

## Numerical analysis of vibration of suspended pipelines in internal and external flow fields

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Received 15 August 2020; Accepted 23 November 2020

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### ABSTRACT

In this paper, firstly, the suspended oil pipelines were numerically simulated under the impact of water flow based on the fluid-structure interaction and vibration theory of acoustic body after taking full account of the influence of internal and external fluid on the natural vibration frequency of the pipeline. In the flow field part, the finite element volume method was used to solve the arbitrary Lagrange Euler N–S equation, and in the structural domain, the finite element method discrete equation was used to transfer the data of the interaction surface. Then, the vibration of suspended pipelines in dry and wet mode was calculated and compared on this basis, and finally, the factors affecting the natural frequency of submarine pipelines and the response frequency of vortex-induced vibration were analyzed. The results showed that the natural frequency was greatly affected by the radius and the length of suspension; the natural frequency in the dry mode was about twice of that in the wet mode; the natural frequency in the wet mode was closer to the frequency of pipeline falling off, so the influence of internal and external fluid on the natural frequency of pipeline must be considered to avoid the resonance effect.

*Keywords:* Vortex-induced vibration; Dry mode; Wet mode; Natural vibration frequency

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### 1. Introduction

After the buried long-distance pipeline is impacted by the water flow, it will emerge from the ground due to the loss of the foundation earth layer. Then, vortex discharge may take place on the pipeline in suspension under the impact of water flow. At this time, if the discharge frequency is close to the natural vibration frequency of the pipeline, resonance effect will occur, causing fatigue damage to the pipeline, which is commonly known as the vortex-induced resonance. Vortex-induced resonance is one of the main causes of fatigue failure of underwater suspended pipelines. Sometimes, even if the pipeline stress and strain are still in the safe range of check, it will still be damaged due to the vortex-induced vibration. Therefore, it is necessary

to study its response characteristics to avoid the occurrence of resonance effect.

In order to analyze, the vibration of the suspended pipelines, first of all, the natural vibration frequency of them should be calculated accurately. A lot of research has been made on the natural vibration frequency of the suspended pipelines. In the early 1990s, Pantazopoulos gave a comprehensive introduction to the theory and development of vortex-induced vibration of pipelines. He presented the vortex-induced vibration model and relevant tests proposed by scholars in the past and summarized the methods of determining parameters such as lift coefficient and frequency bandwidth of vortex shedding frequency by combining experience and experiments [1]. Yao [2] analyzed the mechanical characteristics of marine risers under the action of the flow field, and obtained the natural frequency and

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formation. Eyankware [3] calculated the dynamic response of the internal flow to the pipelines and found that the amplitude of the lateral vibration increased obviously. In China, there is less research on vortex-induced vibration of suspended pipelines by numerical simulation method, most of which focuses on the simulation of pipeline vibration in one-way flow.

In this paper, a numerical calculation model of underwater suspended pipelines was established based on the plane wave equation and acoustic vibration theory. Then, considering the fluid-structure interaction between the internal and external fluid domain and the pipelines, the vibration of the pipelines in the dry and wet modes was compared, and the relevant parameters and change rules affecting the natural vibration frequency of the pipelines were analyzed.

## 2. Principle of vortex-induced resonance in submarine pipelines

The excitation frequency of the submarine pipelines is the vortex shedding frequency when the water flow impacts the suspended pipeline [4]. When the vortex sheds from the surrounding of the pipelines, the hydrodynamic loads varying with the time will be generated along the downstream and transverse directions of the pipelines, forcing them to vibrate in the transverse, and vertical directions.

The shedding frequency of vortex-induced vibration is mainly determined by the Strouhal number  $S_f$ . The relationship between the shedding frequency  $f_i$  and Strouhal number  $S_f$  is  $f_i = S_f U/D$ , where  $U$ -flow velocity and  $D$ -pipeline diameter. In theory, the vortex-induced resonance is easy to occur when the frequency of shedding is close to that of natural vibration. But in reality, resonance often occurs in a confined range. In order to better study the range in which vortex-induced resonance takes place, the reduced velocity  $U_i$  is used to express the relationship between shedding frequency and natural vibration frequency, that is,  $U_i = \frac{1}{S_f} \frac{f_i}{f_n}$ . Generally,

the frequency locking of pipelines occurs when the reduced velocity is 4–8. When the Strouhal number is a fixed value, the value of  $S_f$  is only related to  $f_i/f_n$ .

When  $S_f = 0.23$ , the  $f_i$  of pipelines with the diameter of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8 m at the flow velocity of 1.5 m/s is calculated, which ranges from 0.43 to 3.45 Hz. For the pipelines with a diameter of 0.3 m, according to the range of vortex-induced resonance, the corresponding natural vibration frequency of the pipelines range from 0.625 to 1.25 Hz.

## 3. Establishment and calculation of the mathematical model

Due to the different densities, the natural vibration frequencies of the structure in water and air vary greatly. The natural frequency of suspended pipeline without considering the fluid action is generally considered as the frequency of structure in a vacuum, which is known as the dry mode. However, in the actual environment, the pipeline is in the water with a relatively density, which has an impact on the structural frequency. In the previous

studies, only the natural frequency of the pipeline in the air is considered, rather than the effect of the water flow outside the pipeline and the liquid transported inside the pipeline, which cannot be ignored.

### 3.1. Fluid control equation

Vortex induced vibration is a typical problem of fluid-structure interaction. In this paper, the flow outside and inside the pipeline were considered as acoustic bodies, and then the natural frequency of the pipeline was calculated. For the problem of acoustic fluid-structure interaction, the N–S equation and the continuity equation must be considered together.

Starting from the law of conservation of mass, its continuity equation in the global Cartesian coordinates is expressed as [5]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0 \quad (1)$$

The N–S equations describing the incompressible viscous Newtonian fluid can be expressed as [6]:

$$\begin{aligned} \frac{\partial \rho v}{\partial t} + \frac{\partial(\rho v v)}{\partial x} + \frac{\partial(\rho v v_x)}{\partial y} + \frac{\partial(\rho v v_z)}{\partial z} &= \rho g_x - \frac{\partial P}{\partial x} \\ + R_x + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial v_x}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial v_x}{\partial z} \right) &+ T_x \\ \frac{\partial \rho v_y}{\partial t} + \frac{\partial(\rho v_x v_y)}{\partial x} + \frac{\partial(\rho v_y v_y)}{\partial y} + \frac{\partial(\rho v_z v_y)}{\partial z} &= \rho g_y - \frac{\partial P}{\partial y} \\ + R_y + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial v_y}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial v_y}{\partial z} \right) &+ T_y \\ \frac{\partial \rho v_z}{\partial t} + \frac{\partial(\rho v_x v_z)}{\partial x} + \frac{\partial(\rho v_y v_z)}{\partial y} + \frac{\partial(\rho v_z v_z)}{\partial z} &= \rho g_z - \frac{\partial P}{\partial z} \\ + R_z + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial v_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial v_z}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial v_z}{\partial z} \right) &+ T_z \end{aligned} \quad (2)$$

The acoustic equation can be obtained by simplifying the continuity equation and momentum equation:

$$\begin{aligned} \nabla \cdot \left( \frac{1}{\rho_0} \nabla p \right) - \frac{1}{\rho_0 c^2} \frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left[ \frac{4\mu}{3\rho_0} \nabla \left( \frac{1}{\rho_0 c^2} \frac{\partial p}{\partial t} \right) \right] &= \\ - \frac{\partial}{\partial t} \left( \frac{Q}{\rho_0} \right) + \nabla \cdot \left[ \frac{4\mu}{3\rho_0} \nabla \left( \frac{Q}{\rho_0} \right) \right] & \end{aligned} \quad (3)$$

where  $c$  is the sound velocity in fluid medium;  $\rho_0$  is the average fluid density;  $K$  is the bulk modulus of liquid,  $m^3$ ;  $p$  is the sound pressure;  $Q$  is the mass source.

Starting from the equation of conservation of momentum, the normal velocity on the boundary of sound field is:

$$\frac{\partial v_{n,F}}{\partial T} = \hat{n} \cdot \frac{\partial \bar{v}}{\partial t} = - \left( \frac{1}{\rho_0} + \frac{4\mu}{3\rho_0^2 c^2} \frac{\partial}{\partial t} \right) \hat{n} \cdot \nabla p + \frac{4\mu}{3\rho_0^2} \hat{n} \cdot \nabla Q \quad (4)$$

Positive acceleration of body particles:

$$\frac{\partial v_{n,F}}{\partial t} = \hat{n} \cdot \frac{\partial^2 \bar{u}_F}{\partial t^2} \quad (5)$$

To integrate the wave equation in volume domain:

$$\begin{aligned} & \iiint_{\Omega_f} \frac{1}{\rho_0 c^2} w \frac{\partial^2 p}{\partial t^2} dv + \iiint_{\Omega_f} \nabla w \cdot \frac{4\mu}{3\rho_0^2 c^2} \nabla \frac{\partial p}{\partial t} dv + \iiint_{\Omega_f} \nabla w \cdot \left( \frac{1}{\rho_0} \nabla p \right) dv \\ & + \iint_{\Gamma} w \hat{n} \frac{\partial^2 \bar{u}_F}{\partial t^2} ds = \iiint_{\Omega_f} w \frac{1}{\rho_0} \frac{\partial Q}{\partial t} + \iiint_{\Omega_f} \nabla w \frac{4\mu}{3\rho_0^2} \nabla Q dv \end{aligned} \quad (6)$$

Fluid discretization, approximate shape function of finite element with spatial variation of pressure and displacement components in the element:

$$P = \{N\}^T \{P_e\} \quad (7)$$

$$u = \{N'\}^T \{P_e\} \quad (8)$$

where  $\{N\}$  is the element shape function of pressure;  $\{N'\}$  is the element shape function of displacement;  $\{p_e\}$  is the node pressure vector.

The virtual variation of the second derivative of the variable can be expressed as follows:

$$\frac{\partial^2 P}{\partial t^2} \{u\} = \{N\}^T \{\ddot{P}_e\} \quad (9)$$

$$\frac{\partial^2}{\partial t^2} \{u\} = \{N'\}^T \{\ddot{u}_e\} \quad (10)$$

$$\delta P = \{N\}^T \{\delta P_e\} \quad (11)$$

Substituting Eqs. (7)–(10) into (6) to get:

$$\begin{aligned} & \iiint_{\Omega_f} \frac{1}{\rho_0 c^2} \{N\} \{N\}^T dv \{\ddot{P}_e\} + \iiint_{\Omega_f} \frac{4\mu}{3\rho_0^2 c^2} [\nabla N]^T [\nabla N] dv \{\dot{P}_e\} + \\ & \iiint_{\Omega_f} \frac{1}{\rho_0} [\nabla N]^T [\nabla N] dv \{P_e\} + \iint_{\Gamma} \{N\} \{n\}^T \{N'\}^T ds \{\ddot{u}_{e,F}\} = \\ & \iiint_{\Omega_f} \frac{1}{\rho_0} \{N\} \{N\}^T dv \{q\} + \iiint_{\Omega_f} \frac{4\mu}{3\rho_0^2 c^2} [\nabla N]^T [\nabla N] dv \{q\} \end{aligned} \quad (12)$$

where  $\{n\}$  is the normal vector outside the fluid boundary;  $\{q\}$  is the node mass source vector;  $\{p_e\}$  is the node pressure vector.

The finite element equation matrix of sound field in fluid can be expressed as follows:

$$[M_F] \{\ddot{P}_e\} + [C_F] \{\dot{P}_e\} + [K_F] \{P_e\} + \bar{\rho}_0 [R]^T \{\ddot{u}_{e,F}\} = \{f_F\} \quad (13)$$

where  $[M_F]$  is the acoustic fluid mass matrix;  $[C_F]$  is the acoustic fluid damping matrix;  $[K_F]$  is the acoustic fluid

stiffness matrix;  $[R]^T$  is the acoustic fluid boundary matrix;  $\{f_F\}$  is the acoustic fluid load vector;  $\bar{\rho}_0$  is the acoustic fluid mass density constant.

### 3.2. Structural control equation

The vortex shedding behind the pipeline under the impact of water flow will cause the structure movement of the pipeline. For elastic pipelines, the finite element method is usually used for discretization. The discrete form of structural vibration equation is [7]:

$$[M_F] \{\ddot{u}(t)\} + [C] \{\dot{u}(t)\} + \{F^i(t)\} = \{F^a(t)\} \quad (14)$$

where  $[M_F]$  is the mass matrix;  $[C]$  is the damping matrix;  $\{\ddot{u}\}$  is the acceleration of pipeline movement;  $\{\dot{u}\}$  is the speed of pipeline movement;  $\{u\}$  is the displacement of pipeline movement.

In order to fully describe the problem of fluid-structure interaction, the fluid pressure load acting on the interface is added to the solid structure control equation.

$$\bar{n} \cdot \bar{u}_s - \bar{n} \cdot \bar{u}_F = 0 \quad (15)$$

$$\bar{\delta}(\bar{u}_s) \bar{n} + \rho \bar{n} = 0 \quad (16)$$

Therefore, the structural equation is changed to:

$$([M_F] + [S_F]) \ddot{P} + [C_F] \dot{P} + [K_F] P + \rho_0 [R]^T \ddot{u} = [f_F] \quad (17)$$

The fluid pressure load vector on the interface  $s$  is obtained by integrating the pressure on the surface area:

$$\{f_e^{pr}\} = \iint_{\Gamma_i} \{N'\} \rho \bar{n} ds \quad (18)$$

$\{n'\}$  is the shape function for discrete displacement components  $U$ ,  $V$ , and  $W$ .

Substituting the approximate finite element function of pressure given in Eq. (10) into Eq. (18) to get:

$$\{f_e^{pr}\} = \iint_{\Gamma_i} \{N'\} \{N\}^T \{n\} ds \{P_e\} \quad (19)$$

By comparing the integral in Eq. (19) with the matrix definition of  $[R]^T$  in Eq. (13), the following results are obtained:

$$\{f_e^{pr}\} = [R] \{P_e\} \quad (20)$$

The structural dynamic equation can be obtained by substituting into Eq. (20):

$$[M_s] \{\ddot{u}_e\} + [C_s] \{\dot{u}_e\} + [K_s] \{u_e\} - [R] \{P_e\} = \{f_s\} \quad (21)$$

For the acoustic-structural system, the complete finite element discrete equation of the acoustic-structure problem is described in Eqs. (13) and (21).

### 3.3. Model calculation

In this paper, the ANSYS Workbench ACT was used to analyze the mode of the suspended pipelines [8–10]. The pipeline whose medium was oil and fluid outside the pipeline was water was used for the finite element analysis. In the wet mode analysis, water and oil were considered as acoustic bodies. The sound velocity in the internal and external fields of the pipeline was 1,500 and 1,000 m/s, respectively. The internal and external walls of the pipeline were set as two fluid-structure interfaces (FSIs).

Firstly, the pipeline with inner diameter of 0.305 m and wall thickness of 0.02 m was modeled. The pipeline was made of steel with Poisson's ratio of 0.3, the elastic modulus of  $2.06 \times 10^{11}$  Pa, and density of  $7,850 \text{ kg/m}^3$ . The finite element model is shown in Fig. 1.

The dry and wet modes of the pipeline with a suspension span of 15 m and an external flow field of  $3 \text{ m} \times 3 \text{ m} \times 15 \text{ m}$  were calculated [11,12]. As shown in Table 1, it was found after comparing with the theoretical calculation frequency that the errors were all within 10%, so the numerical model established in this paper can meet the engineering accuracy requirements, and it was also found that the difference between the natural frequency of the wet mode and the dry mode was 35%.

The first 15 natural frequencies are shown in Figs. 2 and 3.

By analyzing the simulation data in Figs. 2–4, it is found that the natural vibration frequency in the wet mode is smaller under the same order. When the interaction of fluid and pipeline is not considered, the natural frequency in dry mode is about twice of that in wet mode considering fluid coupling; the natural frequency in wet mode is closer to that of vortex-induced vibration, and resonance effect is more likely to occur at this time. Therefore, the interaction between internal and external fluids and pipeline must be considered to prevent resonance [13–15].

## 4. Analysis of the factors influencing the vortex-induced vibration failure of submarine suspended pipelines

### 4.1. Fluid-structure interaction analysis of pipes with different span lengths

For the pipeline with diameter of 0.3 m and suspension span length of 15, 20, 30, 40, 50, and 100 m respectively,

the finite element model for fluid-structure interaction is established, and the relationship between natural vibration frequency and suspension span length is shown in Fig. 5.

The higher the order is, the higher the natural vibration frequency of the pipeline is when the suspension length of the pipeline is the same. The larger the span length is, the smaller the natural frequency is under the same order. When the span length is more than 40 m, it is found that the first three steps of the natural frequency coincide with the shedding frequency by comparing with the calculation range of the shedding frequency. At this time, the possibility of resonance is increased, and measures should be taken to avoid resonance.

### 4.2. Analysis on fluid-structure interaction of pipelines with different diameters

The analysis on fluid-structure interaction of the pipeline is carried out when the suspension span length of the pipeline is 40 m and the diameters are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8 m, respectively, and the relationship between the natural vibration frequency and the pipeline diameter is shown in Fig. 6.

It is observed from the curve above that the natural vibration frequency increases with the increase of order when the pipeline diameter is the same, and that the natural frequency increases with the increase of the pipe radius in the same order when the diameter of the pipeline is different. For the pipelines with diameters of 0.1, 0.2, and 0.3 m, the first three natural frequencies range from 0.288 to 1.96 Hz, and for the pipelines with diameters of 0.4–0.8 m, the first natural frequencies range from 0.839 to 1.266 Hz. Then their data are compared with the range of shedding

Table 1  
Comparing with the theoretical calculation frequency

Theoretical dry mode	9.56
Numerical dry mode	8.845
Error	7.5%
Theoretical wet mode	5.69
Numerical wet mode	5.18
Error	8.9%

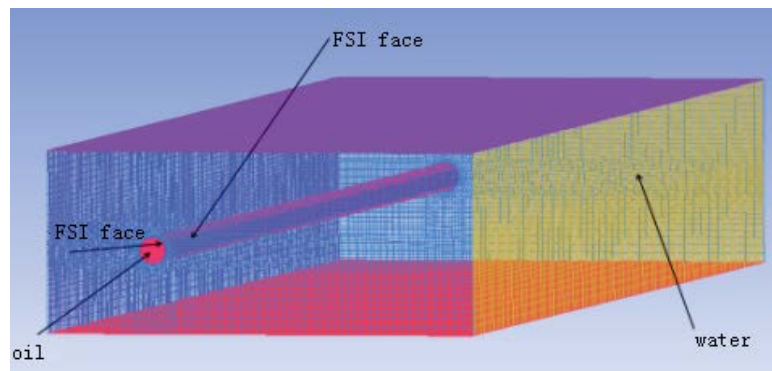


Fig. 1. 3D model.

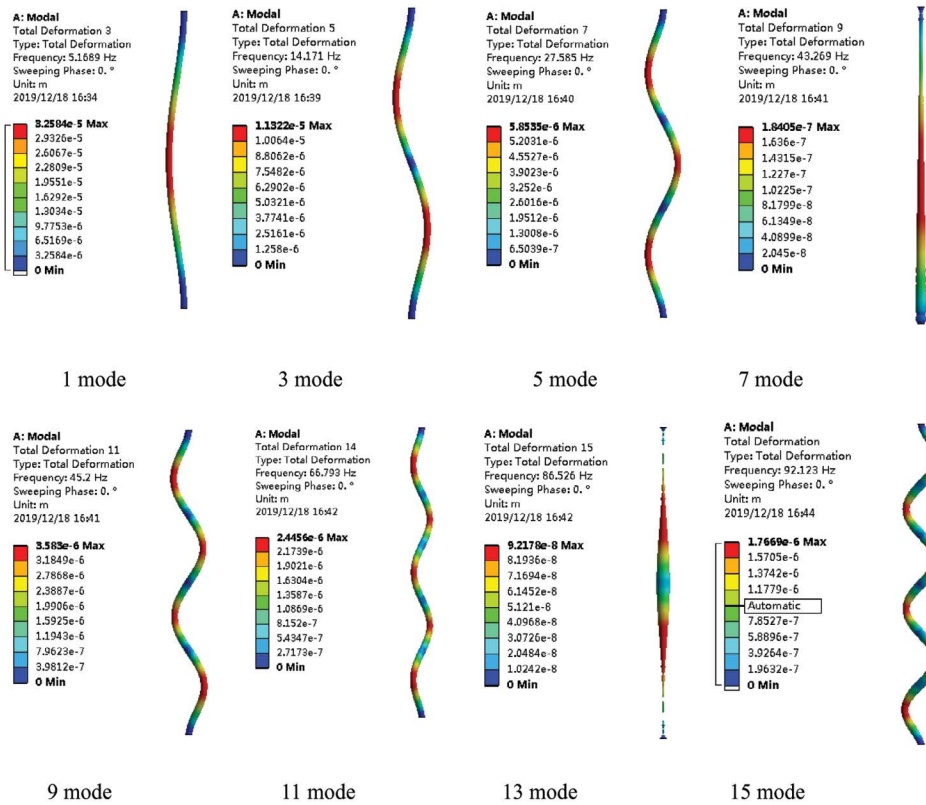


Fig. 2. Dry mode formation.

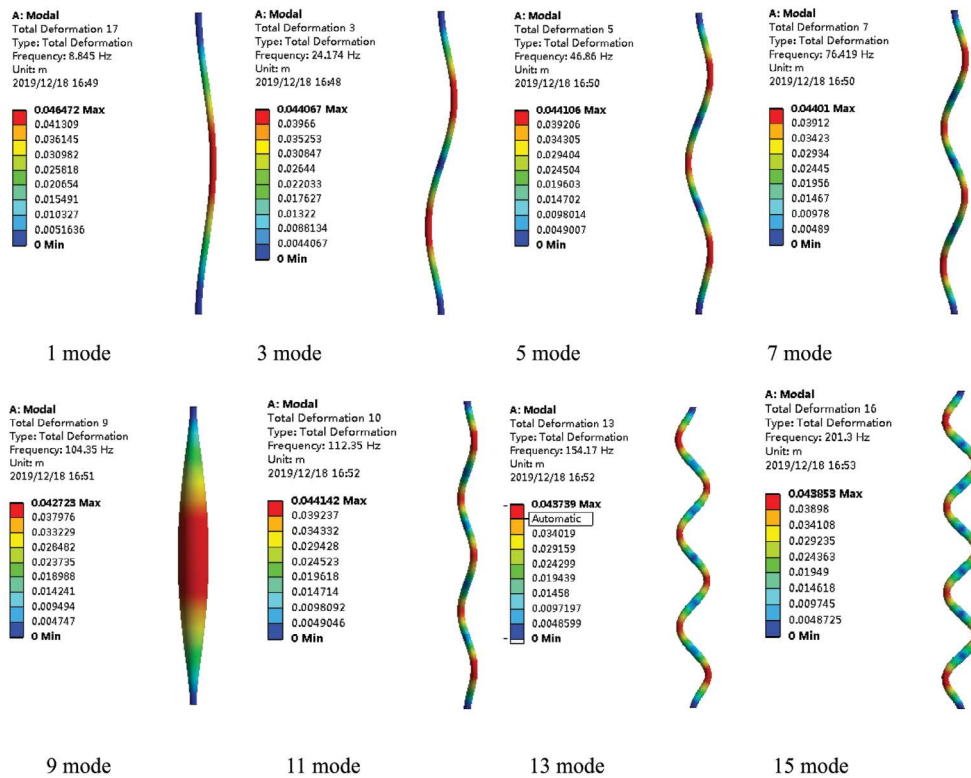


Fig. 3. Wet mode formation.

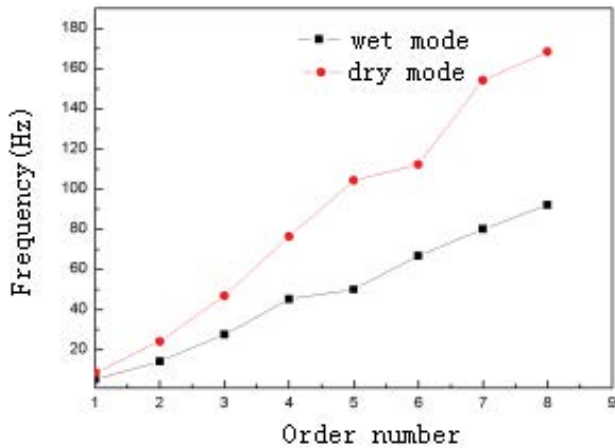


Fig. 4. Dry and wet natural paired frequencies of 15 m pipeline.

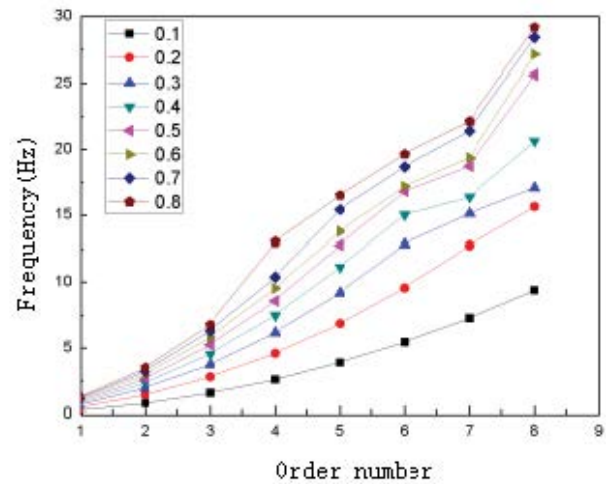


Fig. 6. Variation curve of natural vibration frequency with pipe diameter.

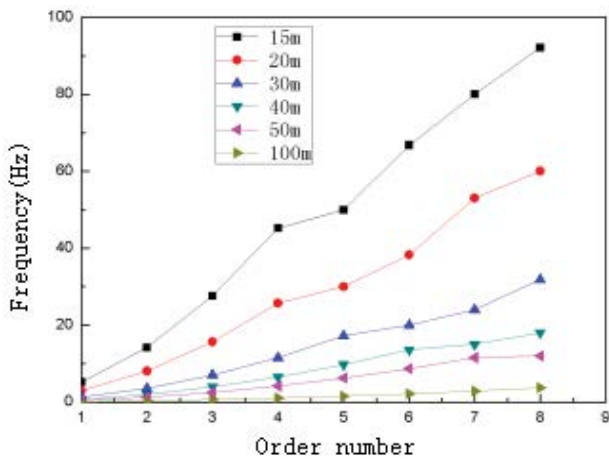


Fig. 5. Variation curve of natural frequency with span length.

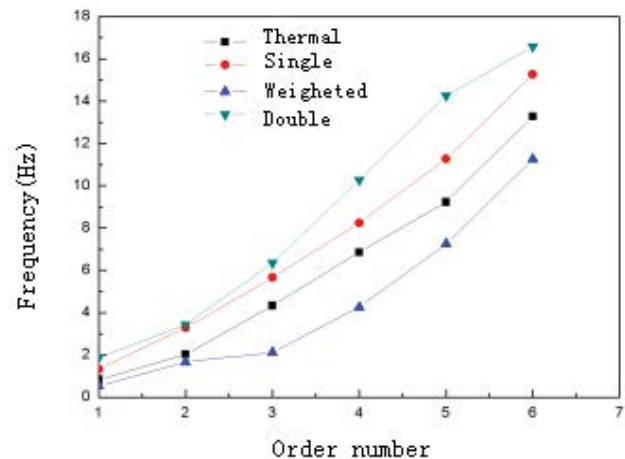


Fig. 7. Variation curve of natural vibration frequency with pipeline type.

frequency of corresponding pipelines, and they finally turn out to be sections with a high risk of resonance. At this time, the occurrence of low-order resonance should be avoided, and the resonance risk can also be reduced by properly increasing the pipeline radius.

4.3. Analysis on fluid-structure interaction of suspended pipelines with different types

The main types of submarine pipelines are single-wall pipeline, double-wall pipeline, weighted pipeline, and heat preservation pipeline. The finite element analysis on fluid-structure interaction was carried out for four kinds of pipelines with a suspension span of 40 m, and the natural vibration frequency is shown in Fig. 7.

It is observed from the data analysis in Fig. 7 that the natural vibration frequency of single-wall pipeline is smaller than that of double-wall pipeline, and the first two frequencies are close to the range of vortex-induced vibration, which is easy to cause resonance. In the same order, the weighted pipelines have the smallest natural vibration frequencies, and the most sections coinciding with the

frequency range of pipelines with vortex-induced resonance. Consequently, attentions should be paid to the prevention of vortex-induced vibration in weighted pipelines.

5. Conclusions

In this paper, a fluid-structure interaction model considering the external water flow and the internal oil body was firstly established by analyzing the vortex-induced vibration of the suspended pipelines impacted by water flow, and then, the influence of the span length, diameter, and type of the pipeline on the resonance of the submarine pipelines were analyzed by comparing the pipelines in the dry and wet modes, and finally, the following conclusions were drawn:

- The natural vibration frequencies of pipelines in the wet mode considering the fluid-structure interaction vary greatly from those in the dry mode without

considering the interaction, which is usually twice, so the influence of internal and external fluids on the vibration of the pipelines must be considered.

- The natural vibration frequencies of pipelines are inversely proportional to the length of the suspension span of the pipeline. The longer the suspension span is, the wider the close range between the natural vibration frequency and the excitation frequency is. For the single-wall pipelines with a diameter of 0.3 m, when the suspension span length is greater than 40 m, there is a coincidence between the natural vibration frequency and the excitation frequency, and measures should be taken to avoid resonance. Compared with the double-wall pipelines, resonance is more likely to occur in the single-wall pipeline, and the same is true for the weighted pipelines. To sum up, measures should be taken to avoid resonance in view of the above problems.

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