



Elaboration of a sewer network modeling tool

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ABSTRACT

This work is a contribution to the elaboration of a decision-making helping tool for the diagnosis of sewage networks. The objective is to realize a modeling approach based on the full equations of St. Venant, in order to simulate the behavior of the sewage networks during rainfall by determining the hydraulic parameters of the flowing in terms of flow, velocity, and height in determined points and moments.

Keywords: Modeling; Shallow water equations; Simulation

1. Introduction

In urban areas, soil sealing provokes a considerable increase in the volume of streaming water during rainfalls and with it comes a huge solicitation of the sewer networks. This, results in overflowing and flooding when these sewer networks are under-sized, threatening the safety of people and assets, and the environment where the human activity takes place.

Sewerage networks managers must make sure they have the right dimensioning for their network either in case of rehabilitation or extension. A good assessment of the transit capacity of a network requires adequate simulation models and a mastery of the calculus formulas suitable for the project area [1].

Generally, in Algeria, the verification of sewer networks for the purposes of rehabilitation is made through the classical method, where flow is considered uniform and permanent, assuming constant

hydraulic parameters. Therefore, the average velocity, height, and the water flow remain invariable. The formula used for calculating these parameters is that of Manning–Strickler. However, this method is only appropriate for the dimensioning; it allows assessment of the peak flow at the outlet but not the real functioning of the network during rainfalls.

The objective of this work is to realize a simulation approach of the sewerage networks based on modeling. This implies taking account of the time parameter among other important parameters in the calculus formulas. This simulation will better understand the functioning of the networks during rainfalls, and will help make more adequate decisions to solve the problems related to sewer networks.

The realized model aims to simulate the hydraulic functioning of the sewerage network. It constitutes a means to make a precise diagnosis of the functioning mode of the network during rainfall in order to:

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- evaluate the rate of the collectors filling,
- detect flooding risks by overflowing,
- verify the flow modalities,
- look for the causes of the dysfunctioning.

It could also be used for contributing to the definition of the arrangements aimed at improving the performance of the network.

Our modeling approach is based on a more subtle resolution of St. Venant equations by introducing narrower discretization steps and the use of a variable discretization. The computational efficiency of any discrete-time numerical simulation algorithm is highly dependent on the time step and the spatial discretization applied in the simulation. Consequently, the time step (Δt) and the spatial discretization step (Δx) must be small enough to resolve non-linear variations in time and space, respectively. Indeed, simulation software uses fixed time and space intervals during the simulation, which can lead to situations of instabilities. Moreover, they increase the discretization steps, to reduce the simulation time. In this context, we want to develop a model based on the resolution of St. Venant system by using a finite difference scheme and by testing the limits of this numerical approximation.

2. Hydraulic modeling

Hydraulic calculus method by deterministic modelling takes into consideration the physical reality of the transient flow by solving the shallow water equations [2].

The mechanistic model of St. Venant is a set of equations that describe a real movement of a fluid in a non-permanent free surface flow [3].

We are interested in a small volume element of liquid in a flowing, small enough so that the physical quantities are homogeneous. This amounts to considering a fluid particle in the flowing. Starting from the balance of forces to which this particle is submitted, and assuming that the fluid incompressible, we obtain the Navier-Stokes equations with Eulerian description [4].

The mass balance equation is:

$$\frac{\partial S(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \quad (1)$$

The momentum equation is obtained through the simplification of Navier-Stokes equation:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \frac{\partial h}{\partial x} = g(I - J) \quad (2)$$

Where S is the wetted area and Q is the flow, U is the component of velocity in the x direction, h is the water height, I represents the slope, and J is the friction loss.

S , Q and h are continuous functions of the variables x and t , with the head loss J . This leads to a four-unknown system.

These equations express the 2 principles of conservation of mass and conservation of momentum if we consider that the fluid is incompressible. This is the case for water in applicable conditions in sewerage [5].

The St. Venant system does not have an analytic solution and must be solved numerically. Many numerical methods are used and these equations are applied using different mathematic tools such as the resolution of the partial differential equations with a discretization in time and space [5].

To resolve the previous equations system, two complementary hypotheses have been made:

- The head loss in transient regime is assumed to be calculable in the same manner as the permanent flow.

The formula used is that of Manning-Strickler:

$$J = \frac{U^2}{K_s^2 R_h^{4/3}} \quad (3)$$

Where K_s is the roughness coefficient (Strickler coefficient), and R_h is the hydraulic radius.

- We use the relations relating wetted area at water height $S = f(h)$ so that $S(x,t) = f(h(x,t))$, we also use the flow expression as a function of the wetted area and the average velocity $Q = SU$.

3. Numerical modeling

Different resolution methods exist depending on numerical schemes with finite differences techniques currently the most used in urban hydrology [5]. The method of resolution by finite differences consists of substituting the differential operators by algebraic operators established from developments in Taylor's series [6].

The explicit scheme is the simplest. However, it is necessary to take a short time step, by effect of the numerical stability condition that imposes [7]:

$$\Delta t \leq \frac{\Delta x}{|U| + c} \text{ or } \Delta t = C_{\text{cfl}} \frac{\Delta x}{|U| + c}$$

C_{eff} represents the coefficient of Courant-Friedrich-Levy or the number of Courant.

Where Δt , Δx are the time step and space step, respectively, C is the Celerity.

In a simplified way, the values of water height and velocity of a sewer network are calculated by the following relations:

$$U_i^{n+1} = U_i^n + g\Delta t (I - J_i^n) - \frac{\Delta t}{2\Delta x} (U_i^n (U_{i+1}^n - U_{i-1}^n) + g (h_{i+1}^n - h_{i-1}^n)) \quad (4)$$

$$h_i^{n+1} = h_i^n - \frac{\Delta t}{2\Delta x} \left(\frac{S_i^n}{\frac{\partial S_i^n}{\partial h}} (U_{i+1}^n - U_{i-1}^n) + U_i^n (h_{i+1}^n - h_{i-1}^n) \right) \quad (5)$$

On the basis of the resolution of St. Venant equations, the developed model represents the sewerage network as a succession of conduits related by nodes. The water flows and depth in these conduits are calculated from mass and momentum conservation equations, admitting the continuity of water flows and water heights at each node. The boundary conditions are shown by the flow hydrograph entering upstream of each conduit of the network, and by a relation height-flow at the downstream.

Our model is divided into two parts: definition of the geometry, then numerical processing of the problem.

4. Calibration and validation of the model

The calibration of the numerical model is an important prior step for the diagnosis of the functioning of the network. Through the adjustment of some factors, it allows to approach by calculus the real flowing modalities in dry and rainy weather [8].

The parameters on which we have intervened to realize the calibrations are the roughness coefficient and the travel time of water in the network. The calibration procedure consists of adjusting these two parameters to be as close as possible to reality.

The validation of the model aims at assessing its robustness, speed of resolution as well as its capacity of reproducing water flows and heights in the sewerage network depending on the entering flows [9]. For this purpose, the models results have been compared with the results of the measuring campaign undertaken by the Water and Sewerage Society of Algiers, which aimed to update blueprint sewerage of Algiers, and that for various rainfalls. The type of these measurements is *velocity and height*, with a piezoresistive

sensor providing the water level, and a Doppler sensor measuring the flow velocity.

For the validation of our model, we have chosen the most significant rainfall events available and, which have generated maximum rainfall intensities. That is why we have decided to model the rainfall events of the 13th and 15th November 2008.

However, for practical reasons i.e. to avoid a lengthy computation time, we have decided to take narrow time intervals during these rainfall events, representing the intense durations of those rainfalls.

5. Results and discussion

In order to assess the performance of our model under different application conditions, and verify its capacity to simulate the flowing in such situations, we have simulated the flowing in Oued Ouchaiah sewage collector on its conduits located in the east of the municipality of Kouba in Algiers. The two measurement points M1_06 and M1_08 include four conduits.

The corresponding characteristics are shown in Table 1 and Fig. 1.

The calibration data correspond to the measurements effectuated downstream of the collector.

The validation phase of the model consists of testing its capacity to reproduce the flows, heights and velocity by comparing these values with the measurements effectuated downstream of the collector.

The elaborated model is conceived to model the flow in sewerage networks. However, the application has been performed on the collector Oued Ouchaiah, because of the availability of measures.

We have used a variable discretization; the simulations have been realized with a time step to the second, whereas the spatial discretization is of the order of $\frac{l_i}{6}$.

In the following section, we show the results of the simulation at the outlet of the collector as well as the spatial variation of water height and the Froude number at the peak moment.

In order to stall the curves, we have estimated the travel time using the following relationship:

Table 1
Characteristics of the collector

Junction	M1_06-A1	A1-A2	A2-A3	A3-M1_08
Length (m)	120.16	140.98	104.68	114.67
Slope (%)	0.65	0.21	0.21	0.55
Diameter (mm)	2,000	2,000	2,000	1,500

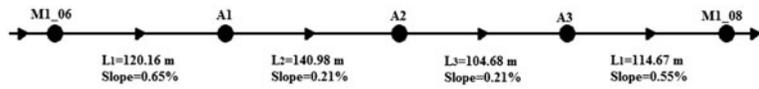


Fig. 1. Network schematization.

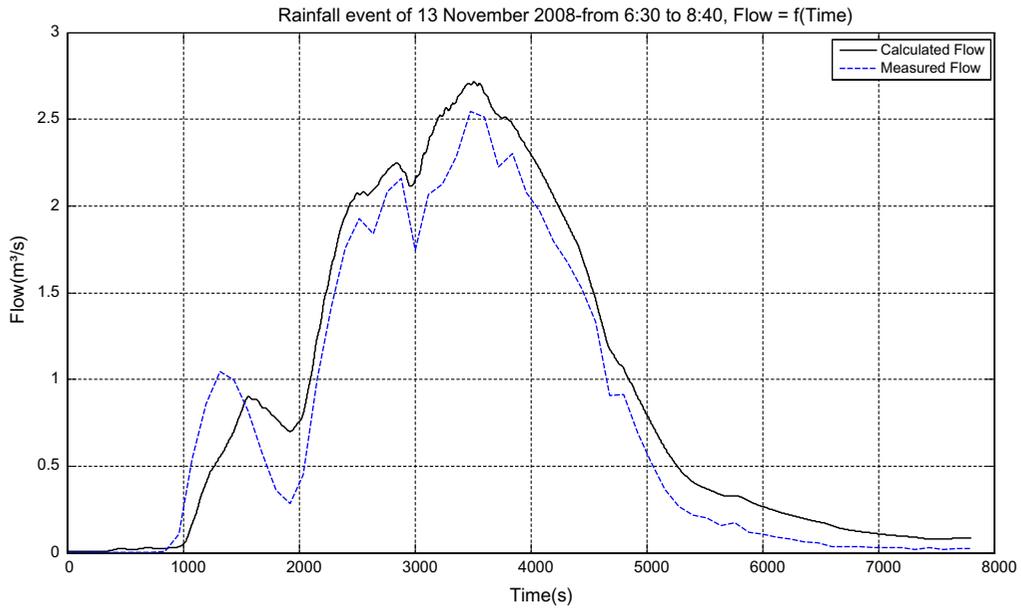


Fig. 2. Flow simulation.

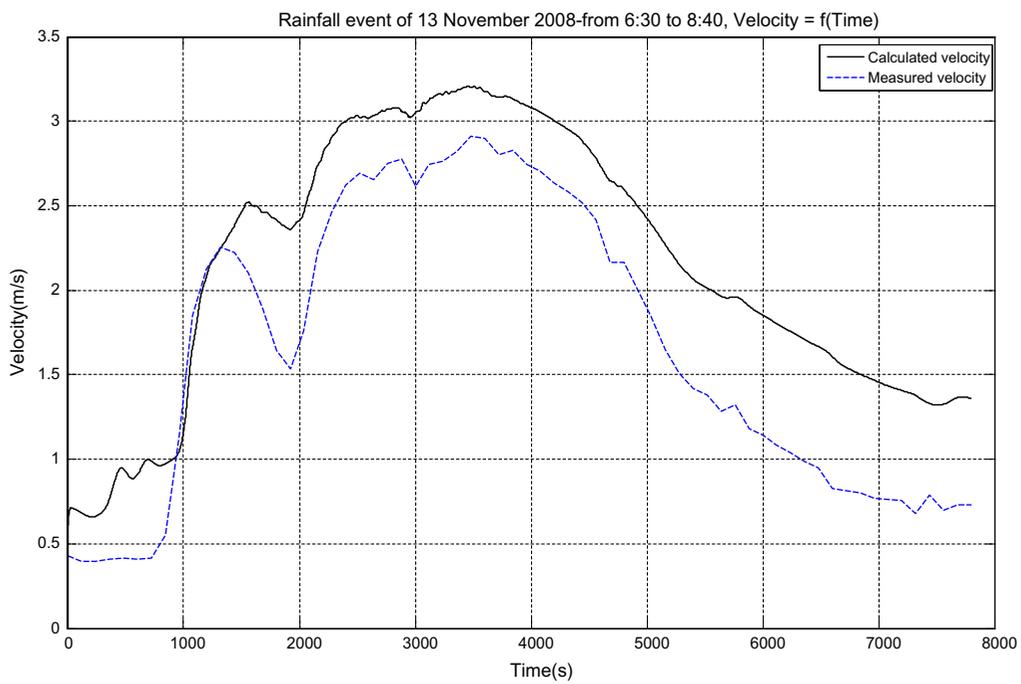


Fig. 3. Velocity simulation.

$T_{\text{travel}} = \sum \frac{l_i}{U_i}$. Where l_i represents the length of conduit i , and U_i is the corresponding average velocity.

The downstream boundary condition does not have an influence on the horizontal offset curves. Indeed, at the downstream of each section, we use iterative Manning equation calculations to determine

the normal depth, or the critical depth in case of change of the flow regime.

5.1. Simulation of the rainfall of 13/11/2008

The shape of the calculated flow curve is similar to one of the measured flows. Likewise, calculated

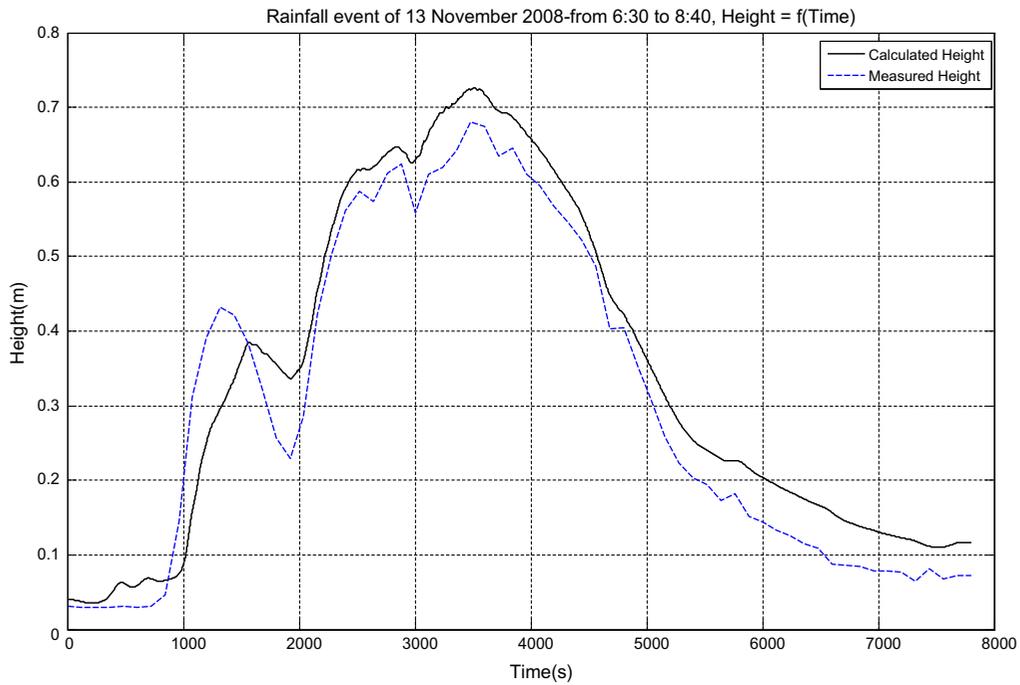


Fig. 4. Height simulation.

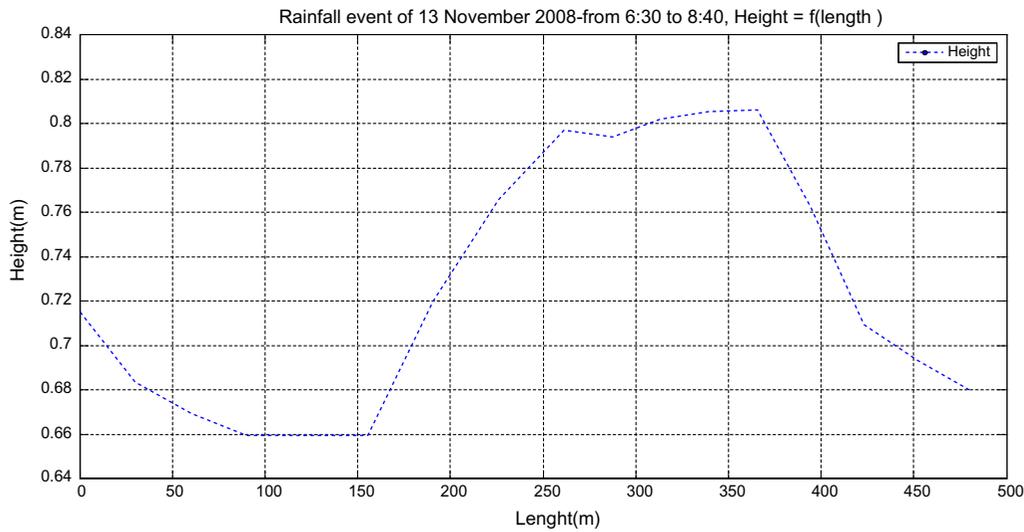


Fig. 5. Water height variation according to length.

velocity and heights curves are the same as those of the measured parameters with a less significant offset (Figs. 2–4).

We notice through this variation, an attenuation of water height in the first and last conduits and an increase in the second and third conduits due to the low slope that these two conduits represent.

The spatial variation curve of Froude number conversely follows that of water height spatial variation whose flow regime is supercritical (Figs. 5 and 6).

In order to model the rainy event of November 13, we needed duration of 12 hours, 55 minutes and 08 seconds.

5.2. Simulation of the rainfall of 15/11/2008

An almost perfect analogy is observed between the curves of measured and calculated flow. The shift is observed at the time of recession and reaches up to $0.35 \text{ m}^3/\text{s}$ (Fig. 7).

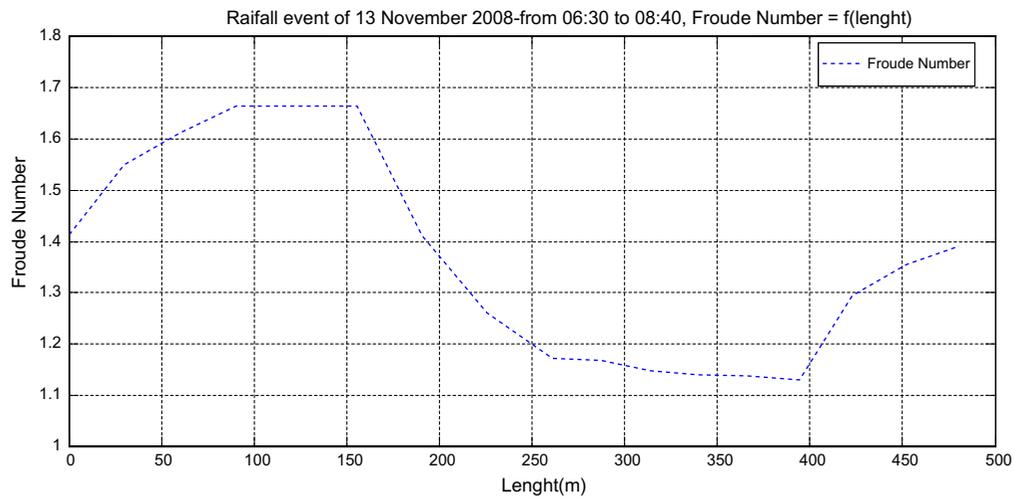


Fig. 6. Froude Number variation according to length.

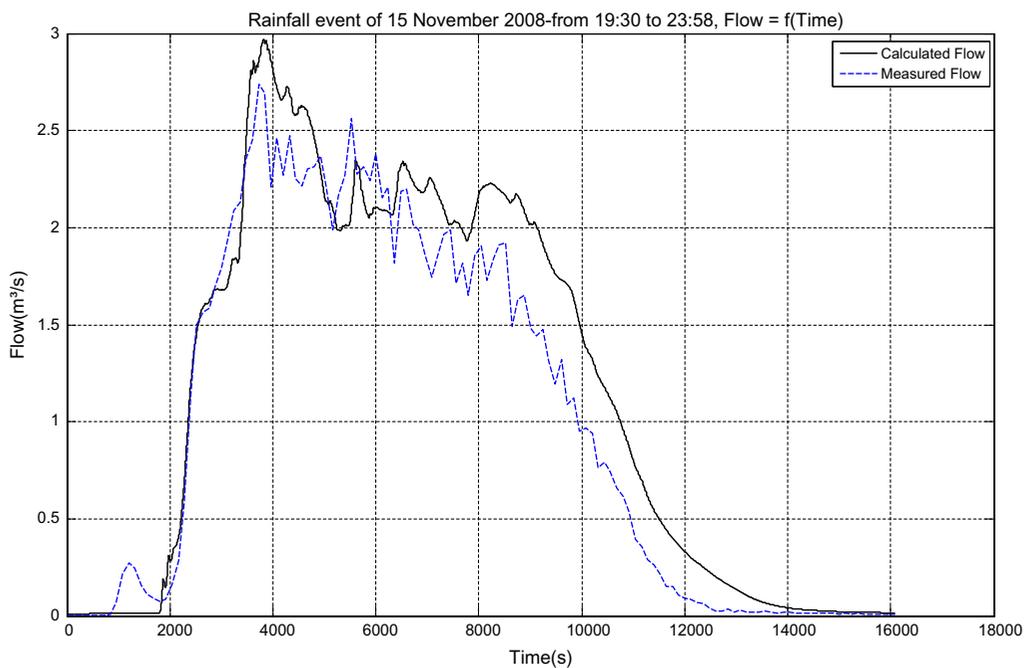


Fig. 7. Flow simulation.

As for flow, the velocity and depth are reproduced with a maximal vertical difference of 0.09 m in terms of height and 0.65 m/s in terms of velocity (Figs. 8 and 9).

Like the rainfall of 13 November, we notice an increase in water velocity in the second and third

conduits of the collector whose slope is unfavorable. A slight attenuation is noticed in the first and last conduits, due to the favorable slope (Fig. 10).

The flow regime remains supercritical along the collector during this rainfall (Fig. 11).

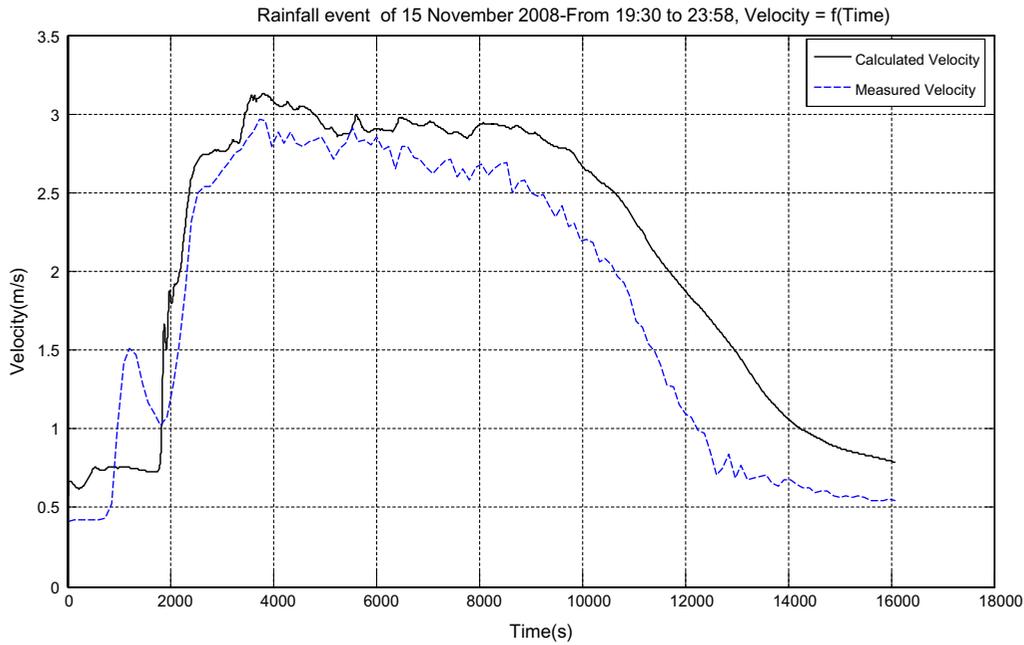


Fig. 8. Velocity simulation.

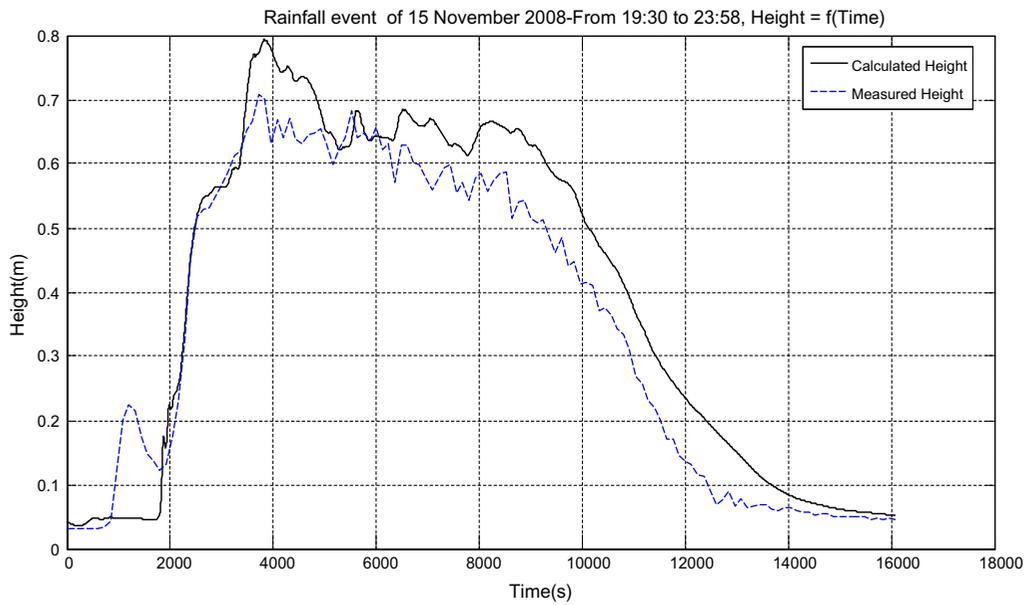


Fig. 9. Height simulation.

Computation time for simulation of the rainy event of 15 November is estimated at 60 hours, 51 minutes and 33 seconds.

5.3. Interpretation

By comparing the results from our model and those of the measurement campaign, we notice some similarity in terms of flow, velocity, and height. On the one hand, this shows the consistency of the developed model, and on the other hand, the convergence of the numerical applied method.

This similarity cannot be perfect in anyway, because of iterative calculus and measurement errors. That is why a slight dissimilarity appears, especially at the beginning of the simulation, due to the choice

of the initial conditions and/or boundary conditions. In fact, the hypothesis of a perfect flowing continuity at the level of the junctions generates some errors.

Moreover, the analysis of the height curve according to length shows a slight solicitation of the collector, whose second and third conduits are the most solicited. This is due to the low slopes present in the conduits and the strangulation in the last conduit.

The spatial variation curve of Froude number helps us to detect the transitions along the collector, which may cause the occurrence of hydraulic jumps. In our case, the regime remains supercritical.

Given that the number of Froude represents the relation between the forces of inertia and the gravity forces [5], the observed flowing regime type is due to

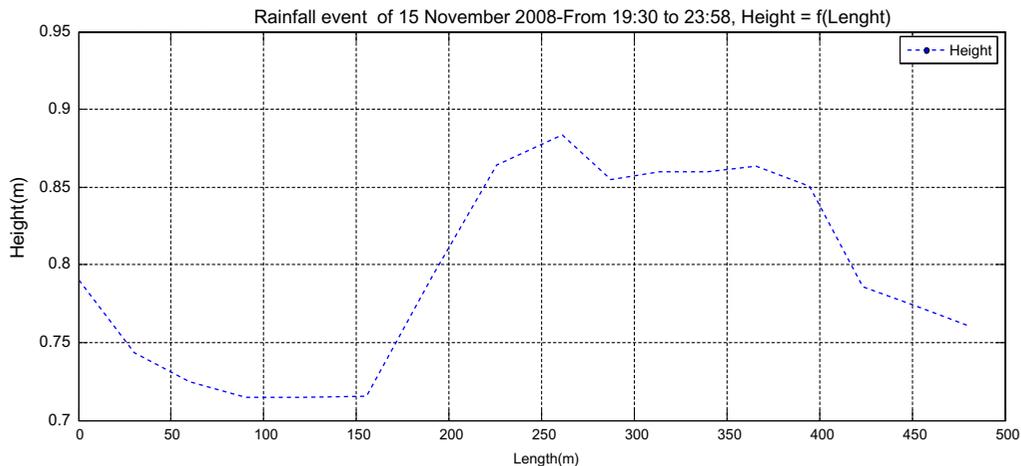


Fig. 10. Water height variation according to length.

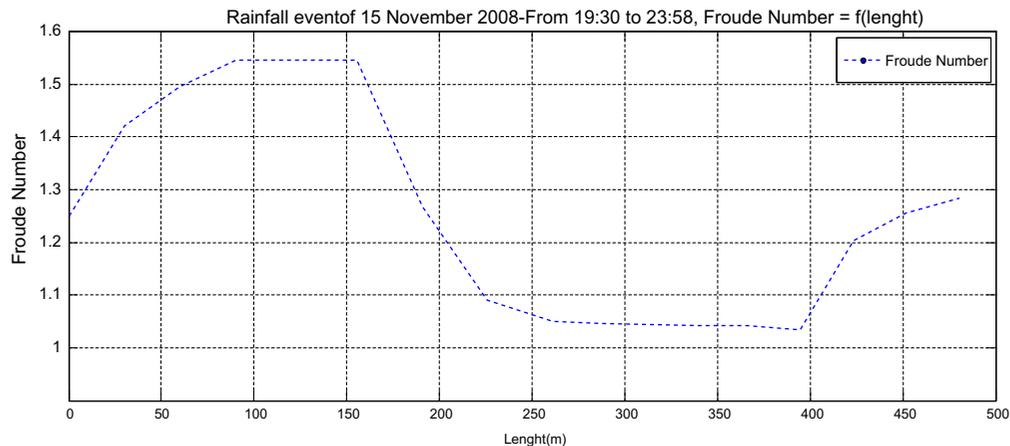


Fig. 11. Variation of Froude number according to length.

the important forces of inertia generating velocity, especially downstream of each conduit.

By analyzing the time of execution of the calculus code during the different simulations, we can say that the calculation time is strongly related to the duration of the modeled rainfall, it is also related to the corresponding spatial and temporal discretization step. The horizontality of the curves observed in the graphs of the spatial height variation and Froude number variation is due to the step size of the spatial discretization.

The realized modeled approach is based on the one-dimensional resolution of the shallow water equations. Indeed, 2D or 3D description is conceivable for a segment but not for the whole network. The calculation time, the computer's capacity, but especially the difficulty of convergence of St. Venant equations consolidates this approach [9]. That is why, in the field of urban hydraulic, the used models are generally 1D.

6. Conclusion

The results of our model remain numerically stable as long as the Courant-Friedrich-Levy stability criterion is verified. In fact, for all the simulations during the validation phase, the space step has been chosen in such a way so as to insure the stability of the results and also reduce the calculation time. This adjustment has been made by gradually decreasing the space step until the appearance of instability signs.

However, the realized simulations show that the modeled portion of the Oued Ouchaiah collector does not have sufficient capacity to face intense rainfalls. The filling rate of the collector has reached 50%, despite the weak frequency of the registered precipitations. In fact, the rainfall events have a return period inferior to two years; this means that the collector will be loaded in case of more intense rainfalls.

Regarding the numerical resolution of the St. Venant equations, we have seen that they can be realized by different numerical schemes. However, the resolution by explicit finite-difference approximations does not seem to be the most appropriate. In fact, the explicit schemes remain submitted to the Courant-Friedrichs-Levy condition and consequently require using a shorter time step in order to guaranty the stability of the results. This, generates a longer calculation time and non-controllable numerical instabilities. In other words, bigger discretization steps would provide shorter computation times at the expense of some acceptable accuracy loss.

In general, due to the complexity of the resolution of de St. Venant equations, we can say that the simulation results obtained from our model represent the

reality of the flow, with an admissible ratio of allowable error.

Due to the type of calculation machine (Standard Personnel Computer) that we have used, the computation time is relatively long. Explicit methods show limits in their ability to manage short discretization steps with the appearance of oscillatory phenomena.

Therefore, it would be more useful to use a numerical scheme of mixed type to resolve the slow computation time, and simultaneously guarantee consistency of the model.

Symbols

C	—	Celerity (m/s)
C_{cfl}	—	Courant-Friedrich-Levy
F	—	Froude number
g	—	acceleration of gravity (m/s)
h	—	water height (m)
h_i^n	—	water depth in $i\Delta x$ space step and $n\Delta t$ time step (m)
i	—	space step index
I	—	slope (m/m)
J	—	friction loss (m/m)
J_i^n	—	friction loss in $i\Delta x$ space step and $n\Delta t$ time step (m/m)
K_s	—	roughness coefficient
n	—	time step index
Q	—	flow (m/s)
R_h	—	hydraulic radius (m)
S	—	wetted area (m)
t	—	time (s)
U	—	water velocity (m/s)
U_i^n	—	water velocity in $i\Delta x$ space step and $n\Delta t$ time step (m/s)
x	—	abscissa (m)
Δx	—	space step (m)
Δt	—	time step (s)

References

- [1] N. El Abboudi, Flow modeling full section, Quebec University, Memory Master of Science (MSc), 2000, 137 p.
- [2] Y. Kovacs, Transient flow simulation models in sewerage networks. National School of Bridges and Roads-Paris, PhD thesis, 1988, 155 p.
- [3] M. Zug, J. Vazquez, Modeling of sewerage systems. Concepts, approaches and steps. National School of Engineering of Water and Environment-Strasbourg (ENGEES), 2006, pp. 13–57.
- [4] M. Carlier, General and Applied Hydraulics. Eyrolles-Paris, 1986, 565 p.
- [5] J.-L. Bertrand-Krajewski, Modélisation des écoulements en réseau d'assainissement, lessons in urban hydrology, National School of Engineering of Water and Environment-Strasbourg (ENGEES), 2006, pp. 25–37.

- [6] D. Euvrard, Numerical Solution of Partial Differential equations Differences Finite difference—Finite element, Masson, 1988.
- [7] M. Abdellah, Methods of numerical discretization of nonlinear hyperbolic equations applied to problems of St. Venant for modeling the hydraulics sewerage, Louis Pasteur University—Strasbourg I, PhD thesis, 2005.
- [8] S. Duchesne, Model for real-time control of sewer networks under overload, Quebec University, PhD thesis, 2001.
- [9] M. Buyer, Transport stream sewerage: 1D model for hydraulic manifolds and spillway taking into account discontinuities, Louis Pasteur University—Strasbourg I, PhD thesis, 2002.