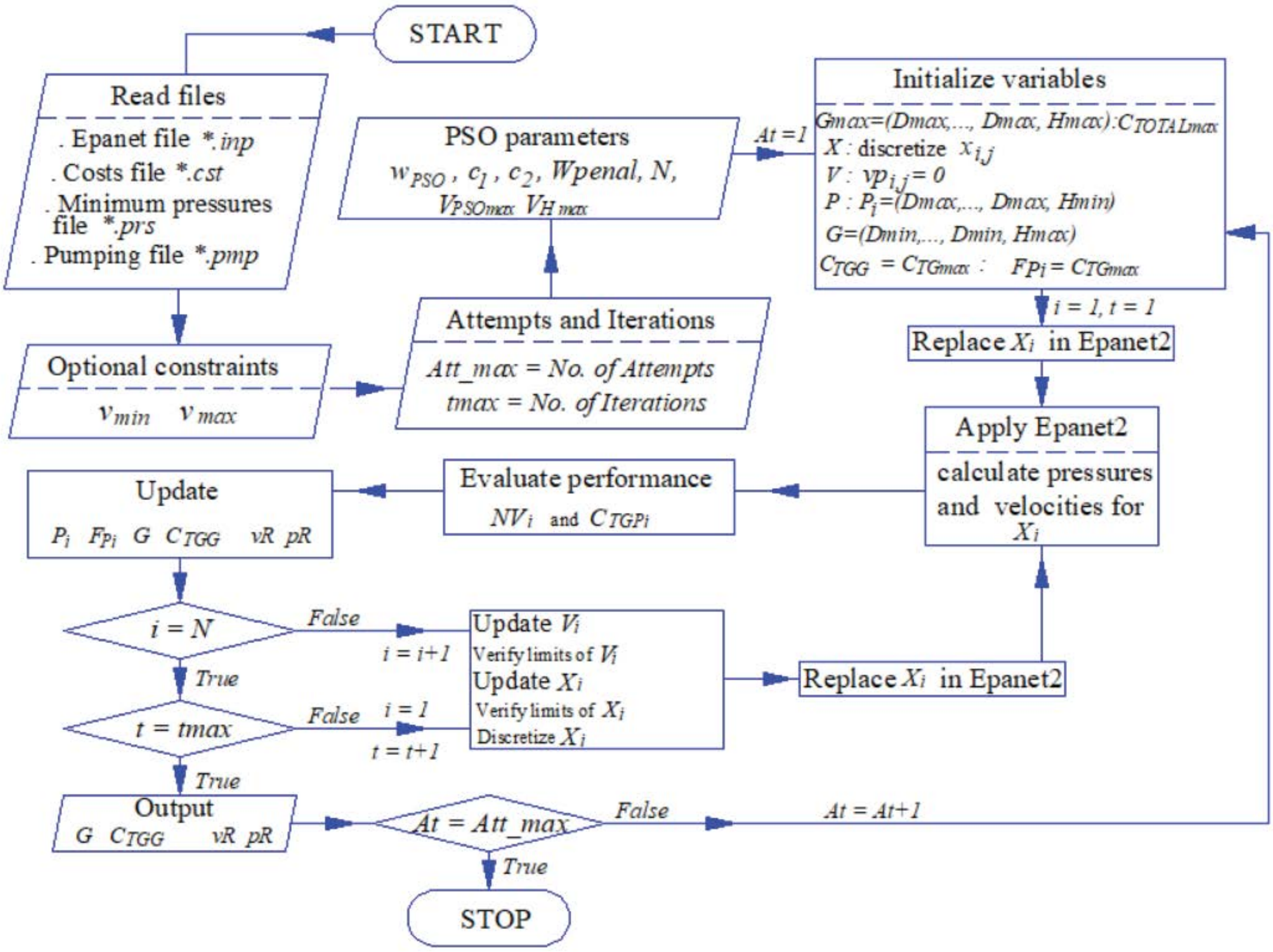

Fig. 2. Block diagram of the developed algorithm.

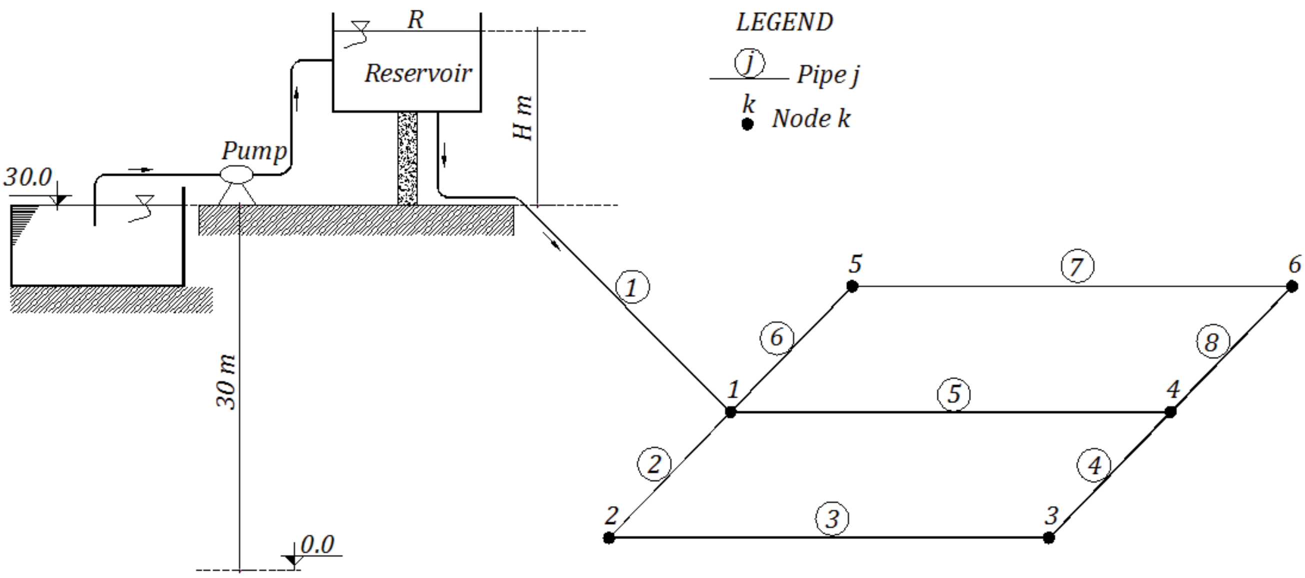

Fig. 3. Grande Setor WDN.

If w_{energy} is greater than w_{pipe}, the minimization of pumping energy costs is preferred and vice-versa. When C_E is preferred, H is lower than in other results. Table 7 presents the results for the pressures in the nodes, as well as the diameters and velocities for each pipe for the combination of weights (0.5,0.5).

The results were compared with those obtained by Gomes et al. [34], according to Table 8. The optimized solution with the proposed algorithm is 2.7% better.

Table 3
Nodal and pipe data for the Grande Setor WDN

Node	Demand (L/s)	Elevation (m)	Pipe	Length (m)
1	0.00	6.00	1	2,540
2	47.78	5.50	2	1,230
3	80.32	5.50	3	1,430
4	208.6	6.00	4	1,300
5	43.44	4.50	5	1,490
6	40.29	4.00	6	1,210
R	0.00	30.00	7	1,460
			8	1,190

Table 4
Pipe data and costs (US$/m) for the Grande Setor WDN

Internal D (mm)	Pipe material	Cost (US$/m)	Roughness coefficient C (H–W)	Nominal D (mm)
108.4	PVC	23.55	145	100
156.4	PVC	31.90	145	150
204.2	PVC	43.81	145	200
252.0	PVC	59.30	145	250
299.8	PVC	76.12	145	300
366.2	Ductile iron	158.93	130	350
416.4	Ductile iron	187.50	130	400
466.6	Ductile iron	218.12	130	450
518.0	Ductile iron	257.80	130	500
619.6	Ductile iron	320.15	130	600

Source: Gomes et al. [34].

Different values of H were selected and the corresponding pipe costs were optimized (Table 9).

5.2. Network 2 with pump-included

Network 2 presented in Fig. 4 was originally presented by Costa et al. [35] and studied by Geem [11]. The network consists of 11 pipes with nine nodes, all with a length of 2,500 m, roughness coefficient (Hazen–Williams) = 130. The minimum pressure imposed is 30 m for all nodes. Table 10 shows the elevation and demand data.

Table 11 shows the ten types of diameters available for network 2.

For this network, there are 10 pumps options. The coefficients of the pump curves and the pumping pressure are shown in Table 12. The discharge pressure of the pump is calculated by Eq. (18).

Pump efficiency is given as a function of Eq. (32):

$$\eta_{nP} = -695.4Q_{nP}^2 + 418.3Q_{nP} + 2.857 \qquad (32)$$

The discharge flow of the system pump is 1,000 m^3/h (0.2778 m^3/s). Replacing this value in Eq. (32) we have the efficiency η_{np} = 65.39%.

Table 5
Pumping costs and conditions for the Grande Setor WDN

Denomination	Value
Number of hours per day pumping (NHPD)	20
Number of days per year (NDPY)	365
Total hours of pumping per year N_{op} = (NHPD)(NDPY)	7,300
Efficiency of the motor-pump unit (η)	0.75
Expected period of service for the network in years (n)	20
Average pumping discharge (Q_T (m^3/s))	0.4204
Electricity cost (Ec (US$/kWh))	0.10
Annual rate of increase in electricity cost (e)	6%
Annual discount rate (j)	12%
Present worth factor (PWF)	11.125
Updated cost of water pumping per elevation meter (E_h (US$/m))	44,657.39

Table 6
Results for different weight vectors

No.	Weight vector (w_{pipe}, w_{energy})	Total cost (US$)	Pipes (US$)	Energy (US$)	H (m)
1	(0.2,0.8)	2,339,255.31	1,866,914.10	472,341.21	10.577
2	(0.3,0.7)	2,321,440.71	1,837,935.15	483,505.56	10.827
3	(0.4,0.6)	2,280,593.35	1,744,302.75	536,290.60	12.009
4	(0.5,0.5)	**2,272,331.75**	**1,662,535.10**	**609,796.65**	**13.655**
5	(0.6,0.4)	2,273,796.85	1,651,362.15	622,434.70	13.938
6	(0.7,0.3)	2,272,438.92	1,651,388.60	621,050.32	13.907
7	(0.8,0.2)	2,272,814.39	1,651,362.15	621,452.24	13.916

Table 7
Results of optimized diameters and pressures in the nodes for the Grande Sector WDN

Nodal heads		Optimal pipe diameters				Gomes et al. [34]	
Node	pr (m)	Pipe	Diameter (mm)	Q (L/s)	v (m/s)	Pipe	Diameter (mm)
1	31.09	1	619.60	420.430	1.39	1	619.60
2	27.24	2	299.80	–80.669	–1.14	2	299.80
3	25.00	3	252.00	32.889	0.66	3	299.80
4	26.22	4	299.80	–47.431	–0.67	4	204.20
5	28.81	5	518.00	291.970	1.39	5	518.00
6	26.03	6	252.00	–47.791	–0.96	6	252.00
R	0	7	108.40	4.351	0.47	7	204.20
		8	252.00	–35.939	–0.72	8	156.40

Table 8
Result comparison for the Grande Setor WDN

Work	H (m)	Total cost (US$)	Energy cost (US$)	Pipe cost (US$)
This work	13.655	2,272,331.75	609,796.65	1,662,535.10
Gomes et al. [34]	15.790	2,335,649.95	705,244.20	1,630,405.75

Table 9
Optimization of pipe installation costs and pumping energy costs for several points of the variable for the Grande Setor WDN

H (m)	Total cost (US$)	Energy cost (US$)	Pipe cost (US$)
8.58	3,514,946.21	383,160.41	3,131,785.80
9.00	3,022,784.21	401,916.51	2,620,867.70
10.00	2,439,508.40	446,573.90	1,992,934.50
11.00	2,329,192.89	491,231.29	1,837,961.60
12.00	2,316,020.68	535,888.68	1,780,132.00
12.50	2,280,356.78	558,217.38	1,722,139.40
13.00	2,285,991.97	580,546.07	1,705,445.90
13.50	2,278,315.97	602,874.77	1,675,441.20
14.00	2,290,880.06	625,203.46	1,665,676.60
14.50	2,287,916.56	647,532.16	1,640,384.40
15.00	2,280,355.85	669,860.85	1,610,495.00
17.00	2,320,320.03	759,175.63	1,561,144.40
20.00	2,401,938.70	893,147.80	1,508,790.90

According to Costa et al. [35], the cost of installing the pump for this network is given by Eq. (33).

$$C_{\mathrm{PM}} = C_{\mathrm{Pump}}\left(Q_{nP}^{\mathrm{Rated}}\right)^{0.7} \cdot \left(H_{nP}^{\mathrm{Rated}}\right)^{0.4} \tag{33}$$

where C_{pump} is a constant (700,743 for this study); Q_{nP}^{Rated}(m^3/s) and H_{np}^{Rated}(m) is the flow and pressure height of the nP pump at the point of maximum efficiency.

Table 13 presents the cost of pumping energy and pumping conditions.

Eq. (34) shows the bi-objective function for optimization.

$$F_P\left(X_i\right) = \sum_{j=1}^{8} w_1 \cdot L_j \cdot \mathrm{Cost}\left(x_{i,j}\right) + w_2 \cdot \left(32{,}719.84\right)\left(H_i\right) + W_{\mathrm{penal}} \cdot NV_i \tag{34}$$

The parameters of the PSO algorithm used were: $w_{\mathrm{PSO}} = 0.9$; $c_1 = c_2 = 2$; penalty W_{penal} = $250,000; population $N = 35$; V_{PSOmax} = 101.6 mm, $V_{H\mathrm{PSOmax}}$ = 8.23 m; $H_{\max}$ = 80.00 m; $H_{\min}$ = 35.78 m; number of iterations $t_{\max} = 45$. Considering all new pipes with the diameter of 762.0 mm, we have the cost of implantation C_P equal to $9,517,750.00 and the maximum cost for the pumping system C_E equal to US$ 2,617,587.20, with a total maximum cost ($C_P + C_E$) C_{TGmax} equal to $12,135,337.20. The lowest overall total cost C_{TG} found is US$ 4,095,257.79, the pump number 4 being the one selected, with discharge pressure of H = 48.48 m. For each combination of weights the time was 1.5 s, on a computer with an Intel processor Core I5 CPU 1.6 GHZ. The results found are shown in Table 14.

The functions C_P (pipe installation cost) and C_E (pumping energy cost) are conflicting functions; already the function C_{PM} (pumping installation cost) is not conflicting with the function C_E. In this study, the pumping installation cost can be calculated according to Eq. (33), considering Q_{nP}^{Rated} = 0.30076 m^3/s and H_{nP}^{Rated} calculated with Eq. (18), so the C_{PM} can be calculated directly. Table 15 presents the optimized result with costs C_P, C_E, and C_{PM}.

The results compared with those of Costa et al. [35] and Geem [36] are identical, with an optimized global cost of $5.505 million. Table 16 presents the optimized diameters and the pressures acting on the nodes.

Table 17 shows the optimization of the installation costs of pipes and pumping energy cost for each pump.

For the optimization of the network 2, the continuous variable H was also considered, the results of which are

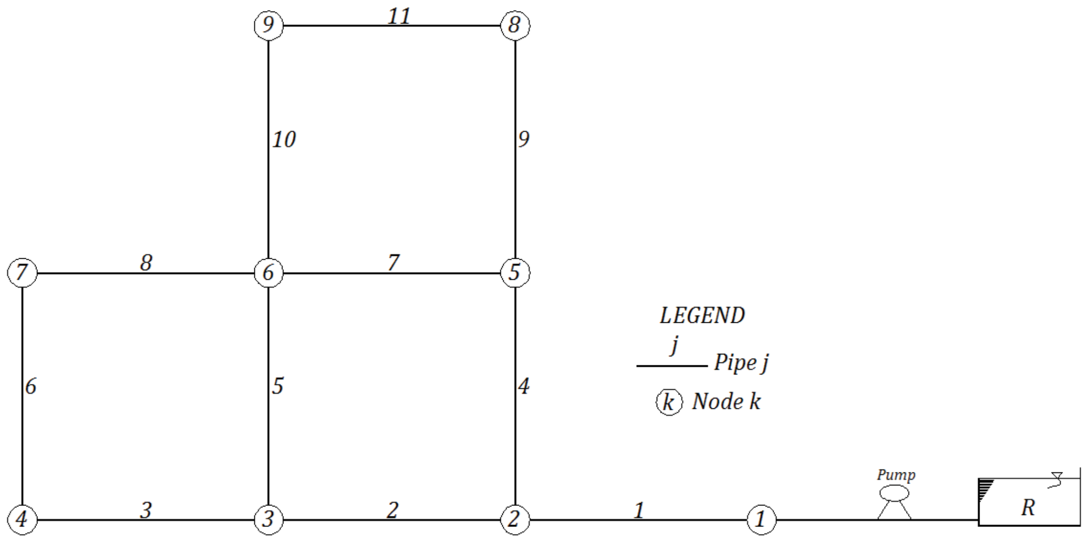

Fig. 4. Network 2 structure.

Table 10
Data for nodes for network 2

Node	1	2	3	4	5	6	7	8	9
Demand (m^3/h)	0	200	100	100	150	150	100	90	110
Elevation (m)	190	200	190	175	180	180	185	185	190

Table 11
Diameters available for network 2 and their costs

Diameter (mm)	152.4	203.2	254.0	304.8	355.6	406.4	457.2	508.0	609.6	762.0
Cost ($/m)	42.00	58.40	73.80	95.80	118.80	143.00	169.00	197.20	252.60	346.10

presented in Table 18. The ideal (most economical) pumping height was 47.872 m.

6. Conclusions

In the present work, an optimization model formulated as a bi-objective MDNLP problem was presented. An algorithm based on PSO was developed in order to solve the problem. The programmer's Toolkit of EPANET v2.1 was used to calculate the pressures, the velocities, and modify pump curve parameters. The model was implemented in a case study for WDN from the literature. The bi-objective function seeks to minimize pipe installation costs and operational costs (relative to energy for the pumping system).

The developed algorithm has proven to be reliable and efficient in the bi-objective optimization of a WDN, obtaining a lower global value than that presented in the literature, considering the studied case 1; for case study 2, the optimal value found is similar to that presented by other researchers. The objective function of the cost of implanting the pump can be calculated directly with the data of the pump selected in the bi-objective optimization process. The ideal elevation, H, relative to the optimal global value that was found, is very close to the global minimum.

Table 12
Pump curve coefficients and discharge pressure

Pump No.	a	b	c	Pumping pressure (m)
1	0	0	0	0
2	–72.0	–24.0	48.0	35.78
3	–81.0	–27.0	54.0	40.25
4	–125.1	10.9	55.1	48.48
5	–126.0	–9.0	65.0	52.78
6	–89.1	1.2	67.0	60.46
7	–103.7	–7.2	75.0	65.00
8	–129.6	0.0	79.0	69.00
9	–162.0	9.0	85.0	75.00
nPT = 10	–136.7	–3.5	91.5	80.00

Table 13
Pumping costs and conditions for network 2

Denomination	Value
Number of hours per day pumping (NHPD)	24
Number of days per year (NDPY)	365
Total of hours of pumping per year N_{op} = (NHPD) (NDPY)	8,760
Efficiency of the motor-pump unit (η)	0.6539
Expected period of service for the network in years (n)	20
Average pumping discharge (Q_T (m^3/s))	0.2778
Power electrical tariff (Ec (US$/kWh))	0.12
Annual rate of increase in the tariff of electrical power (e)	0
Annual discount rate (j)	12%
Present worth factor (PWF)	7.469
Update cost of water pumping per elevation meter (E_h (US$/m))	32,719.84

Table 14
Optimized results for different weight vectors for network 2

No.	Weight vector (w_{pipe}, w_{energy})	Total cost (US$)	Pipe cost (US$)	Energy cost (US$)	Pump No.
1	(0.1,0.9)	**4,095,257.79**	2,509,000.00	1,586,257.79	4
2	(0.2,0.8)	**4,095,257.79**	2,509,000.00	1,586,257.79	4
3	(0.3,0.7)	4,114,257.79	2,528,000.00	1,586,257.79	4
4	(0.4,0.6)	**4,095,257.79**	2,509,000.00	1,586,257.79	4
5	**(0.5,0.5)**	**4,095,257.79**	**2,509,000.00**	**1,586,257.79**	**4**
6	(0.6,0.4)	4,144,953.15	2,418,000.00	1,726,953.15	5
7	(0.7,0.3)	4,283,789.60	2,157,000.00	2,126,789.60	7
8	(0.8,0.2)	4,616,587.20	1,999,000.00	2,617,587.20	10
9	(0.9,0.1)	4,616,587.20	1,999,000.00	2,617,587.20	10

Table 15
Optimized costs (C_P, C_E, and C_{PM}) for network 2

Total ($)	Pipe ($)	Energy ($)	Pump ($)	Pump No.
5,505,785.79	2,509,000.00	1,586,257.79	1,410,528.00	4

Table 16
Optimized diameters and pressures acting on nodes

Pipe	1	2	3	4	5	6	7	8	9	10	11
Diameter (mm)	609.6	254	152.4	457.2	152.4	152.4	355.6	254	254	254	152.4
Node	1	2	3	4	5	6	7	8	9		
Pressure head (m)	48.48	35.11	35.01	34.17	49.48	42.83	31.55	39.79	30.57		

Table 17
Network 2 optimization for each pump

Pump No.	Total cost ($)	Pipe cost ($)	Energy cost ($)
1	–	Infeasible	0.00
2	–	Infeasible	1,170,715.84
3	–	Infeasible	1,316,973.52
4	4,095,257.79	2,509,000.00	1,586,257.79
5	4,097,453.10	2,370,500.00	1,726,953.10
6	4,206,241.47	2,228,000.00	1,978,241.47
7	4,314,289.54	2,187,500.00	2,126,789.54
8	4,407,668.89	2,150,000.00	2,257,668.89
9	4,532,487.93	2,078,500.00	2,453,987.93
10	4,616,587.12	1,999,000.00	2,617,587.12

Table 18
Network 2 results considering variable H as continuous

Total cost ($)	Pipe cost ($)	Energy cost ($)	H (m)
4,076,738.37	2,509,000.00	1,567,738.37	47.872

The selected weight vector affects the results of the global objective function, that is, the greater the weight of energy (w_{energy}), the lower the elevation (H) compared with the results from other weight vectors. The greater the elevation, the greater the pumping energy costs. Consequently, diameter values are lower, leading to a reduction in pipe installation costs.

Symbols

- Att_max — Number of attempts
- c_1, c_2 — Cognitive and social acceleration coefficients
- C_E — Objective function of the energy cost
- C_j — Hazen–Williams roughness coefficient for pipe j
- C_P — Objective function of pipe costs
- C_{PM} — Objective function of pump costs
- C_{TGG} — Value of the global objective function for particle G
- $C_{\text{TG}i}$ — Value of the global objective function for particle i
- C_{TGmax} — Maximum total cost of the WDN
- $C_{\text{TGP}i}$ — Value of the penalized global objective function for particle i
- D_j — Diameter of pipe j
- $D_{\max}$ — Maximum available diameter for the pipes
- $D_{\min}$ — Minimum available diameter for the pipes
- D_{SET} — Set of available diameters for the network
- $d(k)$ — Demand at node k
- e_1 — Annual interest or discount rate
- e_2 — Annual rate of increase in the unit cost of energy
- E_C — Electricity cost per kWh
- E_h — Updated cost of water pressurization per elevation meter
- E_P — Energy delivered by a pump
- F_{Pi} — Value of the penalized objective function for vector P_i
- G — Global best solution vector
- $G_{\max}$ — Vector where all the pipes have diameter $D_{\max}$ and elevation $H_{\max}$
- GOF — Global objective function
- h_f — Head loss
- H_i — Pumping head (m) for solution i
- $H_{\max}$ — Maximum pumping head
- $H_{\min}$ — Minimum pumping head
- N_{op} — Total of hours of pumping per year
- i — Particle i
- j — Pipe j
- k — Node k
- K — Number of demand nodes
- L_j — Length of pipe j
- LP — Linear programming
- M — Total number of pipes
- MOOP — Multi-objective optimization problem
- n — Years of service
- nP — Pump number
- nd — Total number of available diameters
- NLP — Nonlinear programming
- N — Total number of particles in the swarm
- NV — Number of hydraulic violations
- P_i — Vector best position of particle i
- $p_{i,j}$ — Component j of vector P_i
- pR — Solution vector for pressure heads in solution G
- $pr(k)$ — Pressure head on node k
- $pr_{\min}(k)$ — Minimum pressure head on node k
- PSO — Particle swarm optimization
- PWF — Present worth factor
- q — Flow rate
- Q_T — Total flow rate of the system (m^3/s)
- r, r_1, r_2 — Random numbers from uniform distribution [0,1]
- t — Iteration number t
- $t_{\max}$ — Maximum number of iterations
- $V_{H\,max}$ — Maximum velocity of the particle for variable H
- $V_{H\,\min}$ — Minimum velocity of the particle for variable H
- v_{Hi} — Velocity of variable H of particle i
- V_i — Velocity vector of particle i
- $v_{i,j}$ — Water velocity in pipe j belonging to the particle i
- $v_{\max}$, $v_{\min}$ — Maximum and minimum acceptable water velocities
- $v_{Pi,j}$ — Velocity component j of particle i
- V_{PSOmax} — Maximum velocity of the particle
- vR — Solution vector for water velocities in solution G
- W — Weight vector for the MOOP
- WDN — Water distribution network
- W_{penal} — Penalty value
- w_{PSO} — Inertia weight
- X_i — Current position vector of particle i
- $x_{i,j}$ — Diameter of pipe j belonging to particle i

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