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Effect of thermal radiation and heat generation on MHD flow past a uniformly heated vertical plate

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ABSTRACT

In this paper, the effect of heat generation and radiation parameters on MHD flow along a uniformly heated vertical flat plate in the presence of a magnetic field has been investigated numerically. The nonlinear partial differential equations, governing the problem under consideration for this analysis are transferred to simultaneous nonlinear ordinary differential equations of first order and those are further transformed into initial value problem by applying Multi-segment integration technique. Finally, solutions are obtained by integrating the initial value problem using fourth order Runge-Kutta integration scheme. Rosseland approximation is used to describe the radiative heat flux in the energy equation. Comparison with previously published work is performed and excellent agreement with the results is obtained. Numerical results for the details of the temperature profiles are shown graphically with the variation of the governing parameters considering in the present problem.

Keywords: Electrically conducting fluid; Porous medium; Radiation; Similarity solution.

1. Introduction

The importance of the radiation effect on MHD flow and heat transfer problems has found increasing attention in industries. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity with temperature.

Ostrach [1] presented the similarity solution of natural convection along vertical isothermal plate. Kay [2] reported that thermal conductivity of liquids with low Prandtl number varied linearly with temperature in range of 0–400°F. Arunachalam and Rajappa [3] considered forced convection flow of liquid metals (having low Prandtl number) with variable thermal conductivity and derived explicit closed form of analytical solution. Chaim [4] also studied heat transfer in fluid flow of low Prandtl number with variable thermal conductivity. Carey and Mollendorf [5] observed the effect of temperature dependent viscosity on free convective fluid flow. Crepeau and Clarksean [6] discussed similarity solution of natural convection with internal heat generation,

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which decayed exponentially. Chamkha and Khaled [7] obtained similarity solution of natural convection on an inclined plate with internal heat generation or absorption in presence of transverse magnetic field.

The thermal radiation of a gray fluid, which is emitting and absorbing radiation in a non-scattering medium has been examined by Ali *et al.* [8], Ibrahim [9], Mansour [10], Hossain *et al.* [11] and Elbashbeshy and Dimian [12]. In the aspect of convection and radiation, Viskanta and Grosh [13] considered the effects of thermal radiation on the temperature distribution and the heat transfer in an absorbing and emitting media flowing over a wedge by using the Rosseland diffusion approximation. This approximation leads to a considerable simplification in the expression for radiant flux. In Viskanta and Grosh [13] and Raptis [14], the temperature differences within the flow were assumed sufficiently small such that T^4 might be expressed as a linear function of temperature, i.e. $T^4 \approx 4(T_\infty)^3 T - 3(T_\infty)^4$. Hossain *et al.* [15] investigated the natural convection–radiation interaction on a boundary layer flow along a vertical plate with uniform suction. Yih [16] investigated the natural convection flow of an optically dense viscous fluid over an isothermal truncated cone. Recently, Bataller [17] investigated radiation effects in the laminar boundary layer about a flat-plate in a uniform stream of fluid. He found that as the value of radiation parameter increases, a diminution in the thermal radiation's effect occurs. Later, he studied radiation effects for the Blasius and Sakiadis flows with a convective surface boundary condition [18]. A comparison of these two flows was described in this work.

Chen [19] performed an analysis to study the MHD natural convection flow over a permeable inclined surface with variable wall temperature and concentration. The results showed that the velocity was decreased in the presence of a magnetic field and with the increase of the angle of inclination, the effect of buoyancy force decreased. Heat transfer rate was however increased when the Prandtl number was increased. Duwairi [20] investigated the effect of viscous and Joule heating on forced convection flow from radiative isothermal surfaces. He found that the heat transfer rate was decreased when the radiation parameter was increased. Duwairi and Damseh [21] also studied the convection heat transfer problem with radiation effects from vertical surface for buoyancy aiding and opposing flows. They concluded that increasing the conduction – radiation parameter decreased the heat transfer rates for the buoyancy aided flow and increased them for the buoyancy opposing flow. Ibrahim *et al.* [22] investigated similarity reductions for problems of radiative and magnetic field effects on free convection and mass transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using Lie group method. They also presented the numerical results for the non-uniform magnetic field. Seddeek [23] investigated effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow. The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation was further studied by Seddeek [24].

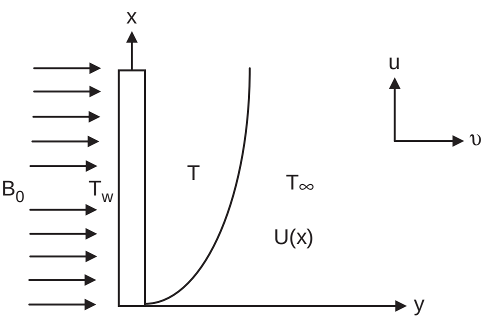

Fig. 1. Flow configuration and coordinate system.

In the present paper, investigation is carried out for the thermal radiation interaction of the boundary layer flow of electrically conducting fluid past a uniformly heated vertical plate embedded in a porous medium. The governing equations are converted into nonlinear system of coupled ordinary differential equations and solved numerically using Multi-segment integration technique. The normalized similarity solutions are then obtained numerically for various parameters entering into the problem and discussed them from the physical point of view.

2. Mathematical formulation of the problem

Let us consider a steady, two-dimensional flow of a viscous, incompressible and electrically conducting fluid of temperature T_∞ past a semi-infinite heated vertical plate having constant temperature T_w (where, $T_w > T_\infty$). A magnetic field of uniform strength, B_o is applied perpendicular to the plate. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field can be neglected. The flow is assumed to be in the *x*-direction, which is taken along the plate in the upward direction and *y*-axis is normal to it. The flow configuration and the coordinate system are shown in Fig. 1. In order to consider the effect of radiation, in terms of the radiative heat flux, the Rosseland approximation is incorporated in the energy equation. The radiative heat flux in the *x*-direction is considered negligible in comparison to the *y*-direction. Within the framework of the above-noted assumptions, it is considered that the boundary layer approximations hold and the governing equations relevant to the problem in the presence of radiation are given by

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K^*}(U-u) \quad (2)$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

where u and υ are the velocity components along x and y coordinates respectively, $U(x)$ is the free stream velocity, $\nu = \mu/\rho$ is the kinematic viscosity, μ is the coefficient of dynamic viscosity, ρ is the mass density of the fluid, σ is the electrical conductivity of the fluid, B_o is the magnetic induction, K^* is the Darcy permeability, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid outside the boundary layer, c_p is the specific heat of the fluid at constant pressure, κ is the thermal conductivity and q_r is the radiative heat flux.

It is assumed that the velocity of the free stream is in the form of

$$U(x) = a\,x + c\,x^2 \quad (4)$$

where a and c are constants.

In the free stream $u = U(x)$, Eq. (2) reduces to

$$U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2}{\rho}U\,. \quad (5)$$

Eliminating $\frac{\partial p}{\partial x}$ between Eqs. (2) and (5), we obtain

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+U\frac{\partial U}{\partial x}+\frac{\sigma B_0^2}{\rho}(U-u)-\frac{\nu}{K^*}(U-u) \quad (6)$$

By using Rosseland approximation, q_r [25] for radiation from an optically thick layer (Ali *et al.* [8]), it can be written as follows

$$q_r=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y}, \quad (7)$$

where σ^* is the Stefan–Boltzmann constant and κ^* is the mean absorption coefficient.

Moreover, the temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor's series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong T_\infty^4+(T-T_\infty).4T_\infty^3=4T_\infty^3T-3T_\infty^4 \quad (8)$$

By using Eqs. (7) and (8), Eq. (3) gives

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty)+\frac{16\sigma^* T_\infty^3}{3\rho c_p\kappa^*}\frac{\partial^2 T}{\partial y^2}. \quad (9)$$

The corresponding boundary conditions for the above problem are given by

$$\left.\begin{array}{ll} u=0,\ \upsilon=0,\ T=T_w & \text{at } y=0 \\ u\to U(x),\ T\to T_\infty & \text{as } y\to\infty \end{array}\right\} \quad (10)$$

In order to obtain a solution of Eqs. (1), (6) and (9), the following transformations are introduced:

$$\eta=\sqrt{\frac{a}{\nu}}\,y \quad (11a)$$

$$u=axf'(\eta)+cx^2g'(\eta) \quad (11b)$$

$$\upsilon=-\sqrt{a\nu}\,f(\eta)-\frac{2cx}{\sqrt{\frac{a}{\nu}}}g(\eta) \quad (11c)$$

$$T=T_w+(T_\infty-T_w)\left[T_0(\eta)+\frac{2cx}{a}T_1(\eta)\right] \quad (11d)$$

$$P=\frac{\rho\nu c_p}{\kappa}\ \text{(Prandtl number)} \quad (11e)$$

$$N=\frac{\sigma B_0^3}{a\rho}\ \text{(Magnetic parameter)} \quad (11f)$$

$$M=\frac{\nu}{aK^*}\ \text{(Darcy parameter)} \quad (11g)$$

$$Q=\frac{Q_0}{a\rho c_p}\ \text{(Heat generation parameter)} \quad (11h)$$

$$K=\frac{\kappa^*\kappa}{4\sigma^* T_\infty^{\ 3}}\ \text{(Radiation parameter)} \quad (11i)$$

$$Pn=\frac{3KP}{3K+4}\ \text{(Radiative Prandtl number)} \quad (11j)$$

where prime denotes differentiation with respect to η.

In view of (11), Eq. (1) is satisfied identically and Eqs. (6) and (9) reduce to

$$f'''+f\,f''-f'^2+N(1-f')-M\,f'+1=0 \quad (12)$$

$$g'''+f\,g''-3f'\,g'+2f''\,g+N(1-g')-M\,g'+3=0 \quad (13)$$

$$T_0''+Pn(f\,T_0'+T_0)+(T_0-1)Q=0 \quad (14)$$

$$T_1''+Pn(-f'\,T_1+f\,T_1'+g\,T_0')+Pn\,T_1\,Q=0\,. \quad (15)$$

The corresponding boundary conditions (10) becomes

$$\left.\begin{array}{ll} f=0,\ f'=0,\ g=0,\ g'=0, T_0=0, T_1=0 & \text{at } \eta=0 \\ f'=1,\ g'=1,\ T_0=1, T_1=0 & \text{as } \eta\to\infty \end{array}\right\}. \quad (16)$$

The nonlinear system of coupled differential Eqs. (12)–(15) together with the boundary conditions (16) are solved numerically using Multi-segment integration technique. First of all, higher order nonlinear differential Eqs. (12)–(15) are converted into simultaneous linear or nonlinear differential equations of order first and they are further transformed into initial value problem by applying Multi-segment integration technique (Kalnins and Lestingi [26]). Once the boundary value problem is reduced to initial value problem, it is then solved using Runge-Kutta fourth order technique (Jain [28]). Now rewrite the governing Eqs. (12)–(15) into a set of first order ordinary differential equations as follows:

$$\Rightarrow \frac{d}{d\eta}\begin{Bmatrix} f \\ f' \\ f'' \\ g \\ g' \\ g'' \\ T_0 \\ T_0' \\ T_1 \\ T_1' \end{Bmatrix} = \begin{Bmatrix} f' \\ f'' \\ (f')^2 - f f'' + N(1-f') - M f' - 1 \\ g' \\ g'' \\ -f g'' + 3f' g' - 2f'' g + N(g'-1) + M g' - 3 \\ T_0' \\ Pn\{Q(T_0 - 1) - f T_0'\} \\ T_1' \\ -Pn(-f' T_1 + f T_1' + g T_0') - Pn Q T_1 \end{Bmatrix} \tag{17}$$

The fundamental set of nonlinear equations (17) together with the boundary conditions (16) has to be integrated over a finite range of the independent variable η. However, the numerical integration of these equations is not possible beyond a very limited range of η due to the loss of accuracy in solving for the unknown initial values, as pointed out by Sepetoski *et al*. [27]. Thus, the multi-segment integration technique developed by Kalnins and Lestingi [26] has been used in this analysis. If the fundamental variables $f, f', f'', g, g', g'', T_0, T_0', T_1, T_1'$ of Eqs. (17) are represented in matrix notation by [w] in a standard form as follows:

$$\frac{dw}{d\eta} = F(\eta, w; P, Q, K, N, M), \tag{18}$$

in which

$$w = [w_i]^T \ where\ 1 \le i \le 10$$

and

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \end{bmatrix} = \begin{Bmatrix} w_2 \\ w_3 \\ w_2^2 - w_1 w_3 + N(1-w_2) - M w_2 - 1 \\ w_5 \\ w_6 \\ -w_1 w_6 + 3w_2 w_5 - 2w_3 w_4 + N(w_5 - 1) + M w_5 - 3 \\ w_8 \\ Pn\{Q(w_7 - 1) - w_1 w_8\} \\ w_{10} \\ -Pn(-w_2 w_9 + w_1 w_{10} + w_4 w_8) - Pn Q w_9 \end{Bmatrix} \tag{19}$$

The boundary conditions Eqs. (16) can be rearranged in the following form as follows:

$$A\, w(0) + B\, w(\eta \to \infty) = C, \tag{20}$$

where

$$A = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Let us consider the initial value problem corresponding to boundary value problems,

$$\frac{dW}{d\eta} = G(\eta, w, W; P, Q, K, N, M) \tag{21}$$

with

$$W(0) = I \ \ and \ \ W = [W_{i,j}]_{10\times 10} \ where\ 1 \le i, j \le 10 \tag{22}$$

$$G(0) = [J] \ where\ J = \left(\frac{F_1,\ F_2,\ F_3,\ F_4,\ F_5,\ F_6,\ F_7,\ F_8,\ F_9,\ F_{10}}{w_1,\ w_2,\ w_3,\ w_4,\ w_5,\ w_6,\ w_7,\ w_8,\ w_9,\ w_{10}} \right) \tag{23}$$

where I is the Identity matrix and J is the Jacobian matrix.

In which,

$$\begin{bmatrix} G_{1j} \\ G_{2j} \\ G_{3j} \\ G_{4j} \\ G_{5j} \\ G_{6j} \\ G_{7j} \\ G_{8j} \\ G_{9j} \\ G_{10j} \end{bmatrix}_{j=1,2,..,10} = \left\{ \begin{matrix} w_{2j} \\ w_{3j} \\ 2w_2 w_{2j} - (w_{1j} w_3 + w_1 w_{3j}) + (N+M) w_{2j} \\ w_{5j} \\ w_{6j} \\ -(w_1 w_{6j} + w_{1j} w_6) + 3(w_2 w_{5j} + w_{2j} w_5) - 2(w_3 w_{4j} + w_{3j} w_4) + (N+M) w_{5j} \\ w_{8j} \\ Pn\left\{ Q w_{7j} - \left(w_1 w_{8j} + w_{1j} w_8 \right) \right\} \\ w_{10j} \\ -Pn\left\{ -\left(w_{2j} w_9 + w_2 w_{9j} \right) + \left(w_1 w_{10j} + w_{1j} w_{10} \right) + \left(w_4 w_{8j} + w_{4j} w_8 \right) \right\} - Pn Q w_{9j} \end{matrix} \right\} \tag{24}$$

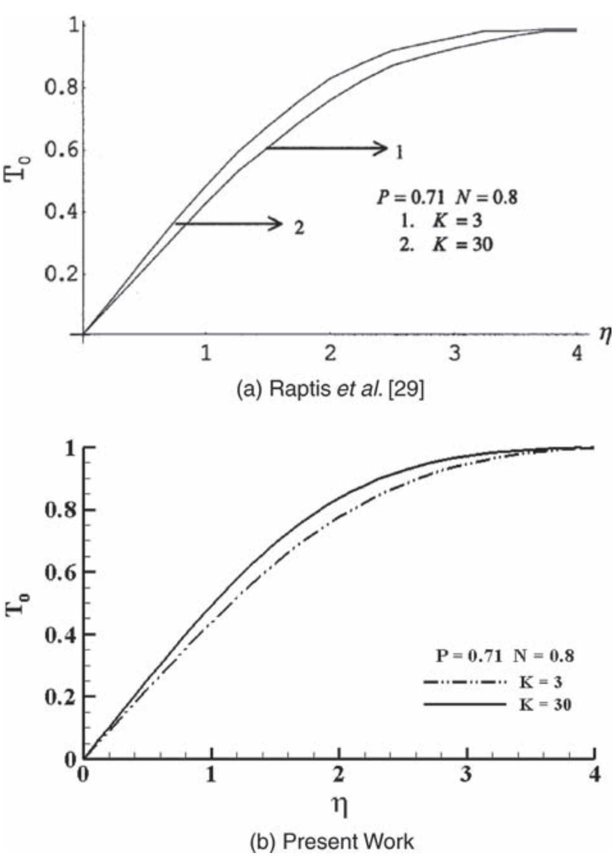

Fig. 2. Comparison of the temperature profiles T_0 and T_1 for $P = 0.71$ and $N = 0.8$.

3. Results and discussion

To assess the accuracy of the present code, the graphs of similarity temperatures $T_0(\eta)$ and $T_1(\eta)$ for different radiation parameter have been plotted where the Prandtl number and magnetic parameter are taken fixed at 0.71 and 0.8 respectively. These graphs are also compared with that of Raptis *et al.* [29] for $K = 3$ and 30. Fig. 2 shows the comparison of the temperature profiles T_0 and T_1 for $P = 0.71$ and $N = 0.8$ produced by the present code and that of Raptis *et al.* [29]. They obtained the solution using Runge-Kutta shooting method whereas the scheme exploited in the present paper is Multi-segment integration technique. Infact, the results show a close agreement and thus give an encouragement for the use of the present code. Hence, the scheme used in this paper is stable and accurate.

The aim of this work is to determine the effects of different parameters on the normalized similarity temperatures $T_0(\eta)$ and $T_1(\eta)$. In the calculations, the values of magnetic parameter *(N)*, Darcy parameter *(M)*, heat generation parameter *(Q)*, Prandtl number *(P)* and radiation parameter *(K)* are chosen arbitrarily. The effect of the Darcy parameter M on the normalized similarity temperatures $T_0(\eta)$ and $T_1(\eta)$ is shown in Fig. 3. From this figure, it is observed that temperature profiles $T_0(\eta)$ increase with the increase of M whereas $T_1(\eta)$ decrease at the same time. As the value of M increases, the resistance to the flow also increases, which means that the temperature field approximates more closely to the equivalent conductive state. Fig. 4 shows the effect of magnetic field parameter *(N)* on the temperature profiles. This figure reveals that the normalized similarity temperature $T_0(\eta)$ shows no effect with the variation of magnetic field parameter but similarity temperature $T_1(\eta)$ increases with the increase of N. This is due to the fact that the magnetic field tends to retard the velocity field, which in turn induces the temperature field and thus results the increase of the temperature profiles. The magnetic field can therefore be used to control the flow characteristics. In Fig. 5, the heat generation parameter *(Q)* is varied keeping all other parameters fixed. It is found that the similarity temperatures $T_0(\eta)$ increase monotonically and similarity temperatures $T_1(\eta)$ also increase close to the plate as Q increases. However, after a short distance from the plate, the profiles overlap and decrease monotonically as Q increases. For different values of Prandtl number *(P)*, significant changes on temperature profiles are observed, which is presented in Fig. 6. From this figure, it can be concluded that in case of cooling the plate, normalized similarity temperature $T_0(\eta)$ increases as P increases and similarity temperatures $T_1(\eta)$ also increases close to the plate. It is apparent that the peak region of each profile move close to the plate as P increases and after a short distance from the plate, these profiles overlap and decrease monotonically.

All the above calculations have been carried out for a fixed radiation parameter *(K)*. Therefore, the effects of radiation parameter on temperature profiles are not clear from the earlier discussions. Fig. 7 shows the effect of radiation parameter *(K)* on the temperature profiles. It is observed that the temperature increases as K increases for both normalized similarity temperatures $T_0(\eta)$ and $T_1(\eta)$. It is also apparent from the figure that for large values of K, the profiles have less significant effect.

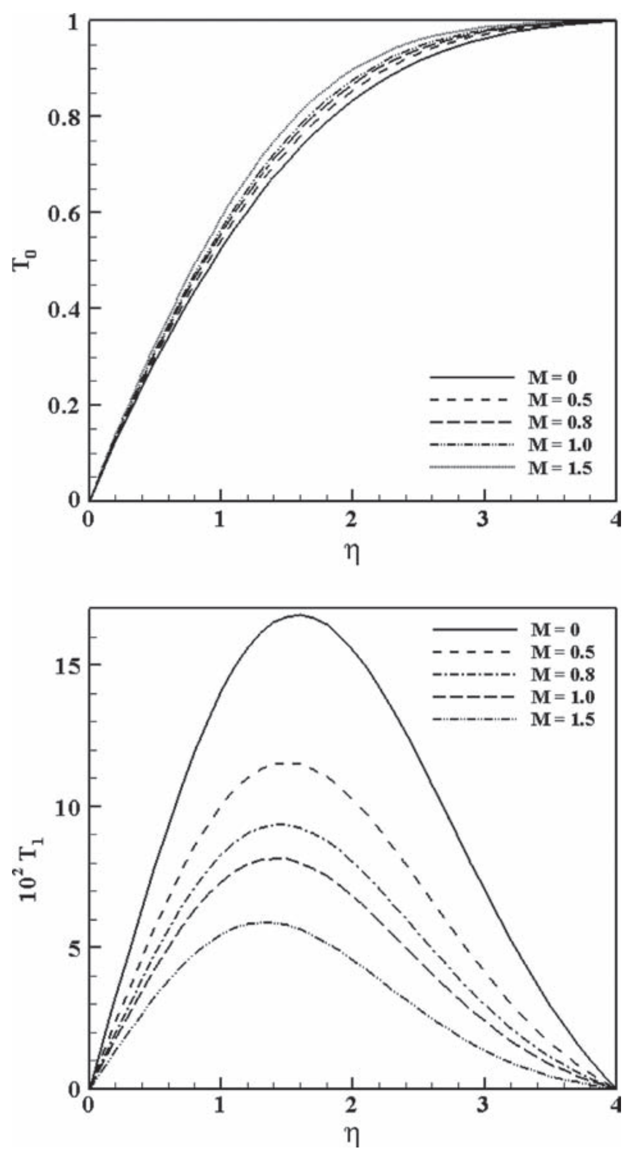

Fig. 3. Temperature profiles with the variation of M for $P = 0.71$, $N = 0.5$, $K = 3$, $Q = 0.5$.

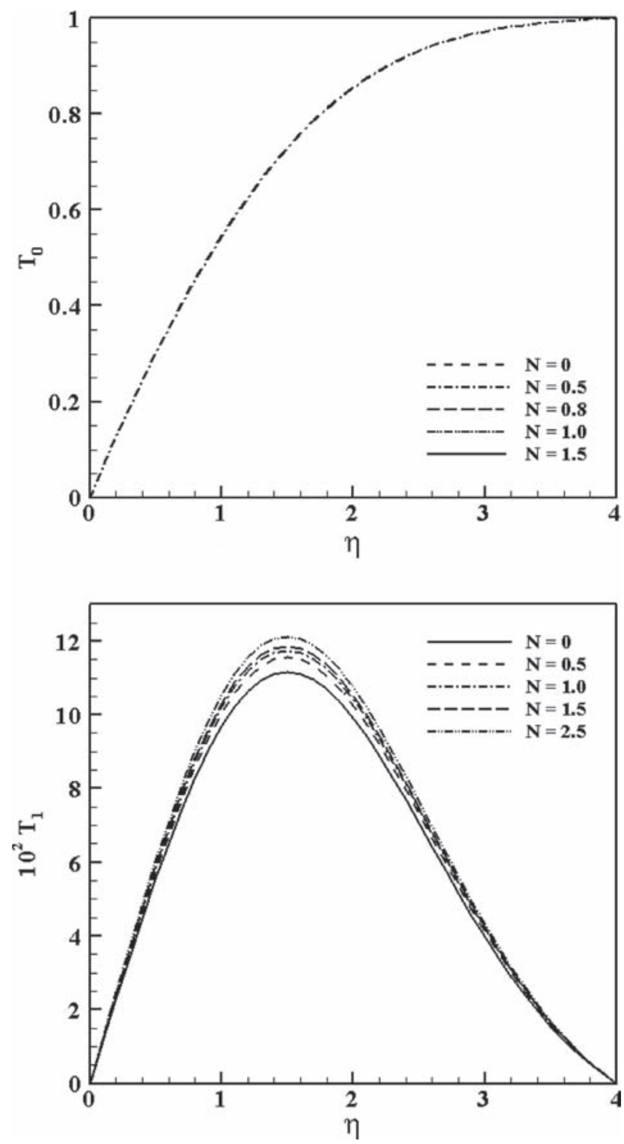

Fig. 4. Temperature profiles with the variation of N for $P = 0.71$, $M = 0.5$, $K = 3$, $Q = 0.5$.

4. Conclusions

An analysis is presented for the problem of MHD flow via a porous medium bounded by a semi-infinite vertical plate. Numerical results are presented to illustrate the details of the heat transfer characteristics of an electrically conducting fluid and its dependence on the material parameters in the presence of radiation. Results show that the normalized similarity temperatures $T_0(\eta)$ *and* $T_1(\eta)$ are greatly influenced by the variations of magnetic parameter *(N)*, Darcy parameter *(M)*, heat generation parameter *(Q)*, Prandtl number *(P)* and radiation parameter *(K)*. From the present

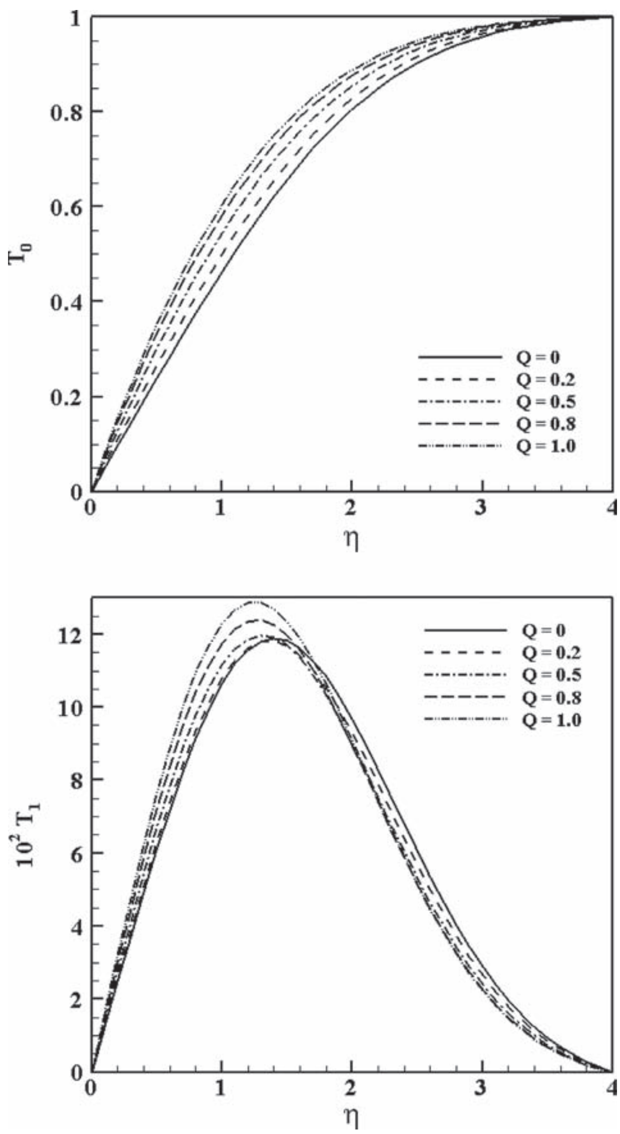

Fig. 5. Temperature profiles with the variation of Q for $P = 0.71$, $M = 0.5$, $K = 3$, $N = 0.5$.

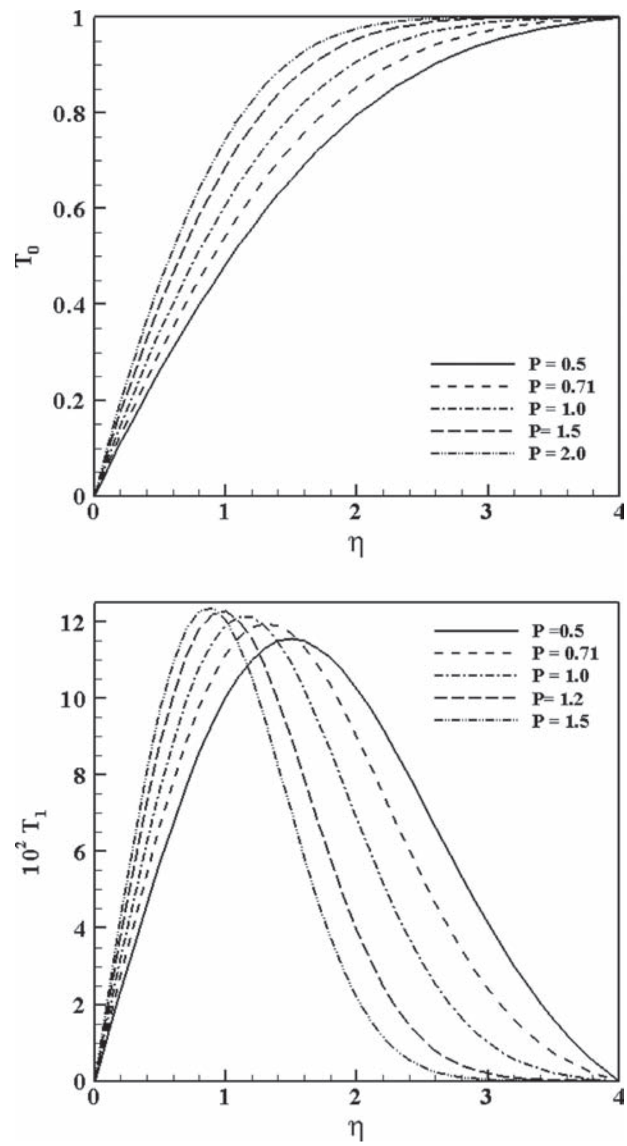

Fig. 6. Temperature profiles with the variation of P for $M = 0.5$, $N = 0.5$, $K = 3$, $Q = 0.5$.

calculation, it is observed that, when the Darcy parameter (M) increases the normalized similarity temperature $T_0(\eta)$ increases whereas the normalized similarity temperature $T_1(\eta)$ deceases. It can be concluded that heat generation and magnetic field parameter have increasing effect on the temperature profiles. Prandtl number and Radiation effect also play a significant role on the temperature profiles.

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Nomenclature

B_0	Magnetic field intensity [T]
c_p	Specific heat at constant pressure [J/Kg K]
I	Identity matrix
J	Jacobian matrix
K^*	Darcy permeability [TL^3/M]
K	Radiation parameter, Eq. (11 i)
M	Darcy number, Eq. (11 g)
N	Magnetic parameter, Eq. (11 f)
p	Pressure [N/m^2]
P	Prandtl number ($= \nu / \alpha$)
Pn	Radiative Prandtl number, Eq. (11 j)
q_r	Radiative heat flux [W/m^2]
Q_0	Volumetric rate of heat generation [W/m^3]

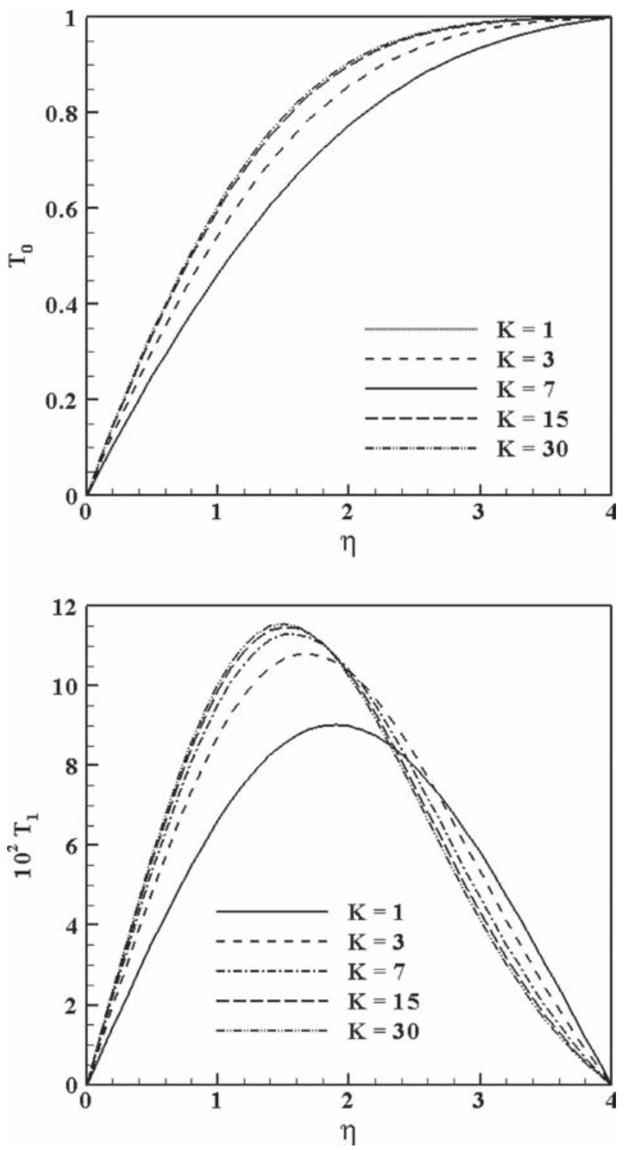

Fig. 7. Temperature profiles with the variation of *K* for $P = 0.71$, $M = 0.5$, $N = 0.5$, $Q = 0.5$.

Q	Heat generation parameter, Eq. (11 h)
T	Temperature within boundary layer [K]
T_w	Temperature at the plate [K]
T_∞	Temperature of the ambient fluid [K]
u	Velocity along x-axis [m/s]
$U(x)$	Free stream velocity ($= a\,x + c\,x^2$)
υ	Velocity along y-axis [m/s]
x	Coordinate along the plate [m]
y	Coordinate normal to the plate [m]

Greek symbols

ρ	Fluid density [kg/m^3]
μ	Coefficient of dynamic viscosity [kg/ms]
ν	Kinematic viscosity [m^2/s]
σ	Electrical conductivity [S/m]
σ^*	Stefan-Boltzmann constant [W/m^{2}K^4]
κ	Thermal conductivity of fluid [W/m^{2}k]
κ^*	Mean absorption coefficient

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