Natural convection from a vertical plate embedded in a stratified medium with uniform heat source

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ABSTRACT

Natural convection flow from an isothermal vertical plate with uniform heat source embedded in a stratified medium has been discussed in this paper. The resulting momentum and energy equations of boundary layer approximation are then made non-similar by introducing the usual non-similarity transformations. Numerical solutions of these equations are obtained by an implicit finite difference method for a wide range of the stratification parameter, X. The solutions are also obtained for different values of pertinent parameters, namely, the Prandtl number, Pr and the heat generation or absorption parameter, λ and are expressed in terms of the local skin-friction and local heat transfer, which are shown as graphical form. Effect of heat generation or absorption on the streamlines and isotherms are also shown graphically for different values of λ.

Keywords: Natural convection; Stratified media; Heat source; Heat Transfer; Vertical plate; Boundary layer flow.

1. Introduction

Every day various free convection processes occur in environments with temperature stratification. Yang [1] first presented a general approach for obtaining similarity solutions for a class of problems with a non-isothermal vertical wall surrounded by an isothermal atmosphere. For laminar free convection along a vertical plate, Cheesewright [2] obtained similarity solutions dealing with various types of non-uniform ambient temperature distributions by using the technique developed by Yang [1]. Fujii et al. [3] presented both analytical and experimental results for a temperature stratification in which the ambient temperature distribution varies with $x^{0.7}$.

Numerical investigation on the natural convection flow along a vertical porous surface placed in thermally stratified media has recently been investigated by Saha [4] and natural convection flow from a vertical flat plate in a stratified medium with effect of viscous dissipation and double diffusion has been studied by Saha and Hossain [5]. More recently, effect of viscous dissipation on the natural convection boundary layer flow along a porous surface considering a temperature stratification in which the ambient temperature distribution varies with x by Hossain et al. [6]. Later, Saha et al. [7] studied the conjugate effect of thermal and mass diffusion on the problem proposed in Ref. [4].

The study of radiation or heat generation/absorption effects in moving fluids is important in view of several physical problems such as those dealing with

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chemical reactions and those concerned with dissociating fluids [8,9]. In fact, the literature is replete with examples dealing with the heat transfer in laminar flow of viscous fluids. Sparrow and Cess [10] investigated the problem of the steady two-dimensional flow and heat transfer of the stagnation point flow taking into account the temperature-dependent heat generation (absorption). Topper [11] analyzed the piston flow in pipes with circular cross-section when the rate of heat generation depends linearly on the local temperature. The author presented an analytical solution both for the case where the wall is isothermal and for the case where the exterior surroundings are isothermal and the heat transfer coefficient between the tube wall and the surroundings is constant. The author claimed that this analysis should be helpful for estimating local temperature and also for predicting the transient response to changes in one of the independent operating variables. Foraboschi and Federico [12] have assumed the volumetric rate of heat generation to be in Eq. (1):

$$Q = \begin{cases} Q_0 \left(T - T_0 \right), & T \geq T_0 \\ 0, & T < T_0 \end{cases} \qquad (1)$$

in their study of the steady state temperature profiles for linear parabolic and piston flow in circular tubes. The above relation as explained by Foraboschi and Federico [12], is valid as an approximation of the state of some exothermic process and having T_0 as the ambient fluid temperature. With the inlet temperatures less than T_0, they used $Q = Q_0(T - T_0)$ and studied its effects on the heat transfer in laminar flow of non-Newtonian heat generating fluids. On the other hand, Moalem [13] studied the effect of temperature-dependent heat sources $[Q \propto 1/(a + bT)]$, where a and b are constants, occurring in electrical heating, on the steady-state heat transfer within a porous medium.

In the present analysis, attention has been given to a study of the natural convection flow of a viscous incompressible fluid from a vertical flat plate embedded in a stratified medium with a distributed heat source which is dependent on the local temperature. Here the ambient temperature is assumed to be a linear function of x that measures the distance from the leading edge in the direction parallel to the surface of the plate. Solutions of the dimensionless boundary layer equations that govern the flow are obtained by introducing the primitive variable formulation as well as the stream-function formulation (SFF) for a wide range of values of the local stratification parameter, X. The effect of varying the heat sources or sinks on the skin friction and heat transfer rates are also shown graphically for a fluids having Prandtl number, Pr equal to 0.7 and 7.0. Finally the effects of varying heat source or sink are also shown in terms of the streamlines and the isotherms patterns.

2. Mathematical formalisms

Consider a steady two-dimensional viscous incompressible fluid on a vertical plate embedded in a stratified medium with uniform heat source. Let $\bar{x}$ be the distance along the surface from the leading edge and $\bar{y}$ the normal distance from the surface. The flow configuration and the co-ordinate system are shown in Fig. 1. The governing equations within the boundary layer subject to the Boussinesq approximation can be written as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (2)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta_T \left(T - T_{\infty,x} \right) \qquad (3)$$

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + Q \left(T - T_{\infty,x} \right) \qquad (4)$$

where $\bar{u}$ and $\bar{v}$ are the $\bar{x}$- and $\bar{y}$-components of the velocity field, respectively, g is the acceleration due to gravity, β_T is the volumetric expansion coefficients for temperature, α is the effective thermal diffusivity. Further, T and $T_{\infty,x}$ are the temperature of the fluid and the ambient temperature, respectively and Q is heat generation (>0) or absorption (<0) coefficient.

The boundary conditions for this problem can be written as:

$$\bar{u} = \bar{v} = 0, T = T_w \quad \text{at} \quad \bar{y} = 0$$
$$\bar{u} = 0, T = T_{\infty,x} = T_0 + B \left(\frac{\bar{x}}{L} \right) \quad \text{as} \quad \bar{y} \to \infty \qquad (5)$$
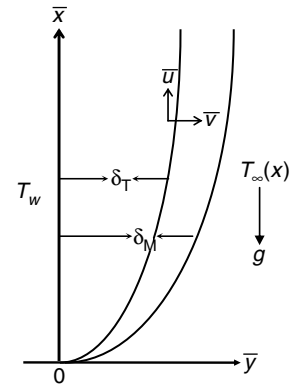

Fig. 1. The flow configuration and the co-ordinate system.

where the wall temperature, T_w is assumed to be constant and the ambient temperature T_∞ vary along the plate as given in Eq. (4), B is constant and L and T_0 being the reference length and reference temperature, respectively:

$$u = \frac{L\bar{u}}{\nu}\,Gr_L^{1/2}, \quad v = \frac{L\bar{v}}{\nu}\,Gr_L^{1/2}, \quad \theta = \frac{T - T_{\infty,x}}{T_w - T_0},$$
$$x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L}\,Gr_L^{-1/4} \tag{6}$$

where the Grashof number for diffusion, Gr_L is defined as:

$$Gr_L = \frac{g\beta_T\,(T_w - T_0)\,L^3}{\nu^2} \tag{7}$$

Using the transformations from Eq. (6), we obtain:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \tag{9}$$

$$u\,\frac{\partial \theta}{\partial x} + v\,\frac{\partial \theta}{\partial y} + Su = \frac{1}{Pr}\,\frac{\partial^2 \theta}{\partial y^2} + \lambda S^{1/2}\theta \tag{10}$$

where the heat generation parameter, λ, the stratification parameter, S and the Prandtl number, Pr are defined respectively by:

$$\lambda = \frac{QL^2}{\nu S^{1/2}}\,Gr_L^{-1/2}, \quad S = \frac{B}{T_w - T_0} \quad \text{and} \quad Pr = \frac{\nu}{\alpha} \tag{11}$$

The Eqs. (8)–(10) satisfy the boundary conditions:

$$u = v = 0, \theta = 1 - Sx, \quad \text{at} \quad y = 0$$
$$u = 0, \theta = 0, \quad \text{as} \quad y \to \infty \tag{12}$$

The important physical quantities are the wall shear stress factor, τ_w, and the heat transfer rate, $q(x)$ which is defined as:

$$\tau_w = \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad q(x) = -\kappa\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \tag{13}$$

The dimensionless shear stress factor or the skin-friction, C_f, can be expressed as follows:

$$C_f = \frac{2\tau_w}{\rho U_\infty^2} \tag{14}$$

where U_∞ is the free stream velocity. Using the quantity (14) in (13) and the transformation (6), we can calculate the local skin-friction in terms of the non-dimensional shearing stress, C_f, given as:

$$\frac{1}{2}\,Gr_L^{1/4}C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{15}$$

We may now define a non-dimensional coefficient of heat transfer in terms of Nusselt number Nu, which is:

$$Nu = \frac{qL}{\kappa\,\Delta T_0} \tag{16}$$

Substituting the transformation (6) in (13) with boundary condition (12), we obtain the rate of heat transfer, in terms of the non-dimensional Nusselt number, given as:

$$\frac{Nu}{Gr_L^{1/4}} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{17}$$

The present problem is proposed to be investigated numerically using the primitive-variable formulation (PVF) as well as the SFF, the details of which are discussed in Sections 2.1 and 2.2.

2.1. Primitive-variable formulation

To find the solutions of the set of Eqs. (8)–(10) together with the boundary conditions (12), we introduce the flowing free variable transformations to reduce the equations into parabolic form:

$$u = x^{1/2}U(Y, X), \quad v = x^{-1/4}V(Y, X),$$
$$\theta = \Theta(Y, X)\ y = x^{-1/4}y, \quad X = Sx \tag{18}$$

Thus from Eqs. (8)–(10), we get:

$$\frac{\partial V}{\partial Y} = -\left[\frac{1}{2}U + X\,\frac{\partial U}{\partial X} - \frac{1}{4}Y\,\frac{\partial U}{\partial Y}\right] \tag{19}$$

$$\frac{1}{2}U^2 + XU\,\frac{\partial U}{\partial X} + \left(V - \frac{1}{4}YU\right)\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \Theta \tag{20}$$

$$XU\frac{\partial\Theta}{\partial X}+\left(V-\frac{1}{4}YU\right)\frac{\partial\Theta}{\partial Y}+XU$$
$$=\frac{1}{\mathrm{Pr}}\frac{\partial^2\Theta}{\partial Y^2}+\lambda X^{1/2}\Theta \tag{21}$$

and the boundary conditions become:

$$U=V=0,\quad \Theta=1-X,\quad\text{at}\quad Y=0$$
$$U=0,\quad \Theta=0\quad\text{as}\quad Y=\infty \tag{22}$$

Now Eqs. (19)–(21) subject to the boundary conditions (22) are discretised by a simple numerical scheme, in which we use central-differencing for diffusion terms and the forward-differencing for the advection terms and thus obtain:

$$V_{i,j}=V_{i+1,j}+\frac{1}{8}\left(Y_j+Y_{j-1}\right)\left(U_{i+1,j}+U_{i-1,j}\right)$$
$$-\frac{\Delta Y}{2}\left(U_{i,j}+U_{i,j-1}\right)$$
$$-\frac{\Delta Y}{\Delta X}X_i\left(U_{i,j}-U_{i,j-1}+U_{i-1,j}-U_{i-1,j-1}\right) \tag{23}$$

$$\left[1+\left(V_{i,j}-\frac{Y_j\mu_{i,j}}{4}\right)\frac{\Delta Y}{2}\right]U_{i-1,j}-\left[\begin{array}{c}2+(\Delta Y)^2\\\left(\frac{X_i}{\Delta X}+\frac{1}{2}\right)\end{array}U_{i,j}\right]U_{i,j}$$
$$+\left[1-\left(V_{i,j}-\frac{Y_j\mu_{i,j}}{4}\right)\frac{\Delta Y}{2}\right]U_{i+1,j}$$
$$=-(\Delta Y)^2\left[\frac{X_i}{\Delta X}U_{i,j}\mu_{i,j-1}+\Theta_{i,j}+X_i\right] \tag{24}$$

$$\left[\frac{1}{\mathrm{Pr}}+\left(V_{i,j}-\frac{Y_j\mu_{i,j}}{4}\right)\frac{\Delta Y}{2}\right]\Theta_{i-1,j}-\left[\begin{array}{c}\frac{2}{\mathrm{Pr}}+\frac{(\Delta Y)^2}{\Delta X}\\X_iU_{i,j}-\lambda X_i^{1/2}\end{array}\right]\Theta_{i,j}$$
$$+\left[\frac{1}{\mathrm{Pr}}-\left(V_{i,j}-\frac{Y_jU_{i,j}}{4}\right)\frac{\Delta Y}{2}\right]\Theta_{i+1,j}$$
$$=-(\Delta Y)^2\frac{X_i}{\Delta X}U_{i,j}\Theta_{i,j-1}+(\Delta Y)^2XU_{i,j} \tag{25}$$

The resulted system of tri-diagonal algebraic is solved by double-sweep technique. The computation is started at $X=0$, and then marches downstream implicitly. The ordinary differential equations governing the upstream condition at $X=0$ can be obtained by taking the limit of Eqs. (23)–(25) that X approaches zero. The associated boundary conditions are Eq. (22) with $X=0$. This is,

in fact, the similarity solution of a natural convection boundary layer along a vertical plate with the y-axis. After several testing runs for convergence, the Y-and X-grids are set at 0.01 in the following computations.

Once we know the values of U, V, and Θ and their derivatives, we are at the position to find the values of skin-friction and the rate of heat-transfer from the relations given, respectively as:

$$\frac{1}{2}Gr_x^{1/4}C_f=\left(\frac{\partial U}{\partial Y}\right)_{Y=0}\quad\text{and}\quad\frac{Nu}{Gr_X^{1/4}}=-\left(\frac{\partial\Theta}{\partial Y}\right)_{Y=0} \tag{26}$$

where Gr_X is the local Grashof number.

2.2. Stream-function formulation

Here we introduce the conventional free-stream transformations on the set of Eqs. (8)–(10) as given below:

$$\psi=x^{3/4}f(Y,X),\quad\theta=\Theta(Y,X)$$
$$\eta=x^{-1/4}y,\quad X=Sx \tag{27}$$

where, ψ is the stream-function that satisfies the equation of continuity and η is the similarity same as Y.

Thus from Eqs. (8)–(10), we get:

$$f'''+\frac{3}{4}ff''-\frac{1}{2}f'^2+\Theta=X\left(f'\frac{\partial f'}{\partial X}-f''\frac{\partial f}{\partial X}\right) \tag{28}$$

$$\frac{1}{\mathrm{Pr}}\Theta''+\frac{3}{4}f\Theta'+\lambda X^{1/2}\Theta=X\left(f'\frac{\partial\Theta}{\partial X}-\Theta'\frac{\partial f}{\partial X}+f'\right) \tag{29}$$

where $'$ denotes the differentiation with respect to η. The boundary conditions become:

$$f(0,X)=f'(0,X)=0,\quad\Theta(0,X)=1-X$$
$$f'(\infty,X)=0,\quad\Theta(\infty,X)=0 \tag{30}$$

Now, we are at the position to employ one of the most efficient and accurate implicit finite difference method together with the Keller-box elimination technique (also known as Keller box method), introduced by Keller [14] and described more detail in Hossain [15] and Hossain et al. [16,17]. To employ this method, the set of Eqs. (28) and (29) is written in terms of a system of first order equations in Y, which are then expressed in finite difference form by approximating the functions and their derivatives in terms of the central differences in both co-ordinate directions. Denoting the mesh points in the (X, Y) plane by X_i and Y_j, where $I=1, 2, 3, …, M$

and $j = 1, 2, 3, \ldots, N$, central difference approximations are made such that the equations involving X explicitly are centred at $(X_{i-1/2}, X_{j-1/2})$ and the remainder at $(X_i, Y_{j-1/2})$, where $Y_{j-1/2} = (Y_j + Y_{j-1})/2$, etc. This results in a set of non-linear difference equations for the unknowns at X_i in terms of their values at X_{i-1}. These equations are then linearised by the Newton's quasi-linearization technique and are solved using a block-tridiagonal algorithm, taking as the initial iteration of the converged solution at $X = X_{i-1}$. Now to initiate the process at $X = 0$, we first provide guess profiles for all five variables (arising the reduction to the first order form) and use the Keller box method to solve the governing ordinary differential equations. Having obtained the lower stagnation point solution it is possible to march step by step along the boundary layer. For a given value of X, the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than 10^{-6}, that is, when $|\delta f^i| \leq 10^{-6}$, where the superscript denotes the iteration number. The

computations were not performed using a uniform grid in the y direction, but a non uniform grid was used and defined by $Y_j = \sinh((j-1)/p)$, with $j = 1, 2, \ldots, 301$ and $p = 100$, the measures $Y_{max} = 7.0$ for $Pr = 0.7$ and $Y_{max} = 5.0$ for $Pr = 7.0$.

As before, we calculate the local skin-friction and local heat transfer from the following relations:

$$\frac{1}{2} Gr_x^{1/4} C_f = f''(0, X) \quad \text{and} \quad \frac{Nu}{Gr_X^{1/4}} = -\Theta'(0, X) \quad (31)$$

Solutions thus obtained for $Pr = 0.7$ and 7.0 are presented graphically in Figs. 2 and 3 in terms of local shear-stress and local heat-transfer coefficients. In Fig. 2, the dashed curves represent the results obtained for the case $\lambda = 0$, which was investigated by Venkatachala and Nath [18] by three methods, namely, (1) Keller-box method, (2) local non-similarity method and (3) the regular perturbation method.
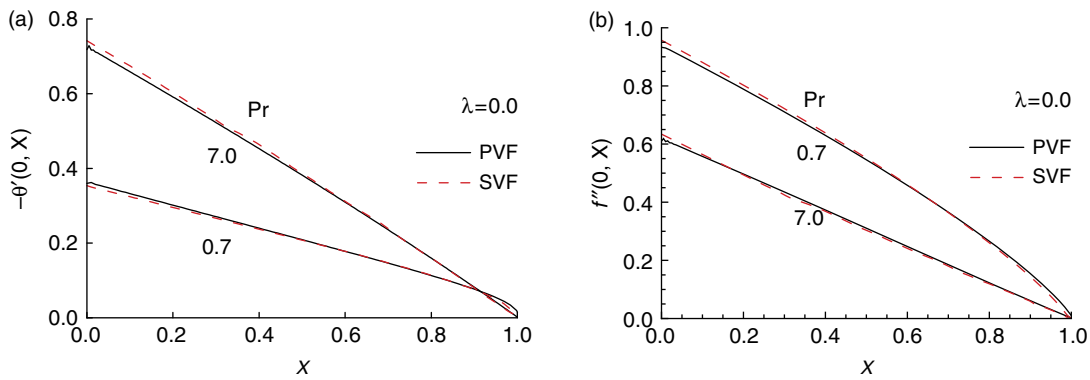

Fig. 2. Numerical results of (a) heat-transfer coefficient and (b) shear-stress coefficient obtained by two methods against X for $Pr = 0.7$ and 7.0 while $\lambda = 0.0$.
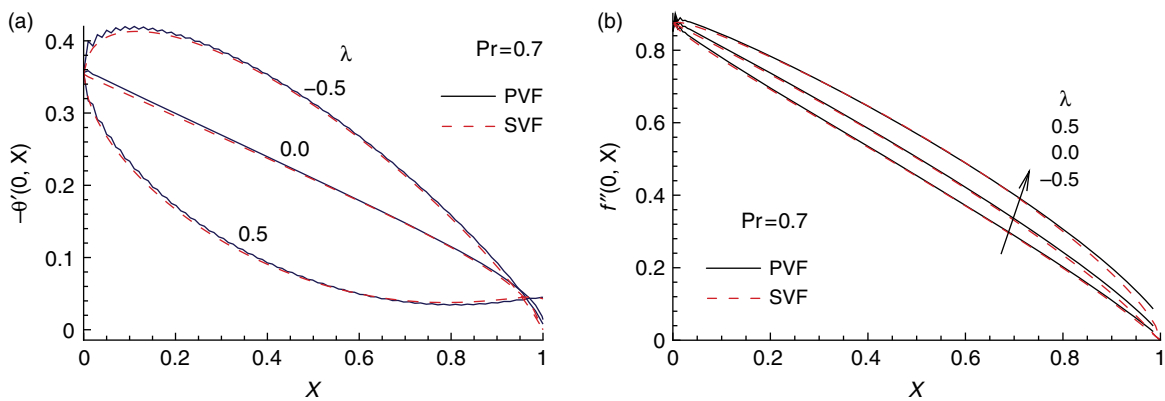

Fig. 3. Numerical results of (a) heat-transfer coefficient and (b) shear-stress coefficient obtained by two methods against X for $\lambda = -0.5, 0$ and 0.5 while $Pr = 0.7$.

3. Results and discussion

We have investigated the problems of the steady two-dimensional flow of viscous incompressible fluid along a vertical flat plate embedded in a stratified medium with temperature dependent heat source as well as heat-sink.

We obtained numerical results for a variety of cases, in terms of heat-transfer coefficient, $-\Theta'(0, X)$, and shear-stress coefficient, $f''(0, X)$, a selection of which are shown in Figs. 2 and 3. For similar section of physical parameters, Figs. 4 and 5 plot the isotherms and contours of stream functions respectively. Near the leading edge,

the surface temperature is higher than the local ambient temperature (i.e., that far from the surface but at the same value of X), and this generates an upward moving boundary layer due to buoyancy forces. However, as X increases, the local ambient temperature also increases since the thermal environment is stratified. Thus the magnitude of the buoyancy force decreases which serves to decelerate and thicken the boundary layer, with a consequent reduction in the surface shear stress and rate of heat transfer. These effects are seen quite clearly in both Figs. 2 and 3.
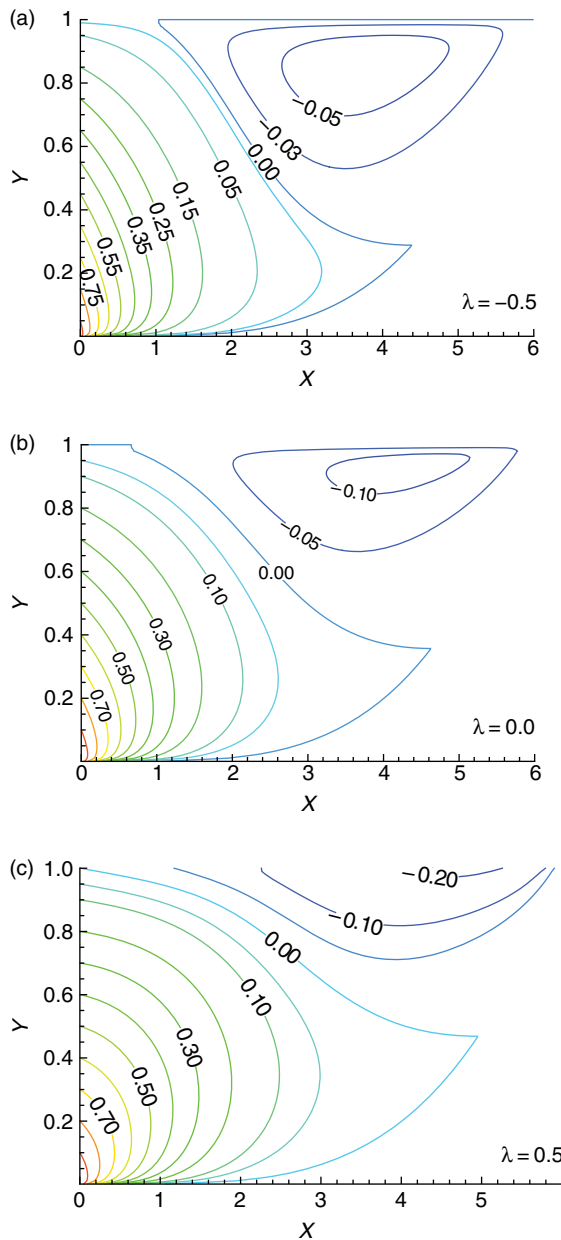

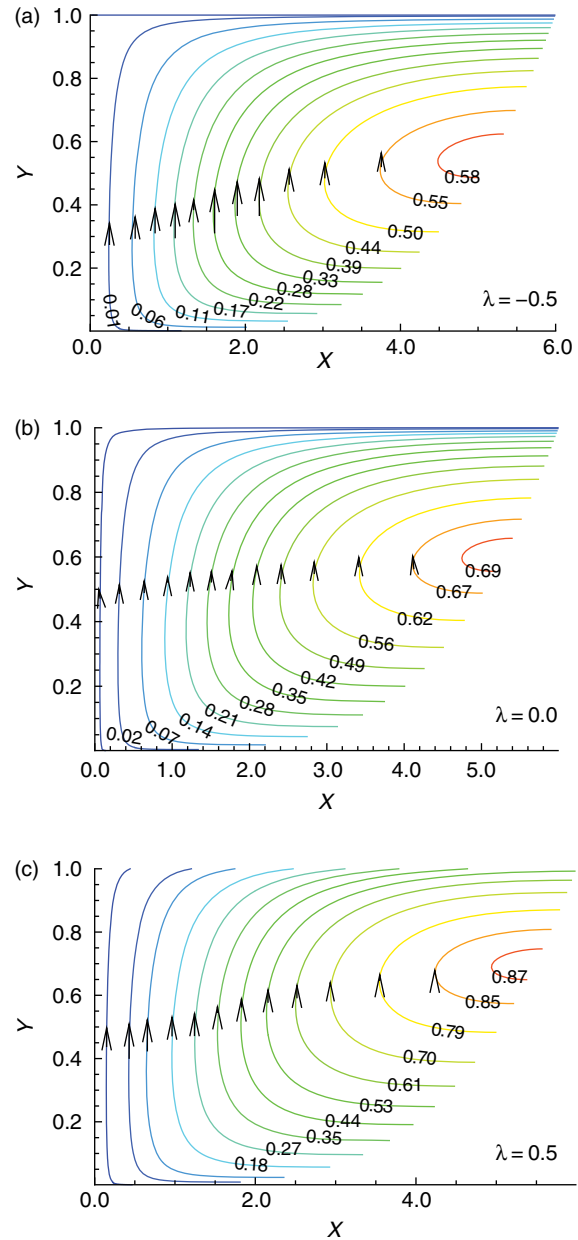Fig. 4. Numerical results of isolines of temperature at Pr = 0.7: (a) λ = −0.5, (b) λ = 0.0 and (c) λ = 0.5.

Fig. 5. Numerical results of streamlines at Pr = 0.7: (a) λ = −0.5, (b) λ = 0.0 and (c) λ = 0.5.

Fig. 3 also displays quite a variation in the surface rate of heat transfer as λ, the source coefficient, varies. This may be understood from the fact that negative values of heat source parameter, λ correspond to the removal of heat from the flow, thereby thinning the thermal boundary layer and increasing the surface temperature gradient. The opposite occurs when heat source parameter, λ is positive.

Figs. 4 and 5 represent the streamlines and isotherms respectively for different values of heat source parameter, λ (λ = 0.5, 0.0, −0.5) while Prandtl number, Pr = 0.7. The numerical simulations reveal that at high heat source parameters, there is a strong reversal of flow, while for low heat source parameters the flow is weaker. The reversal of temperature was found to be stronger at high heat source parameter, λ and weaker for its lower value. The boundary layer thickness increases with the increasing values of λ.

At X = 1, the surface temperature and local ambient temperature are equal, and there is no overall buoyancy force to drive the flow. However the fluid is still moving upwards as it received its momentum from the buoyancy forces existing nearer the leading edge. This is why the surface shear stress has not reduced to zero
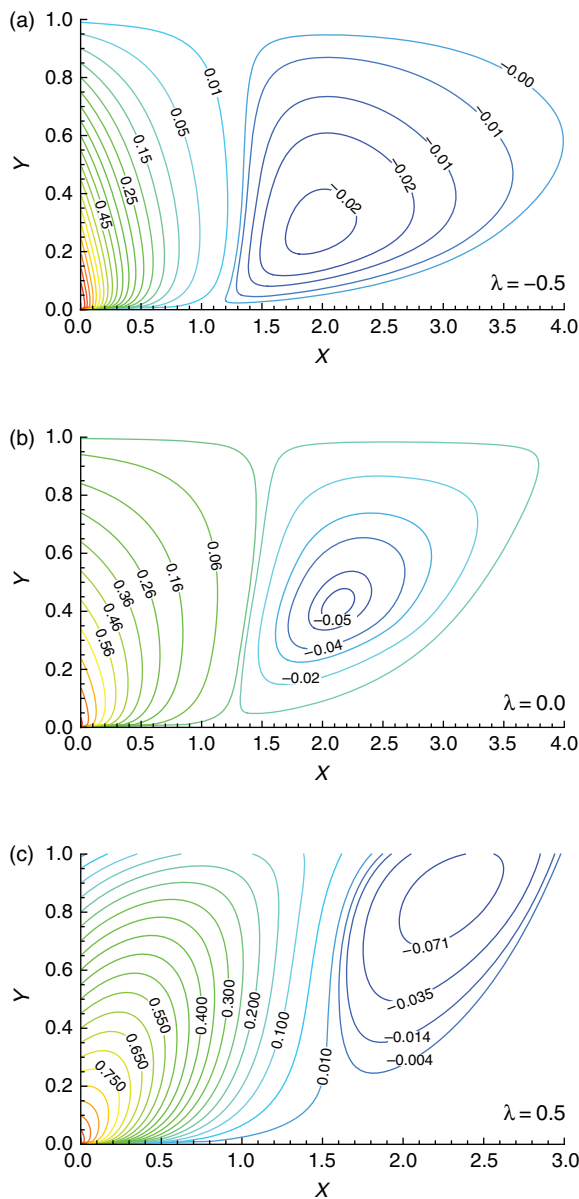

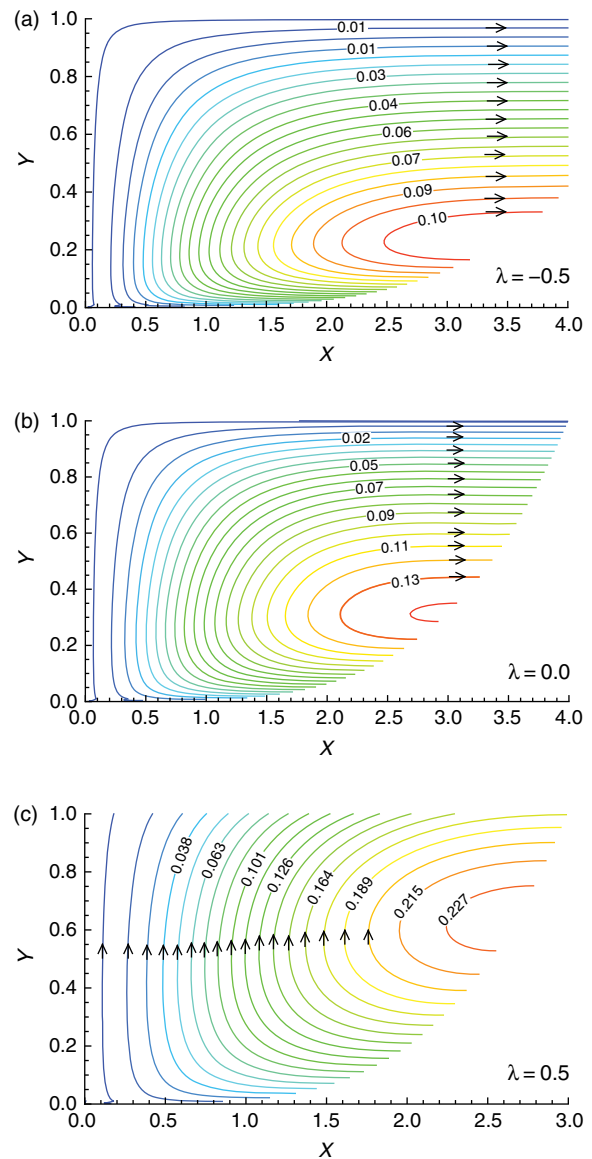Fig. 6. Numerical results of isolines of temperature at Pr = 7.0: (a) λ = −0.5, (b) λ = 0.0 and (c) λ = 0.5.

Fig. 7. Numerical results of streamlines at Pr = 7.0: (a) λ = −0.5, (b) λ = 0.0 and (c) λ = 0.5.

at $X = 1$. When $X > 1$, buoyancy force acts downwards and therefore, we would expect an overall downward fluid motion. A detailed discussion of this aspect would require a numerical analysis of the fully elliptic equations of motion. At some point near $X = 1$, two boundary layers moving in opposite directions will collide and erupt from the surface as a jet. The formation of that jet is seen in Figs. 4 (a, b and c); where entertainment (or inflow) of fluid, which is a normal characteristic of boundary layer flows, is seen relatively close to the leading edge, but close to $X = 1$ there is net outflow.

The defect in the temperature occurs because the cooler fluid from the bottom overshoots upward to a level where the ambient temperature is higher. Isotherms and streamlines are also shown in Figs. 6 and 7 for the same heat source parameter, λ as of Figs. 4 and 5 but for Pr = 7.0. The similar flow patterns can be observed. However, the strong flow reversal can be seen for Pr = 7.0.

4. Conclusions

In the present study, we have investigated the problem of the steady two-dimensional flow of viscous incompressible fluid along a vertical flat plate embedded in a stratified medium with uniform heat source. We have solved the boundary layer equations in finite difference method together with the Keller-box technique. Numerical results thus we obtained for the different values of the Prandtl number (Pr = 0.7 and 7.0 for air and water, respectively), while the values of the source parameter, $\lambda = -0.5$, 0.0 and 0.5. The effect of the heat source parameters on the Nusselt number and share stress as well as streamlines and isotherms have been shown graphically. From the present investigation, we may conclude the followings:

- Near the leading edge, the surface temperature is higher than the local ambient temperature and this generates an upward moving boundary layer.
- As X increases the magnitude of the buoyancy force decreases which serves to decelerate and thicken the boundary layer, with a consequent reduction in the surface shear stress and rate of heat transfer.
- The negative values of heat source parameter, λ corresponds to the removal of heat from the flow, thereby thinning the thermal boundary layer and increasing the surface temperature gradient. The opposite occurs when heat source parameter, λ is positive.

- The reversal of temperature happened to be stronger at high heat source parameter, and weaker for lower value of λ.

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