Desalination and Water Treatment
www.deswater.com

36 (2011) 354–362
December

1944-3994/1944-3986
doi: 10.5004/dwt.2011.2639

Theoretical analysis of sliding vane energy recovery device

O.M. Al-Hawaj*

Department of Mechanical Engineering, Kuwait University, P.O. Box: 5969 Safat – 13060, Kuwait
Fax: +9654847131; email:osamah@kuc01.kuniv.edu.kw

ABSTRACT

This paper presents a theoretical analysis of a novel energy recovery device, termed 'sliding vane work exchanger (SVWE)'. The device operates as a combined positive displacement pump and a positive displacement turbine whereby hydraulic energy recovered from the brine in the turbine section of the device is conveyed to the feed in the pump section by means of dual sliding vane rotor assembly disposed within an elliptical chamber. The paper presents models for flow variation, friction and leakage losses, and overall volumetric and hydraulic efficiencies of the device. Furthermore, a parametric study was carried out to investigate the effect of geometrical, physical, and operational parameters on the performance of the device. The study indicates that the viability of the SVWE as an energy recovery device is highly dependent on having low values of vane tip friction and vane tip leakage.

Keywords: Work-exchanger; Energy recovery; Reverse osmosis; Seawater; Desalination

1. Introduction

Energy recovery (ER) is a method used in major industries to minimize the energy input to the overall system by the utilization the energy exchange from one sub-system to another sub-system of the overall system. Energy exchange can either be in thermal form such as sensible or latent energy, or mechanical form such as kinetic, potential or pressure energy. A common application of the ER principle is between exhaust and intake subsystems whereby portion of the energy available in the waste stream is transferred via an ER device to the input material flow. Water industry is an industry concerned with producing potable water with acceptable quality at minimum cost. Desalination of sea water is one of those industries that employ expensive processes to produce potable water because of its high energy demand. The reverse osmosis (RO) desalination has been demonstrated to be one of the least costly methods of desalination even without the usage of ER devices. Although the power consumption in RO plants is not the major problems when compared to membrane life, it is still one of the important aspects of the RO process. Since the advent of RO in 1970s, tremendous effort was undertaken to find a way to reduce the associated operating costs. Among the recent advances to reduce the cost of desalinated water are the application of energy recovery devices (ERDs) and the improvement in RO membrane. Therefore, reducing the energy cost by reducing the energy wastage in the high energy brine is of vital importance. ERD can be classified as energy recovery turbines (ERT) or work exchangers (WE). Examples of the first class include Francis Turbine (FT) or reverse running turbine, Pelton impulse turbine, PIT, and hydraulic turbocharger (HTC). These devices operate on the whole flow with reported efficiency range (70–85%) depending on capacity. Examples of the second class include devices using floating reciprocating pistons in stationary cylinders such as (DWEER) system, other uses direct contact between feed and brine separated by a reciprocating buffer in a rotating

*Corresponding author

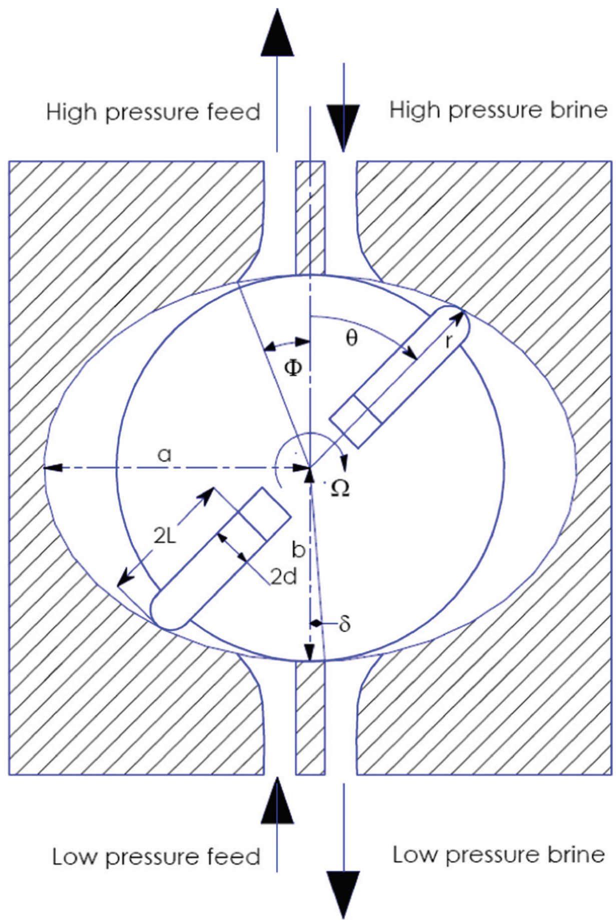

Fig. 1. Sliding vane work exchanger.

in multi-channel cylindrical rotor such as (PX) system [1–3], and rotary work exchanger (RWE) [4]. These devices operate on portion of the feed equal to the brine rate with reported efficiencies above 90%. In particular, the direct contact devices according to [5,6], the size of the reciprocating buffer zone can potentially limit the volumetric efficiency of such devices.

This paper presents a theoretical analysis of a new concept of positive displacement ER device termed Sliding Vane Work Exchanger (SVWE). The paper begins with a description of the working principle of the device, followed by theoretical flow prediction, leakage modeling, volumetric efficiency prediction, friction loss modeling, overall efficiency prediction, a case study, a parametric study, and end up with conclusions and suggestions.

2. How the SVWE works

The SVWE device is a positive displacement ERD which operates on the principle of pressure exchange between two differently pressurized fluids without direct contact. A sectional view of the SVWE device is schematically shown in Fig. 1 as one component of seawater reverse osmosis (SWRO) system. The SVWE device comprises an oval-shaped enclosure enclosing a cylindrical rotor fitted with two sliding vanes in respective slots. The enclosure and rotor together define two separate mirror-image crescent-shaped volumes; each volume connected to an inlet at one tapered end and an outlet at the other tapered end, such the two exchanging streams flow through the device in a counter-current direction without intermixing. The pressure exchange occurs as the brine side act as a turbine and the feed side act a pump, whereby the work transmitted mechanically through vanes and rotor from the brine side to the feed side. The SVWE can be made in two possible arrangements, a floating rotor or motor-driven rotor arrangement. In the floating rotor case, a booster pump is needed to bring the pressure level to that required at membrane inlet feed. In the other hand, the motor-driven rotor no need for booster pump since the motor provide the necessary input work for the additional pressure boost. The motor-driven SVWE is more complex as a sub-system due to inclusion of a motor, but it simpler when considered as one component of the whole SWRO due to the elimination of booster pump.

3. Capacity of SVWE

Considering a single rotor SVWE rotor as schematically shown in Fig. 1, concentrically disposed within an elliptical chamber having and minor radius along vertical direction, b, defining small clearance with rotor, a major radius, a, an eccentricity, e, width equal to rotor radius, b, and rotating with an angular velocity, Ω, it can be shown from geometry that the volumetric flow rate variation with vane angle, θ, measured from vertical is given by the relation:

$$Q_1 = \frac{0.5b^3e^2\Omega\sin^2(\theta)}{1 - e^2\sin^2(\theta)}. \quad (1)$$

Due to the pulsating flow nature of the single rotor SVWE, which can cause cavitation problems and vibrations, it is more practical to employ multi-rotor SVWE, as schematically depicted in Fig. 2, with n rotors operating in parallel and synchronized to a common shaft with equally distributed vane angular shift. Such arrangement can advantageously be used to increase the capacity and eliminate flow fluctuations associated with single rotor SVWE, as illustrated in Fig. 3. The capacity for n-rotor SVWE can be expressed by the relation:

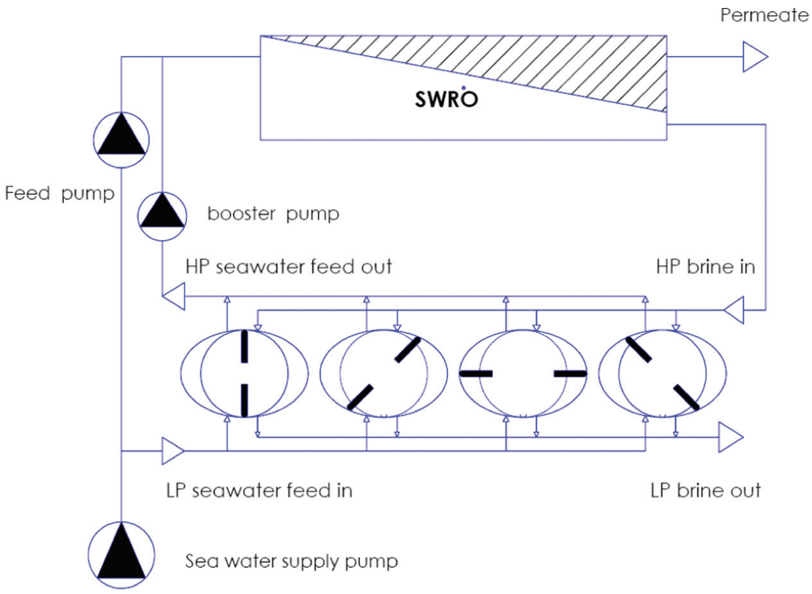

Fig. 2. Schematic of RO plant with four-rotor SVWE.

$$Q_n = \sum_{i=1}^{n} \frac{0.5e^2b^3\Omega \sin^2(\theta + \pi(i-1)/n)}{1 - e^2 \sin^2(\theta + \pi(i-1)/n)}, \; n = 1, 2, 3, \ldots \tag{2}$$

And the average theoretical flow rate for n-rotor SVWE operating with rotor speed, N, can be given by the relation

$$Q_{\mathrm{th}} = n\pi b^3 N \left(\frac{1}{\sqrt{1-e^2}} - 1 \right). \tag{3}$$

As depicted in Fig. 3, the use of multi-rotor reduces this pulsation significantly, for example a 4-rotor SVWE reduces pulsation to less than 2% of the main flow and average flow rate increases four times that for a single rotor.

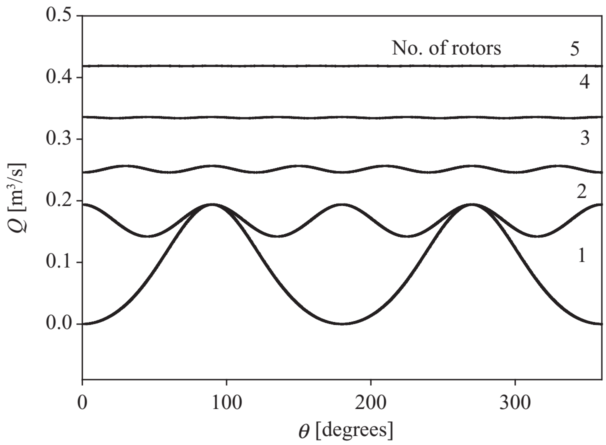

Fig. 3. Theoretical flow variation for n-rotor SVWE.

4. Leakages in SVWE

Under ideal flow conditions of zero-leakage and zero pressure drop across inlet feed and brine manifolds, both feed and brine streams through the device will have equal flow rates, equal to the average theoretical flow rate. However, due to the presence of clearances between moving parts and high pressure differential, leakages do take place making the actual streams deviate from the theoretical one; the actual brine flow rate, tends to be greater while the actual feed flow, tends to be lower than the theoretical flow rate. This deficit and surplus in respective feed and brine flows is a consequence of brine leakage, feed leakage, such that mass balance for each stream can be expressed as:

$$Q_{\mathrm{Fd}} = Q_{\mathrm{Fi}} = Q_{\mathrm{th}} - q_{\mathrm{LkF}}, \tag{4}$$

$$Q_{\mathrm{Bd}} = Q_{\mathrm{Bi}} = Q_{\mathrm{th}} + q_{\mathrm{LkB}} \tag{5}$$

Leakages in SVWE can occur in several paths including end face, vane side, vane tip, and short-circuit leakages. In this analysis, only the last two leakages are considered significant since the first two leakages can be made practically small by appropriate design. A modified flow through an orifice is used to model these leakages as expressed by the relations:

$$q_{LkF} = n\ cd_F A_{vF}\left(\sqrt{\frac{2(P_{Fd} - P_{Fi})}{\rho_F}} + \frac{(a+b)\Omega}{2}\right), \tag{6}$$

$$q_{LkB} = n\ cd_B A_{vB}\left(\sqrt{\frac{2(P_{Bi} - P_{Bd})}{\rho_B}} - \frac{(a+b)\Omega}{2}\right) + n\ cd_{sc} A_{eff}\sqrt{\frac{2(P_{Bi} - P_{Bd})}{\rho_B}}. \tag{7}$$

The first term in brackets in the above equations is the leakage contribution due leakage across vane tip, and the second term in Eq. (7) is the leakage contribution due to short-circuit leakage. The vane tip leakage is expressed as a flow through a moving orifice, whereby the effect of moving vane is accounted for through a correction as expressed in the second term in brackets in the above equations. The sign of the correction term reflects a larger leakage occurring in the feed side due to vane motion opposite to the direction of leakage flow, and a lesser leakage flow occurring in the brine side due to vane movement in the same direction of the leakage flow. The short-circuit leakage takes place as the high pressure brine inlet and the low pressure brine outlet becomes in direct communication as vane start sweeping through the outlet brine port, which takes place over a brief angular interval, Φ, measured from vertical to the beginning edge of the port. On the feed side of device, as vane sweeps through the feed outlet port, the inlet feed becomes exposed to the outlet feed resulting in a reversed or backflow driven by the presence of the high pressure differential. Fortunately, this feed reversed flow can be prevented using check valve installed downstream of the inlet feed. Unlike vane tip leakage which occurs continuously throughout the vane cycle, the brine short circuit occurs only over an angular interval, equal to difference between port angle, Φ, and half vane angle, δ. To account for such intermittency nature of the short-circuit leakage flow, an effective leakage area, A_{eff}, is defined as half the actual leakage area port area corrected by an intermittence factor, (Φ/π), and is expressed by the relation:

$$A_{eff} = 0.5\left(\frac{\Phi - \delta}{\pi}\right) b \int_{\delta}^{\Phi} r\,d\theta, \tag{8}$$

$$\delta = \arcsin\left(\frac{d}{R}\right). \tag{9}$$

Also the vane tip clearance areas for both feed and brine sides are assumed to be equal and given by the relations:

$$A_{vB} = A_{vF} = by_{vt}. \tag{10}$$

Due to the lack of experimental data on the device discharge, discharge coefficients for both vane tip and short circuit leakages are assumed equal, and a value for a typical conventional orifice is used, (i.e., $cd_{\underline{B}} = cd_B = cd_{SC} = 0.6$).

5. Volumetric efficiency of SVWE

Defining brine and feed leakage efficiencies, (η_{LkB}, η_{LkF}), two respective measures of volumetric leakage efficiencies (η_{VB}, η_{VF}) can be defined by the following expressions:

$$\eta_{VB} = \frac{Q_{th}}{Q_{Bi}} = \frac{1}{1 + \eta_{LkB}}, \tag{11}$$

$$\eta_{VF} = \frac{Q_{Fd}}{Q_{th}} = 1 - \eta_{LkF}, \tag{12}$$

where

$$\eta_{LkB} = \frac{q_{LkB}}{Q_{th}}, \quad \eta_{LkF} = \frac{q_{LkF}}{Q_{th}}, \tag{13a, b}$$

Defining the WE volumetric efficiency as the ratio of feed discharge rate to the inlet brine rate, this can be written as

$$\eta_V = \frac{Q_{Fd}}{Q_{Bi}} = \eta_{VB}\eta_{VF}. \tag{14}$$

The above equation expresses the volumetric efficiency of the SVWE as the product of the brine side volumetric efficiency times the feed side volumetric efficiency.

6. Friction losses in SVWE

The friction losses in the SVWE device are primarily resulting from contact forces acting on vane tip and vane sides. These forces, as shown in Fig. 4, are determined by solving the dynamic problem. Solution of the dynamic problem, not given here, takes the following assumptions under consideration:

- Each vane makes three point contacts, one vane tip contact and two side vane contacts.
- Each vane tip makes contact with lateral stator wall at all times.
- Each vane is subjected at base to a base force resulting from the transmittance of pressurized feed.

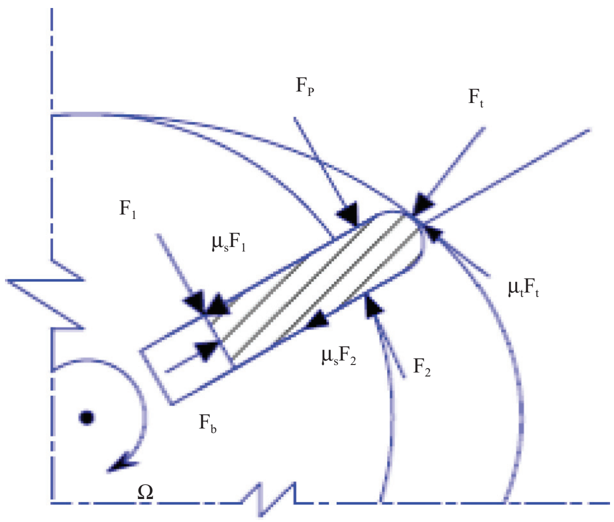

Fig. 4. Forces acting on vane.

- Friction forces are described by Coulomb's law of friction formula.
- Manifolds pressure losses are negligible.
- Constant rotor speed.

Hence, with reference to Fig. 4, the forces acting on the vane include vane tip, F_t, lower vane side, F_1, upper vane side, F_2, pressure force, F_P, vane base force, F_b, vane tip friction, $\mu_t F_t$, side frictions $\mu_s F_1$ and $\mu_s F_1$. Hence, the vane tip and side friction losses can be expressed by the following relations:

$$W_{fvt} = 2\mu_t \int_0^{2\pi} rF_t \mathrm{d}\theta, \tag{15}$$

$$W_{fvs} = 2\mu_s \int_0^{2\pi} (|F_1| + |F_2|)\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)\mathrm{d}\theta. \tag{16}$$

Other friction power losses include end face, bearing, rotor lateral wall are obtained assuming Couette flow conditions from the following relations, respectively

$$W_{fef} = \frac{2\pi^2\mu\Omega\left(R^4 - R_{sh}^4\right)}{y_{ef}}, \tag{17}$$

$$W_{fbe} = \frac{8\pi^2\mu\Omega R_{sh}^3 L_{sh}}{y_{be}}, \tag{18}$$

$$W_{frs} = \frac{4\pi\mu\Omega\Phi R^4}{y_{rs}}. \tag{19}$$

Hence, the total frictional power losses becomes

$$W_f = W_{fvt} + W_{fvs} + W_{fbe} + W_{fef} + W_{frs}. \tag{20}$$

7. Overall efficiency of SVWE

The SVWE can be considered as a combination of a turbine and a pump within a single device, hence energy balance can be considered separately.

Turbine energy balance: The decrease in available power of brine from inlet to outlet is the sum of mechanical power loss, leakage power loss, and mechanical shaft power delivered to the pump portion of the SVWE. This can be expressed mathematically by relation.

$$Q_{Bi}(P_{Bi} - P_{Bd}) = W_{fB} + q_{LkB}(P_{Bi} - P_{Bd}) + W_{sh}. \tag{21}$$

Turbine efficiency can then be expressed as the ratio of mechanical shaft power to the decrease in hydraulic power of the brine, such that

$$\eta_t = \frac{W_{sh}}{Q_{Bi}(P_{Bi} - P_{Bd})}. \tag{22}$$

Pump energy balance: The mechanical shaft power transmitted to the pump section is partly dissipated as mechanical friction power loss, another portion as a leakage or recirculation power loss, and the remainder delivered as hydraulic power in the form of pressurized feed gain. Hence,

$$W_{sh} = W_{fB} + q_{LkF}(P_{Fd} - P_{Fi}) + Q_{Fd}(P_{Fd} - P_{Fi}). \tag{23}$$

The pump efficiency is defined as the ratio of the gain in hydraulic energy of the feed to shaft input power, and this can be expressed mathematically by the relation below

$$\eta_p = \frac{Q_{Fd}(P_{Fd} - P_{Fi})}{W_{sh}}. \tag{24}$$

An overall energy balance for the SVWE can be obtained by eliminating shaft power in Eq. (20) using Eq. (22), hence

$$Q_{Bi}(P_{Bi} - P_{Bd}) = W_{TL} + Q_{Fd}(P_{Fd} - P_{Fi}). \tag{25}$$

where the total power loss, W_{TL}, is the sum of friction, brine leakage, and feed leakage power losses, such that:

$$W_{TL} = W_f + q_{LkB}(P_{Bi} - P_{Bd}) + q_{LkF}(P_{Fd} - P_{Fi}). \tag{26}$$

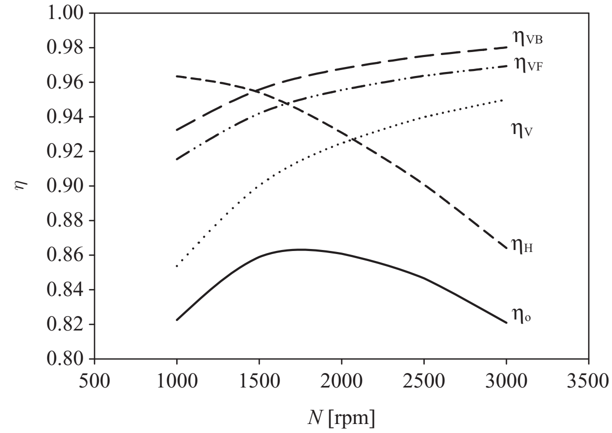

Fig. 5. Effect of rotor speed on efficiency.

And the friction power loss is the sum of friction losses in feed and brine sides, such that

$$W_f = W_{fB} + W_{fF}. \tag{27}$$

The SVWE efficiency is defined as ratio of the hydraulic energy gain in the feed to the hydraulic energy loss in the brine, which is equivalent to the product of turbine and pump efficiencies such that:

$$\eta_o = \eta_t\eta_P = \frac{Q_{Fd}(P_{Fd} - P_{Fi})}{Q_{Bi}(P_{Bi} - P_{Bd})} = 1 - \frac{W_{TL}}{Q_{Bi}(P_{Bi} - P_{Bd})}. \tag{28}$$

Defining hydraulic efficiency, η_H, of WE as the ratio of pressure feed gain in pump section of the device to the brine pressure loss in turbine section, an alternative expression for overall efficiency, η_o, can then be expressed as the product of volumetric and hydraulic efficiencies of the WE such that

$$\eta_H = \frac{(P_{Fd} - P_{Fi})}{(P_{Bi} - P_{Bd})}, \tag{29}$$

$$\eta_o = \eta_V\eta_H. \tag{30}$$

8. A case study

Consider a hydrodynamically lubricated 4-rotor module SVWE device having the following parameters:
(a) Geometrical parameters:

Rotor radius, $R = b$	150 mm
Elliptical eccentricity, e	0.65
Major radius, a	197 mm
(continued)	
No of rotors, n	4
Port angle, Φ	
Vane half thickness, d	11.25 mm
Vane half length, L	37.5 mm
Vane tip leakage clearance, y_{cl}	0.1 mm
(b) Operating parameters:	
Rotor speed [rpm]	1,500
Brine supply pressure [kPa]	6,000
Feed supply pressure [kPa]	300
Brine discharge pressure [kPa]	300
(c) Physical parameters:	
Vane tip and side friction coefficient, f	0.05
Leakage discharge coefficient, cd	0.6
(d) Calculated results:	
Theoretical flow rate, Q_{th} [m^3/h]	1,206
Actual discharge feed rat, Q_{Fd} [m^3/h]	1136.4
Feed discharge pressure, P_{Fd} [kPa]	5,738
Volumetric efficiency, η_{VF}	0.94
Volumetric efficiency, η_{VB}	0.96
Volumetric efficiency, η_V	0.90
Hydraulic efficiency, η_H	0.95
Overall efficiency, η_o	0.86

9. Influence of design parameters

Considering a reference SVWE device with operating conditions as listed in the figures, the influence of different parameters are studied. The approach is to solve for the discharge feed pressure using an iterative method, which requires the solution of the energy balance, and dynamic problem for the contact forces, estimation of frictional losses, leakage losses, and the overall energy balance. Once convergence is achieved various efficiency parameters are calculated.

9.1. Influence of rotor speed

According to Fig. 5, considering a well hydrostatically lubricated SVWE device, the effect of increasing rotor speed results in a decrease in hydraulic efficiency due to higher frictional losses associated with higher speeds. On the other hand, the volumetric efficiencies for both feed and brine improve with increasing speed due to the decreasing brine and feed leakage contribution, (η_{LkB} & η_{LkF}), with increasing rotor speed. Also, according to the figure, the feed volumetric efficiency, η_{FV}, is consistently less than the brine volumetric efficiency, η_{VB}, due to the vane tip leakage enhancement associated with the advancing vane movement opposite to direction of leakage flow in the feed side. Also, the overall efficiency, η_o, of the SVWE, as influenced by the oppositely behaving trends of both the volumetric efficiency and hydraulic efficiency, indicates an optimum operating speed that results in a maximum overall efficiency.

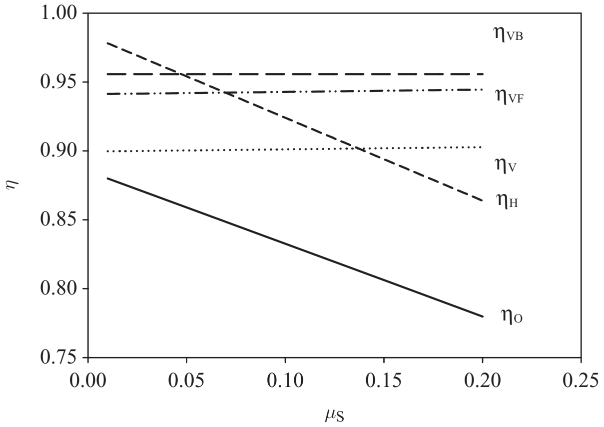

Fig. 6. Effect of stator vane tip friction on efficiency.

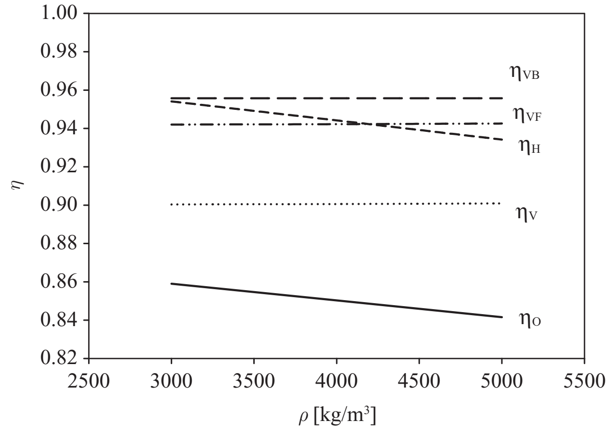

Fig. 8. Effect of material vane density on efficiency.

9.2. *Influence of vane friction coefficients*

According to Figs. 6 and Fig. 7, vane tip and vane side frictional coefficients have negligible effect on volumetric efficiency. On the other hand, this effect on hydraulic efficiency and consequently on the overall efficiency are significant. This is true assuming an all-time vane tip contact or the maintenance of small vane tip clearance with lateral stator. This condition can be enhanced by application pressure force at the base of vanes through the transmission of high pressure brine or feed. As expected, frictional losses increase proportionally with increase in the magnitude of friction coefficients. This increase in frictional losses causes a decrease in the available hydraulic energy transmitted to the feed as exhibited by lower hydraulic efficiency, η_H, and consequently lower overall efficiency, η_o.

9.3. *Influence of vane material density and vane thickness*

The effect of increase of both vane material density, ρ, and vane thickness, *d*, is to increase centrifugal forces on vanes as a result of increase in the mass of vanes. Also, larger vane thicknesses results in larger applied vane base forces as a result of increase in vane cross-sectional areas. Both effects results in larger vane tip forces and thereby larger vane tip frictional losses, and consequently a decrease in the available hydraulic energy transmitted to the feed. As depicted in Figs. 8 and Fig. 9, the hydraulic efficiency, η_H, indicates a slight decline. On the other hand, the effect of vane thickness on volumetric efficiency as depicted on Fig. 9, indicates significant improvement in volumetric efficiencies for both feed and brine side. This improvement is due to decreasing short-circuit leakages resulting from decreasing inlet to outlet port exposure as the high and low pressure sides of brine and feed approach isolation.

9.4. *Influence of port angle*

As expected and as depicted in Fig. 10, the effect of increasing port angle parameter, Φ, influences only the short circuit leakage occurring in the brine and feed sides, as a result of larger angular interval of inlet to

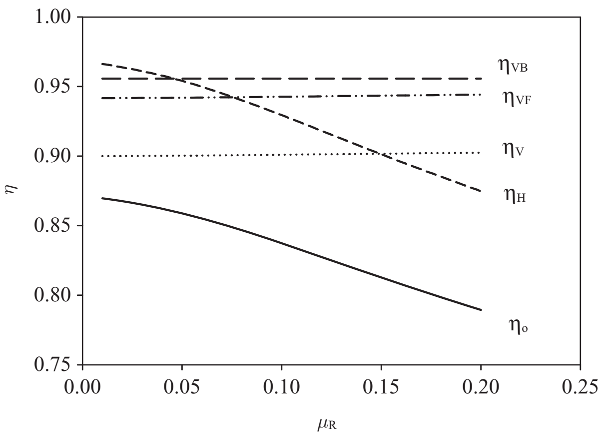

Fig. 7. Effect of side vane friction on efficiency.

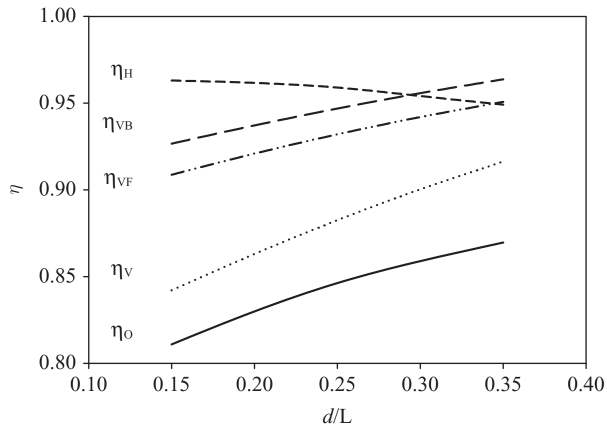

Fig. 9. Effect of vane thickness on efficiency.

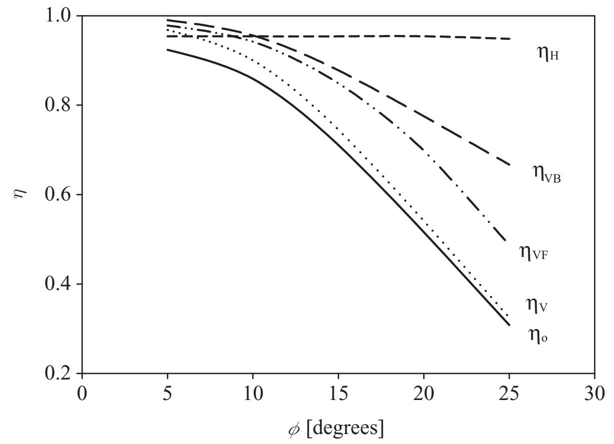

Fig. 10. Effect of port angle on efficiency.

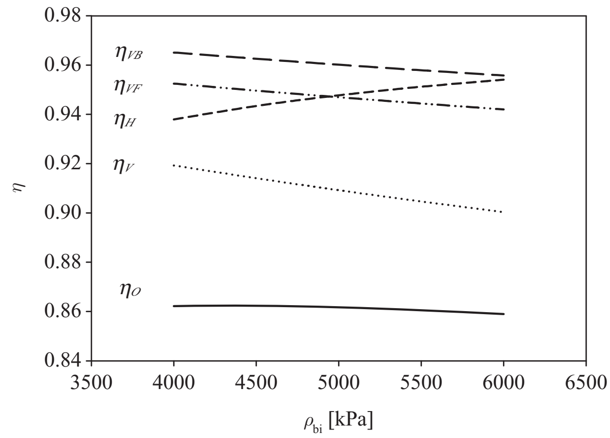

Fig. 12. Effect of inlet brine pressure on efficiency.

outlet exposure between high and low pressure for both feed and brine sides. As a result, the brine volumetric efficiency, η_{VB}, feed volumetric efficiency, η_{VF}, and total volumetric efficiency, η_V, and consequently the overall efficiency, η_o, exhibit significant decrease with increase in port angle parameter.

9.5. Influence of vane tip-stator clearance

Although an all time vane tip-stator contact is one of the constraining assumptions considered in this study, vane tip leakage is bound to take place due to either wear or due to an allowed vane tip design in which portion of the vane making all-time contact and the rest of the vane tip portion making a small designed clearance with stator as a mean to reduce vane tip friction losses. According to Fig. 11, the effect of increasing vane tip clearance yields a significant linear decreasing trend in both volumetric and consequently the overall efficiency of the device.

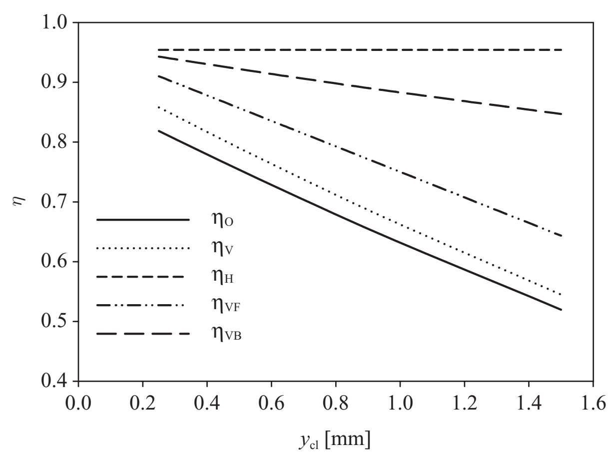

Fig. 11. Effect of vane tip clearance on efficiency.

9.6. Influence of inlet brine pressure

According to Fig. 12, the effect of operating the device with higher inlet brine pressure results in slight improvement in the hydraulic efficiency and approximately equal decline in the volumetric efficiency such that the net effect in on the overall efficiency, η_o, is small.

10. Conclusions and recommendations

The SVWE is a positive displacement ER device characterized by high volumetric and hydraulic efficiencies, above 90%, with a typical overall efficiency in access of 85%. The device is also characterized by minimal feed-brine mixing and flow pulsation. According to the theoretical analysis and parametric study, the rotor speed, vane tip leakage, short circuit brine leakage, and vane tip friction, vane thickness, port angle are among the parameters that significantly influence the performance of the device, while vane material density and inlet brine pressure are among the parameters with the least influence on the performance of the device.

The study also suggests that further improvement in performance can be made by a combination of factors which include improving vane tip seal to reduce leakage across vane tip through transmission of pressurized feed at base of vanes to provide a net outward pressure force, reduction of vane tip friction through the use of hydrostatic lubrication and material selection with surface properties having least friction coefficients, and the reduction of short-circuit leakage through optimization of dimensions of vane thickness and port angle.

Nomenclature

A_{eff} — effective short-circuit leakage area
A_{vB} — average vane tip brine side leakage area

A_{vF} average vane tip feed side leakage area
a major radius of ellipse
b minor radius of ellipse also width of rotor
cd_B brine side discharge coefficient
cd_F feed side discharge coefficient
cd_{sc} short-circuit discharge coefficient
d vane half thickness
e eccentricity of ellipse
F_1 first contact force of vane side at edge of rotor slot
F_2 second contact force of vane side at vane lower edge
F_b vane base force
F_p vane pressure force
F_t vane tip force
L vane half length
L_{sh} total shaft length
N rotor speed [rpm]
n number of rotors
P_{Bd} pressure of discharge brine
P_{Bi} pressure of inlet brine
P_{Fd} pressure of discharge feed
P_{Fi} pressure of inlet feed
Q volumetric flow rate [m^3/s]
Q_1 instantaneous volumetric flow rate of single rotor SVWE [m^3/s]
Q_n instantaneous volumetric flow rate of n-rotor SVWE [m^3/s]
Q_{th} average theoretical volumetric flow rate of n-rotor SVWE [m^3/s]
Q_{Bd} actual brine discharge volumetric flow rate [m^3/s]
Q_{Bi} actual brine inlet volumetric flow rate [m^3/s]
Q_{Fd} actual feed discharge volumetric flow rate [m^3/s]
Q_{Fi} actual feed inlet volumetric flow rate [m^3/s]
q_{LkF} feed leakage flow rate [m^3/s]
q_{LkB} brine leakage flow rate [m^3/s]
r radial coordinate of ellipse profile
R rotor radius
R_{sh} shaft radius
W_f power loss due to friction
W_{fB} power loss due to brine side friction
W_{fF} power loss due to feed side friction
W_{fbe} power loss due to bearing friction
W_{fef} power loss due to end face fluid friction
W_{fvt} power loss due to vane tip friction
W_{fvs} power loss due to vane side friction
W_{frs} power loss due to rotor side fluid friction
W_{sh} shaft power output of turbine side
W_{TL} total power loss
y_{be} bearing gap clearance
y_{ef} end face clearance
y_{rs} rotor side clearance
y_{vt} vane tip gap clearance
δ port lower edge angular limit [rad]
η efficiency
η_H hydraulic efficiency
η_{LkB} brine leakage efficiency
η_{LkF} feed leakage efficiency
η_o overall efficiency
η_t turbine efficiency
η_V volumetric efficiency
η_{VB} volumetric brine efficiency
η_{VF} volumetric feed efficiency
μ dynamic viscosity of feed [$N\ s\ m^{-2}$]
μ_s Coulomb's friction coefficient of vane side-rotor slot [-]
μ_t Coulomb's friction coefficient of vane tip-stator slot [–]
Φ port upper edge angular limit [rad]
Ω angular speed of rotor [rad/s]
θ angular position of vane measured from vertical [rad]

References

[1] R.L Stover and J. Martin, Reverse osmosis and osmotic power generation with isobaric energy recovery, Desal. Water Treat., 15 (2010) 267–270.
[2] R.L Stover and J. Martin, Titan PX-1200 Energy Recovery device – test results from the Inima Los Cabos, Maxico, sweater RO facility, Desal. Water Treat., 3 (2009) 179–182.
[3] R.L Stover, Development of a fourth generation energy recovery device, A'CTO's Notebook, Desalination, 165 (2004) 313–321.
[4] O.M. Al-Hawaj, Rotary work exchanger and method, US Patent No. 6,773,226, Patent and Trademark Office, Washington, D.C., August 10, 2004.
[5] O.M. Al-Hawaj, The work exchanger for reverse osmosis plants, Desalination, 157 (2003) 23.
[6] Z. Yihui, D. Xinewei, J. Maowei and C. Yuqing, Numerical simulation on a dynamic mixing process in ducts of a rotary pressure exchanger for SWRO, Desal. Water Treat., 1 (2009) 107–113.