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Oil spill at sea based on non-hydrostatic three-dimensional hydrodynamic mathematical model

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ABSTRACT

A non-hydrostatic three-dimensional hydrodynamic mathematical model for solving three-dimensional free-surface flow based on unstructured grid was established. The model used a planar unstructured grid and a vertical stratified grid. The model equations were discretized by the finite volume method and the finite difference method, and the simulation results of the tidal process were in good agreement with the measured data after the comparison validation. Based on the calculation results of three-dimensional surface flow field and the consideration of the migration and diffusion range of oil film under the action of real-time wind field and flow field, a mathematical model of oil spill at sea was established. The three-dimensional hydrodynamic mathematic model under non-hydrostatic assumption is suitable for solving free-surface hydrodynamic problems under complicated conditions, which can be used to simulate the diffusion process of oil spills as well as the leakage of harmful substances whose density is close to the oil.

Keywords: Oil spill; Fire; Three-dimensional model; Non-hydrostatic

1. Introduction

The forecast model for oil spills started in the 1960s and many patterns have been developed so far. Blokker [1] established an oil diffusion and volatilization model in 1964. Shen and Yapa [2] made in-depth theoretical and experimental work in terms of the physical properties of oil, the interaction between oil and seawater, etc., and established the empirical formulas to predict the oil spill path, area and residual amount. Fay [3] proposed the three-stage theory of oil film spread, which has made groundbreaking achievements in the research and application of oil spill spread model. After Fay's theory was put forward, many scholars improved their models [4,5] and combined the self-spreading process and diffusion process of oil spill on the existing basis to establish various spreading models. For example, Mackay et al. [6,7] considered the influence of wind in Fay's second stage formula and established the spreading models of thick oil film and thin oil film, respectively, based on the actual observation results. Lehr et al. [8] revised Fay's theory, taking the influence of wind field and flow field on the asymmetry of the oil film into consideration, Lehr held that the oil film spread to the periphery in the form of an ellipse rather than a circle in the ocean and its long axis direction corresponded to the wind direction. The oil particle method was first proposed by Johansen [9] and Elliot [10] according to which the actual physical phenomena of the diffusion process can be directly simulated by dividing the oil spill into many discrete oil droplets to simulate the drift and diffusion process of the oil spill in seawater [11–14].

The early three-dimensional hydrodynamic model is based on the hydrostatic model of the three-dimensional Reynolds equation [15]. However, when we solve the water flow affected by large topography fluctuations, slight

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free-surface fluctuations, large density gradients, shortwave movements and other factors, the vertical motion of the water flow is not negligible compared with the horizontal motion, so the hydrostatic hypothesis is no longer applicable at this time [16,17]. In such case, a more elaborate three-dimensional non-hydrostatic pressure flow model shall be introduced that considers the influence of non-hydrostatic pressure.

The objective of this paper was establishing a non-hydrostatic numerical model for solving three-dimensional free-surface flow based on unstructured grid. Based on the model, the governing equation was spatially discretized by finite difference method and finite volume method, and the pressure Poisson equation was solved by the method of fractional steps to decompose the pressure term into hydrostatic pressure term and hydrodynamic pressure term for separate treatment. Based on the discrete solution and verification of the model, the mathematical model of oil spill at sea was established considering the range of migration and diffusion of oil film under the action of real-time wind field and flow field.

2. Theoretical basis of mathematical model

2.1. Hydrodynamic mathematical model theory

The three-dimensional incompressible Navier–Stokes equations in Cartesian coordinates are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}&=fv-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}(\gamma\frac{\partial u}{\partial x})\\&+\frac{\partial}{\partial y}(\gamma\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\gamma\frac{\partial u}{\partial z})\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial vw}{\partial z}&=-fu-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}(\gamma\frac{\partial v}{\partial x})\\&+\frac{\partial}{\partial y}(\gamma\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\gamma\frac{\partial v}{\partial z})\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial y}+\frac{\partial w^2}{\partial z}&=-g-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\partial}{\partial x}(\gamma\frac{\partial w}{\partial x})\\&+\frac{\partial}{\partial y}(\gamma\frac{\partial w}{\partial y})+\frac{\partial}{\partial z}(\gamma\frac{\partial w}{\partial z})\end{aligned} \tag{4}$$

where u, v, and w are the components of the velocity vector, respectively, along the three coordinate axes x, y, z; ρ is the density of water; g is the acceleration of gravity; P is the pressure; f is the Coriolis coefficient; and γ is the eddy viscosity coefficient.

Integrating the continuity Eq. (1) from bottom $-h(x, y)$ to surface $\eta(x, y, t)$, the water level evolution equation is obtained as follows:

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{\eta}u\mathrm{d}z+\frac{\partial}{\partial y}\int_{-h}^{\eta}v\mathrm{d}z=0 \tag{5}$$

The standard κ-ε turbulence model can be expressed as follows:

$$\frac{\mathrm{D}k}{\mathrm{D}t}-\nabla\left[\frac{\nu_t}{\sigma_k}\nabla k\right]=c_\mu\frac{k^2}{\varepsilon}G-\varepsilon \tag{6}$$

$$\frac{\mathrm{D}\varepsilon}{\mathrm{D}t}-\nabla\left[\frac{\nu_t}{\sigma_\varepsilon}\nabla \varepsilon\right]=c_1\frac{\varepsilon}{k}G-c_2\frac{\varepsilon^2}{k} \tag{7}$$

where c_1 = 1.14, c_2 = 1.92, c_μ = 0.09, σ_k = 1.0, σ_ε = 1.3, and G is the generated term of turbulent kinetic energy and can be expressed as follows:

$$G=\left(\partial u_i/\partial x_j+\partial u_j/\partial x_i\right)\left(\partial u_i/\partial x_j\right) \tag{8}$$

The pressure term P in Eqs. (2)–(4) can be decomposed into hydrostatic pressure term and hydrodynamic pressure term. That is,

$$\begin{aligned}P(x,y,z,t)&=p_a(x,y,t)\\&+g\left[\eta(x,y,t)-z\right]\\&+g\int_z^{\eta}\frac{\rho-\rho_0}{\rho_0}\mathrm{d}z+q(x,y,z,t)\end{aligned} \tag{9}$$

where $p_a(x, y, t)$ is the atmospheric pressure; $g[\eta(x, y, t)–z]$ and $g\int_z^{\eta}\frac{\rho-\rho_0}{\rho_0}\mathrm{d}z$ are, respectively, the positive pressure term and baroclinic term of the hydrostatic pressure; and $q(x, y, z, t)$ is the hydrodynamic pressure term.

2.2. Definite conditions of governing equation

2.2.1. Initial condition

One of the initial conditions is to set the still water or set the uniform flow. The other one is to determine the spatial distribution of the physical quantity in the computational domain by the interpolation method based on model observation data. No matter which method gives initial conditions, there is always an error. Under normal circumstances, the error of the initial value will be quickly decreased with time and thus will not affect the final calculation results.

Because of the fast adjustment of water dynamic (flow field) process, the initial value is generally taken as zero.

$$u(x,y,z,0)=0 \tag{10}$$

$$v(x,y,z,0)=0 \tag{11}$$

$$w(x,y,z,0)=0 \tag{12}$$

$$\eta(x,y,0)=\eta_0(x,y) \tag{13}$$

$$k(x,y,z,0)=k_0(x,y,z) \tag{14}$$

$$\varepsilon(x,y,z,0)=\varepsilon_0(x,y,z) \tag{15}$$

2.2.2. Boundary conditions

For the flow with free surface, the boundary conditions to be considered include free-surface conditions, bottom conditions, inflow boundary conditions, and open boundary conditions.

2.2.2.1. Free-surface boundary condition For turbulent variables, κ and ε are usually given by the following formula:

$$\frac{\partial k}{\partial z}=0,\ \varepsilon=(k\sqrt{c_\mu})^{1.5}/(0.07\kappa h) \tag{16}$$

2.2.2.2. Undersurface boundary condition At the bottom boundary, the speed parallel to the undersurface is found by the law of logarithm:

$$\frac{V_\tau}{V_*}=\frac{1}{\kappa}\log_e C \tag{17}$$

where V_τ is the speed parallel to the undersurface, V_* is the shear velocity, and

$$C=\begin{cases}\dfrac{30.0}{k_s}\Delta y & \text{Rough Undersurface}\\[2ex] \dfrac{9.05V_*}{\nu}\Delta y & \text{Smooth Undersurface}\end{cases} \tag{18}$$

where k_s is the equivalent roughness.

For turbulent variables, κ and ε are usually given by the following formula:

$$k=\frac{V_*^2}{\sqrt{c_\mu}},\ \varepsilon=\frac{|V_*|^3}{\kappa\Delta y} \tag{19}$$

2.2.2.3. Inflow boundary condition For the water level inflow boundary, the measured water level data or the water level value calculated by a wider range of mathematical models are usually used as the control conditions. As to the open channel flow calculation, the upstream side gives the flow rate (velocity) boundary generally. If the given flow velocity (rate) is as a boundary condition, the distributed computing shall be conducted according to the section width and the water depth based on the section width function and the vertical velocity water-depth logarithmic distribution function. If the given water level is the boundary condition, the velocity boundary is usually treated with a normal gradient of zero.

At the inflow boundary, the turbulence variable value is given as follows:

$$u=\text{constant} \tag{20}$$

$$k=0.03u^2,\ \varepsilon=c_\mu\frac{k^{1.5}}{0.09h} \tag{21}$$

2.2.2.4. Outflow boundary condition At the exit boundary, the normal gradient of κ and ε is 0.

2.2.3. Discretization of equation

The three-dimensional mathematical model adopted a planar unstructured grid and a vertical σ hierarchical grid to discretize the model equation by the finite volume method and the finite difference method. The water level gradient term was discretized by semi-implicit finite difference, and the vertical viscosity term was implicitly discretized. The finite volume method was used to discretize the continuity equation. The pressure Poisson equation was solved by the method of fractional steps to decompose the pressure term into the hydrostatic pressure term and the hydrodynamic pressure term for separate treatment and thus to solve the final variable value by pressure correction.

2.3. Oil spill mathematical model theory

2.3.1. Oil spill spread

The spreading process refers to the increase of the area of the oil film due to its own characteristics, and hereby the third-stage theoretical model proposed by Fay [3] was adopted, which is a classical theory describing the spreading process and has been widely used. According to the theory, the spreading of the oil film can be divided into three stages: at the first stage, the gravity and inertial force play the main role; at the second stage, the gravity and viscosity force exert the influence; and at the final stage, the surface tension and viscosity force play the leading role. Assuming that the oil film spreads radioactively when the oil film is approximately circular and linearly spreads when the oil film is long striped (i.e., the length-width ratio is greater than 3), the observation results of the actual oil spill events reveal that the spreading process dominates for the first dozens of hours, and such dominance quickly weakens with the passage of time, and then with the influence of weathering process, the oil film gradually reaches its maximum radius and the spreading process eventually stops. At this time, the oil film area A_f is as follows:

$$A_f = 10^5 \forall^{3/4} \quad (22)$$

where $\forall$ is the volume of the oil film and the spreading stops when the thickness of oil film reduces to $10^{-5}V^{1/4}$ m.

2.3.2. Oil spill diffusion

The diffusion process of oil spill at sea is divided into two parts: horizontal spreading and vertical diffusion.

2.3.2.1. Horizontal spreading process The random walk of oil particles leads the size and shape of oil particle cloud cluster to change over time. For a three-dimensional case, the distance of the random walk can be written as follows:

$$\Delta\alpha = R \cdot \sqrt{6K_\alpha \Delta t} \quad (23)$$

where $\Delta\alpha$ is the turbulent diffusion distance in the α direction (α represents the x, y, or z direction), R is the uniformly distributed random number between –1 and 1, K_α is the turbulent diffusion coefficient in the α direction, and Δt is the time step.

2.3.2.2. Vertical diffusion process The random motion of oil particles in the vertical direction is mainly manifested as follows: after entering the interior of water body under the disturbance of waves, the oil particles do the random walk in the vertical director under the action of turbulence. The distance of the vertical random walk of the oil droplets at a certain depth inside the water body under the action of turbulence can be expressed as follows:

$$\Delta Z = \xi \cdot \sqrt{6K_Z \Delta t} \quad (24)$$

where K_Z is the vertical diffusion coefficient and ε is the uniformly distributed random number between –1 and 1.

2.3.2.3. Treatment of oil particle diffusion boundary conditions During the particles movement, they can reach the seabed or land boundary or float out the sea. The treatment here is that when the oil particles reach the land boundary, the particles are considered to stick to the land and no longer participate in the calculation.

2.3.3. Oil spill drift

The action of the wind field and surface flow field is the main driving force of oil film drift. Under the action of waves, the drift speed of oil film is as follows:

$$u_t' = u_c' + u_w' \quad (25)$$

Among which $u_c' = K_1 u_c$ is the surface flow velocity term; $u_w' = K_2 u_f$ is the wind field term; where K_1 is the correction factor of surface flow velocity, which takes into account the influence of the internal stress, thickness, and other factors of oil film on the surface velocity and can be determined by empirical formula, and generally K_1 is 1.0; u_c is the sea surface velocity; K_2 is the correction factor of wind field term and determined by the empirical formula; and u_f is the real-time wind speed over the sea surface, obtained by interpolating the measured data.

3. Bohai Sea oil spill mathematical model

3.1. Model parameter

Taking the Bohai Sea as the range of the whole mathematical model, the control entrance boundary is from Dalian (38°52′N, 121°41′E) to Yantai (37°33′N, 121°23′E), with the north-south length of 425.5 km, the east-west width of 417.1 km, and the model range of about 8.57 × 10^4 km^2.

The computing domain is divided by the triangular net. The coarse meshes are used in a large area of water, and the local densification method is used for the meshes of Bohai Bay area in order to accurately reflect the outline of the shorelines and buildings. The nested grid technology is utilized to densify the grids gradually, from the largest grid scale of 6,000 m to the smallest grid scale of 20 m, in order to ensure the analog computation accuracy in the model. The grid node number is 26762, and the total number of units is 52,253 for this calculation (see Fig. 1 for detailed information).

For the tide level verification, in order to ensure the correctness of the large-range overall model, the model was validated by the tide level process from 2–17 April, 2005 (15 d) to test the adaptability of the model to spring, moderate, and neap tides. The synchronous tide level data of eight tidal stations which have the hour-by-hour tidal data along the Bohai Sea were used to validate the tide level of the model. The tidal level verification results at Bayuquan area station are shown in Fig. 2. The calculated results are in good agreement with the measured data, which shows that the mathematical model achievement can be used for the prediction studies on the marine oil spill hydrodynamic environment.

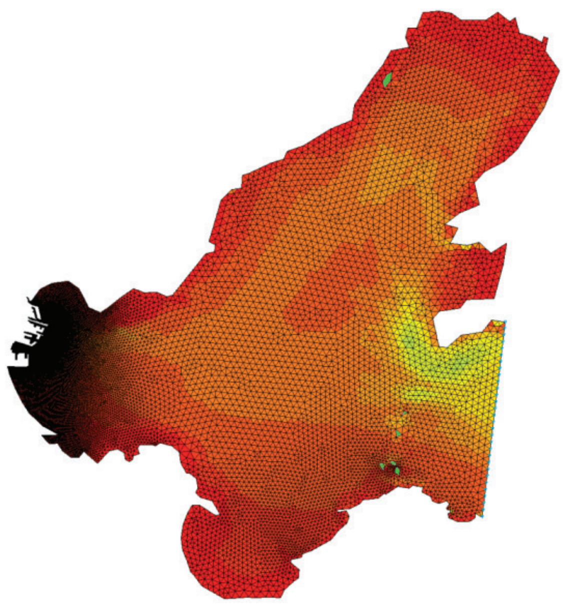

Fig. 1. Mesh generation of 2-d numerical model for tidal current in Bohai Sea.

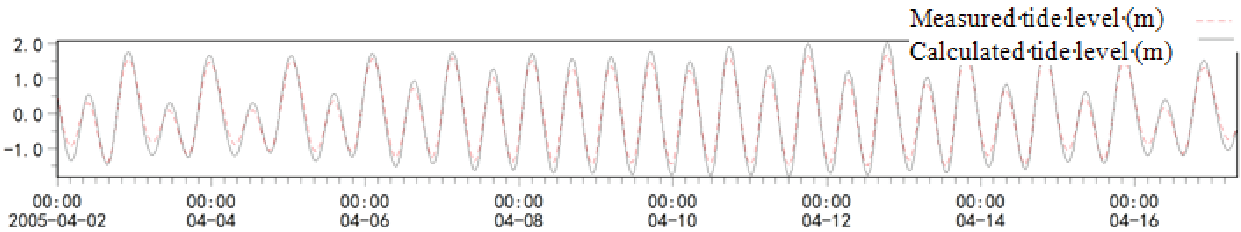

Fig. 2. Whole-process tide level verification of Bayuquan station.

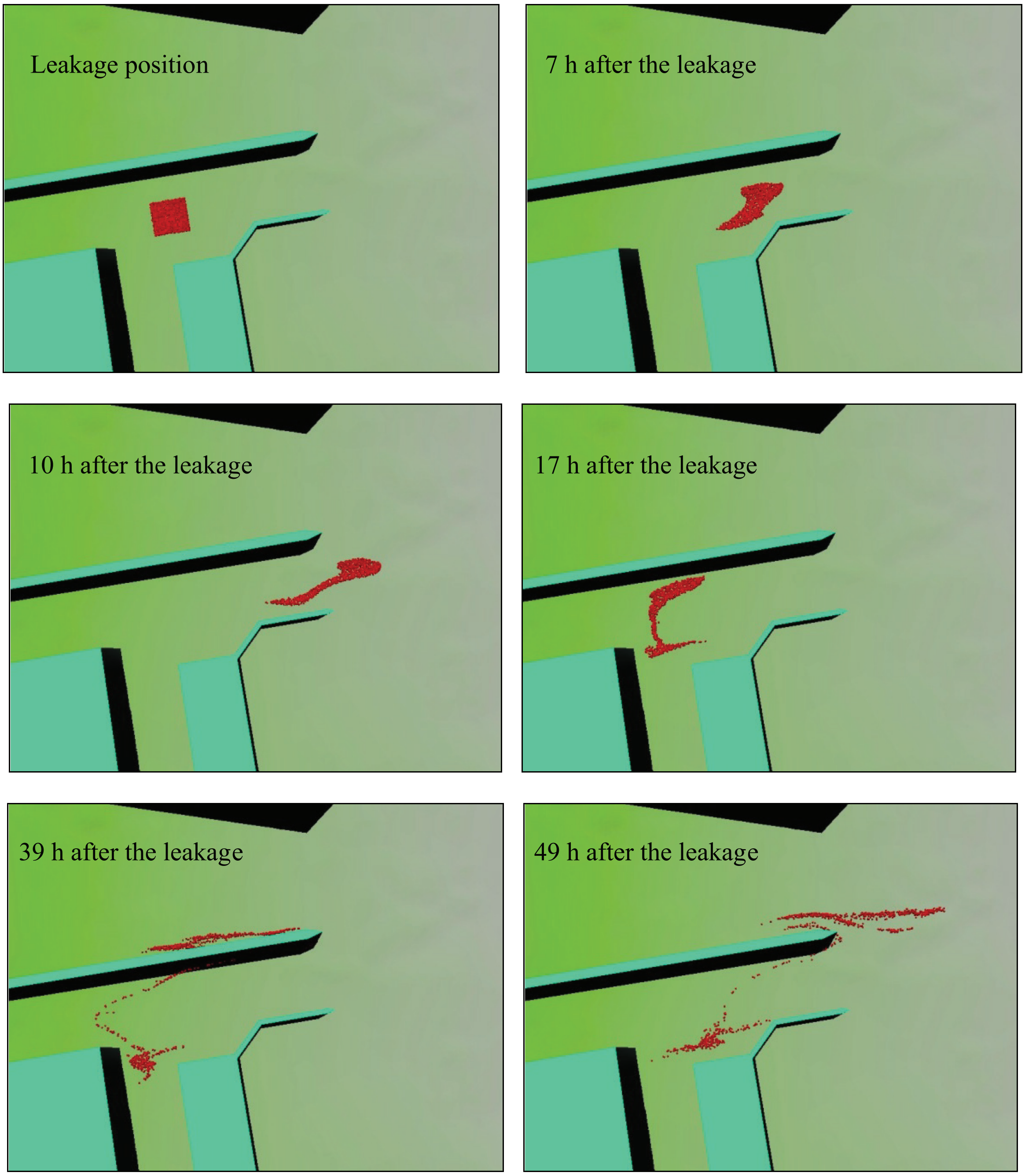

Fig. 3. Range of influence of oil spill with the tidal movement.

3.2. Cognitive test results and analysis

It is assumed that an oil spill accident occurs at the junction of the north exit of the second harbor basin of Southport and the fairway, with 200 tons of oil spilling in 1 h, the leaking oil's specific gravity of 0.88 kg/m^3, and volume of 227,273 m^3. Then the tide shape of the spring tide on 17–18 October, 2011 is selected to be simulated, with the wind speed of 2 m/s and the direction of southeast, and the simulation of the range of convective diffusion within 24 h after the leakage chooses the high flood moment as the leaking start time.

Oil is an insoluble material with a lower specific gravity than sea water. It floats on the surface of the water and is mainly affected by surface currents and winds. The simulation calculation was carried out in a non-hydrostatic three-dimensional manner. The model was vertically divided into three layers, and the surface flow field was obtained through the simulation of the mathematical model. And then based on the surface flow field, the particle method was used to obtain the range of influence of the oil film diffusion under the action of the rising/falling tide currents and winds. Fig. 3 shows the range of oil film diffusion at several times after oil spill. The sea area swept by the oil film during this period was counted. The affected sea area was 18.3 km^2, mainly distributed in the oil spill harbor basin, the entrance, and the northern water areas of the guide levee.

4. Conclusions

Based on the assumption of non-hydrostatic pressure, and the consideration of the migration and diffusion range of oil film under the action of real-time wind field and flow field, a three-dimensional hydrodynamic mathematical model was established.

The three-dimensional hydrodynamic mathematical model under the non-hydrostatic hypothesis is suitable for solving free-surface flow hydrodynamic problems in complex states. The non-hydrostatic hypothesis-based three-dimensional flow model can solve practical engineering problems, which will effectively promote the application of three-dimensional flow calculation in practical engineering and drive the development of three-dimensional mathematical model in the field of water transport engineering.

The model can be used to simulate the diffusion process of oil spills as well as the leakage of harmful substances whose density is close to the oil.

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