Desalination and Water Treatment
www.deswater.com
doi: 10.5004/dwt.2018.22661

122 (2018) 91–99
August

Multi-factor nonlinear time-series ecological modelling for algae bloom forecasting

Li Wang[a], Tianrui Zhang[a], Xuebo Jin[a], Jiping Xu[a], Xiaoyi Wang[a,*], Huiyan Zhang[a], Jiabin Yu[a], Qian Sun[a], Zhiyao Zhao[b], Lei Zheng[b]

[a]*School of Computer and Information Engineering, Beijing Key Laboratory of Big Data Technology for Food Safety, Beijing Technology and Business University, Beijing 100048, China, emails: 2424163121@qq.com (X. Wang), wangli@th.btbu.edu.cn (L. Wang), 634639293@qq.com (T. Zhang), jinxuebo@th.btbu.edu.cn (X. Jin), xujiping@139.com (J. Xu), zhanghuiyan369@126.com (H. Zhang), yujiabin_1984@hotmail.com (J. Yu), sunqian0403@hotmail.com (Q. Sun)*
[b]*College of Water Sciences, Beijing Normal University, Beijing 100875, China, emails: zzy_btbu@163.com (Z. Zhao), zhengleilei@bnu.edu.cn (L. Zheng)*

Received 24 February 2018; Accepted 25 June 2018

ABSTRACT

The current mechanism analysis method of algae bloom fails to take into consideration multi-factor nonlinear time-series characteristics of the ecological dynamics system in water, which leads to the low accuracy of algae bloom forecasting. In this paper, based on monitoring data of algae biomass (chlorophyll a concentration) and nutrient concentration, the nonlinear ecological dynamics model of algae bloom is constructed, which contains algae feeding and nutrient circulation, and model parameter optimization method is put forward by the combination of intelligent evolutionary algorithm and numerical algorithm. On this foundation, the effects of multiple factors time series such as illumination and temperature, which are the main influence factors of algae bloom, are considered into the algae bloom ecological system modelling. By using multi-factor time-series model to describe the variation of multiple influence factors, the algae multi-factor nonlinear time-series ecological dynamics model is constructed. A new method for algae bloom forecasting is put forward by multi-factor nonlinear time-series dynamic analysis. The example of Taihu Lake monitoring data shows that, compared with the current mechanism analysis method of algae bloom, multi-factor nonlinear time-series ecological dynamics model can better reflect dynamic characteristics of the algae bloom influence factors variation with time, and compared with the current forecasting methods, the forecasting results of algae bloom by the new method in this paper are more accurate.

Keywords: Multi-factor; Algae bloom; Time series; Nonlinear ecological modelling; Forecasting

1. Introduction

With the rapid development of industrialization and urbanization, rivers and lakes eutrophication is becoming more and more serious and the outbreak of algae bloom has become a prominent problem [1]. Large-scale algae bloom reduces the utilization efficiency of water resources, which causes serious ecological damage and huge economic losses. At the same time, the production of algae toxin brings great hidden danger to public health [2–4]. Algae bloom has become one of the main problems of water pollution in the world.

Algae bloom forecasting has been a difficult problem in the work of algae bloom pollution prevention and control.

* Corresponding author.

Presented at the 3rd International Conference on Recent Advancements in Chemical, Environmental and Energy Engineering, 15–16 February, Chennai, India, 2018.

The forecasting model of algae bloom in the current research is mainly divided into data-driven model and mechanism-driven model.

Data-driven model includes artificial intelligence model [5–8] and mathematical statistical model [9–12], which digs the inherent law that hides in the system from a large amount of data information. However, the model is limited by the monitoring of the data and it cannot reflect the essence of algae growth. Mechanism-driven model is the ecological dynamics simulation of algae growth process. With the development of water study, mechanism-driven model has been developed from single TP concentration to the whole phosphorus system circulation in water [13–15], from simple nutrient circulation to the exchanging process of nutrients between sediment and water interface [16–18], from the linear dynamics process of algae growth to the nonlinear dynamics process [19–23]. However, algae growth dynamics system is regarded as the time invariant system by most of the current mechanism-driven models, and the model parameters are set to be constant, which neglects the influence of some factors changing with time, such as water temperature, illumination and other factors, on the growth rate and death rate of algae in the actual water. Hence, it is difficult to effectively describe the dynamic characteristics of the time-varying system of algae growth, and to realize the accurate forecasting of algae bloom.

In this article, based on the monitoring data of algae biomass (chlorophyll a concentration), nutrient concentration, water temperature and illumination, the mechanism-driven modelling and data-driven modelling method are combined to construct the nonlinear ecological dynamics model of algae growth, and genetic algorithm and numerical algorithm are used for parameter optimization. A nonlinear time-varying ecological dynamic model for algae growth with multi-factor time-varying parameters is proposed. By multi-variate time-series modelling of these parameters and time-varying nonlinear dynamics systems analysis, the problem of algae blooms forecasting is resolved.

2. Algae growth dynamics modelling and analysis

2.1. Algae growth nonlinear ecological dynamics modelling

Considering the algae feeding behaviour and nutrient circulation characteristics of algae growth, the algae feeding model and nutrient circulation model are expressed by the Lotka–Volterra equation. The feeding of zooplankton to algae is ignored for the toxicity in most species of algae. The biomass of algae is expressed by chlorophyll a concentration and the algae growth dynamics model is established as follows:

$$\begin{cases} \dfrac{\mathrm{d}c_a}{\mathrm{d}t} = GNc_a - Dc_a \\ \dfrac{\mathrm{d}N}{\mathrm{d}t} = N_0 - g_N GNc_a - d_N N \end{cases} \tag{1}$$

Here, c_a denotes the concentration of chlorophyll a and G denotes the growth rate of algae. N denotes the nutrient concentration and D denotes the death rate of algae. N_0 denotes the initial value of nutrient concentration, and g_N denotes the rate of absorption of nutrients by algae. d_N denotes the loss rate of nutrients.

In the actual water body, the growth rate (G) and death rate (D) of algae are determined by the factors such as water temperature (T), illumination (I) and so on. Their relationship is as follows:

$$\begin{aligned} G &= G_T(T)\cdot G_I(I) \\ G_T(T) &= g_{\max}\times 1.066^{T-20} \\ G_I(I) &= \frac{I}{I+k_I} \\ D &= d_{\max}\times 1.08^{T-20} \end{aligned} \tag{2}$$

Here, $G_T(T)$ and $G_I(I)$ denotes the influence function of water temperature and illumination on the growth rate of algae. $g_{\max}$ denotes the maximum growth rate of algae. k_I denotes the half-saturation concentration of illumination. $d_{\max}$ denotes the maximum death rate of algae.

2.2. Model parameter optimization and calibration

The algae growth dynamics model involves many parameters and the determination of parameters has a great influence on the results of model analysis. Parameters of most current algae growth dynamics model are determined by experience, and their applicability is influenced. Moreover, the conventional model parameter optimization method is limited to the monitoring information of the data and fails to provide the global optimal solution. Genetic algorithm is a kind of artificial intelligence algorithm based on natural selection and genetic theory, which is widely used in parameter optimization. However, at present, the parameter calibration of genetic algorithm is mainly based on single variable differential equation, and Eq. (1) is a multi-variate differential equation with two variables, which cannot be directly applied. Hence, this paper optimizes and calibrates the parameters (N_0, g_N, d_N, $g_{\max}$, k_I, $d_{\max}$) in Eq. (1) based on sensor monitoring data of the actual water and combining with genetic algorithm and numerical algorithm. The parameter optimization and calibration process is shown in Fig. 1 and specific steps are as follows:

(1) *Initialization conditions setting*. The number of individuals, the maximum genetic algebra, the number of parameters to be optimized, the generation gap and the threshold of fitness are determined.
(2) *Population initialization*. Multi-parameter cascade floating point coding is adopted. A number of different parameter combinations are randomly generated as the initial population, which constitute the space of the parameter to be chosen.
(3) *Fitness evaluation*. Set up the fitness function

$$F = \sum_{t=1}^{n}\left(c_{at} - c_a(t)\right)^2 + \sum_{t=1}^{n}\left(N_t - N(t)\right)^2 \tag{3}$$

Here, F denotes the fitness value. c_{at} denotes the true value of chlorophyll a at time t and $c_a(t)$ denotes a function value of chlorophyll a at time t. N_t denotes the true

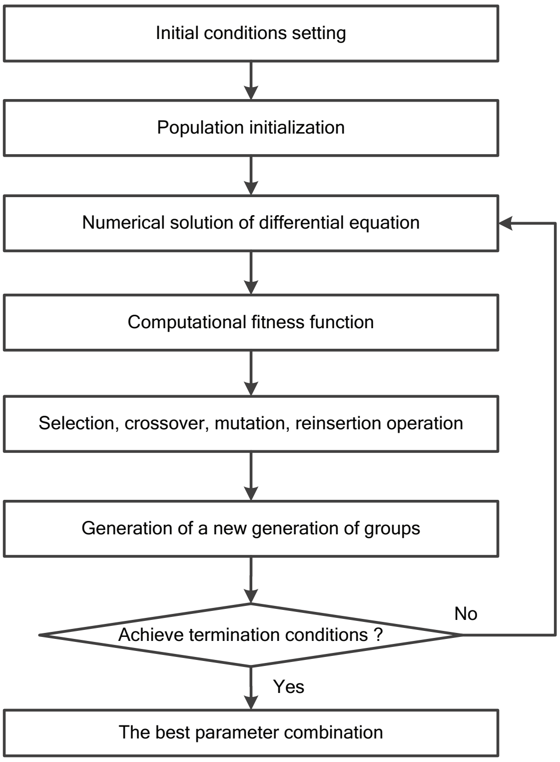

Fig. 1. Parameter optimization and calibration process by genetic algorithm and numerical algorithm.

value of nutrients in time t and $N(t)$ denotes a function of nutrient at time t.

Due to the complex structure of multiple differential equations, analytic solutions of ca (t) and N (t) are difficult to be obtained. Hence, it is necessary to use numerical algorithm. The fourth-order Runge-Kutta numerical algorithm is used in this paper. Its numerical integral expression is as follows:

$$\begin{aligned} c_a(t+1) &= c_a(t) + h \times (k_1 + 2k_2 + 2k_3 + k_4)/6 \\ N(t+1) &= N(t) + h \times (m_1 + 2m_2 + 2m_3 + m_4)/6 \end{aligned} \tag{4}$$

The specific expression of the parameter k_i is as follows:

$$\begin{aligned} k_1 &= f_1\left(c_a(t), N(t)\right) \\ k_2 &= f_1\left(c_a(t) + k_1 \times h/2, N(t) + m_1 \times h/2\right) \\ k_3 &= f_1\left(c_a(t) + k_2 \times h/2, N(t) + m_2 \times h/2\right) \\ k_4 &= f_1\left(c_a(t) + k_3 \times h, N(t) + m_3 \times h\right) \\ f_1(x,y) &= Gyx - Dx \end{aligned} \tag{5}$$

The specific expression of the parameter m_i is as follows:

$$\begin{aligned} m_1 &= f_2\left(c_a(t), N(t)\right) \\ m_2 &= f_2\left(c_a(t) + k_1 \times h/2, N(t) + m_1 \times h/2\right) \\ m_3 &= f_2\left(c_a(t) + k_2 \times h/2, N(t) + m_2 \times h/2\right) \\ m_4 &= f_2\left(c_a(t) + k_3 \times h, N(t) + m_3 \times h\right) \\ f_2(x,y) &= N_0 - g_N Gyx - d_N y \end{aligned} \tag{6}$$

The fitness of each individual is calculated by the fitness function equation (3).

(4) Selection, crossover, mutation, and reinsertion operation.
(5) *Genetic algorithm termination*. Eqs. (3) and (4) step in the formation of the new individual is repeated constantly based on individual fitness value, and the termination condition is that repeated times reach maximum genetic algebra or the fitness function value of parameters combination is less than or equal to threshold. When the termination condition is reached, the minimum fitness function value of the parameter combination is taken as the fitness value of the model and the parameters combination is the optimal parameter combination of the model.

2.3. Nonlinear dynamics analysis of algae growth based on bifurcation theory

Algae bloom is caused by the explosive growth of algae, which is analyzed and explained by the intrinsic nonlinear dynamics of algae growth system based on Eq. (1).

The equilibrium point of algae growth system is solved by Eq. (1).

$$\begin{cases} c_{a1}^* = 0 \\ N_1^* = \dfrac{N_0}{d_N} \end{cases} \qquad \begin{cases} c_{a2}^* = \dfrac{N_0 G - d_N D}{g_N G D} \\ N_2^* = \dfrac{D}{G} \end{cases} \tag{7}$$

The stability of the two equilibriums is analyzed. First of all, Eq. (1) is transformed by the coordinate transformation. Set $\begin{cases} u = c_a - c_a^* \\ v = N - N^* \end{cases}$ and put them into Eq. (1) and the linear part of the matrix is obtained as follows:

$$\text{Jacobin} = \begin{pmatrix} -d_N - g_N G c_a^* & -g_N G N^* \\ G c_a^* & -D + G N^* \end{pmatrix} \tag{8}$$

When the first equilibrium point (c_{a1}^*, N_1^*) is put into the Jacobin matrix, the characteristic value of the Jacobin matrix is $\lambda_{1,2} = \left(-d_N, -D + \dfrac{GN_0}{d_N}\right)$. When $\dfrac{GN_0}{d_N} < D$, the equilibrium point is a stable node and when $\dfrac{GN_0}{d_N} > D$, the equilibrium point is a saddle point by the theory of stability. Hence, small fluctuations cause changes in the state of the system.

When the second equilibrium point (c_{a2}^*, N_2^*) is put into the Jacobin matrix, the c eigenvalue of the Jacobin matrix is $\lambda_{1,2} = \dfrac{-\dfrac{GN_0}{D} \pm \sqrt{\left(\dfrac{GN_0}{D}\right)^2 - 4\left(N_0 G - d_N D\right)}}{2}$. When $N_0G - d_N D > 0$, the equilibrium point is a stable node; when $N_0G - d_N D = 0$, the system occurs critical bifurcation and when $N_0G - d_N D < 0$, the equilibrium point is not stable. The real part of the eigenvalue cannot be zero by analytic equation of Jacobin eigenvalue. Hence, pure imaginary roots cannot appear. In other words, the system does not undergo Hopf bifurcation. When

$$\left(\frac{GN_0}{D}\right)^2 - 4\left(N_0G - d_ND\right) < 0 \tag{9}$$

there is imaginary part in the eigenvalue and the system oscillates. After a period of oscillation, the system approaches to second equilibrium point.

By the above nonlinear dynamic analysis, there is an intersection of the two equilibrium points when $N_0G - d_ND = 0$. When $N_0G - d_ND < 0$, the first equilibrium point is a stable node and when $N_0G - d_ND > 0$, the second equilibrium point is the stable node. Since the chlorophyll a concentration of the first equilibrium point is zero, it is meaningless to study. Hence, it is important to study the second equilibrium points. By the definition of second equilibrium points, it is known that when it is a stable node, the concentration of chlorophyll a is greater than zero, which has physical meaning. When the eigenvalue of the matrix appears imaginary part, the system oscillates. Its physical meaning is that the peak of chlorophyll a concentration and nutrient concentration is alternated with time, which is the behaviour of algae bloom. Hence, when all the parameters of Eq. (1) meet the conditions that the second equilibrium point is stable and the system oscillates, the algae bloom outbreaks.

3. Nonlinear time-varying dynamic analysis of algae growth system and bloom forecasting

3.1. Nonlinear time-varying dynamics modelling of algae growth system

Algae growth dynamics system is defined as the time invariant system in Eq. (1) and the parameters of growth rate (G) and death rate (D) are not changed with time. However, water temperature and illumination factors change with time in the actual water, which lead to algae growth rate (G) and death rate (D) changing with time. They are not constants as defined in Eq. (1). Hence, algae growth dynamics system is a time-varying system and growth rate ($G(t)$) and death rate ($D(t)$) of algae are time-varying parameters in Eq. (1). The algae growth dynamics model of time-varying parameters is constructed as follows:

$$\begin{cases} \dfrac{\mathrm{d}c_a}{\mathrm{d}t} = G(t)Nc_a - D(t)c_a \\ \dfrac{\mathrm{d}N}{\mathrm{d}t} = N_0 - g_NG(t)Nc_a - d_NN \end{cases} \tag{10}$$

Here, the equation of the time-varying growth rate and death rate is as follows.

$$\begin{aligned} &G(t) = G_T(T(t))\cdot G_I(I(t)) \\ &G_T(T(t)) = g_{\max}\times 1.066^{T(t)-20} \\ &G_I(I(t)) = \frac{I(t)}{I(t)+k_I} \\ &D(t) = d_{\max}\times 1.08^{T(t)-20} \end{aligned} \tag{11}$$

Here, $T(t)$ and $I(t)$ are the time-varying water temperature and illumination.

3.2. Influence factors time-series modelling

For the water temperature and illumination changing with time and considering the correlation between the two factors, sensor monitor data of the two influencing factors are bivariate time series.

Time cumulative trend, seasonal variation and environmental random disturbance of the two influencing factors are considered in water. Hence, the bivariate time series is decomposed into trend term, periodic term and random term. They are as follows:

$$\begin{pmatrix} T(t) \\ I(t) \end{pmatrix} = \begin{pmatrix} F_T(t) \\ F_I(t) \end{pmatrix} + \begin{pmatrix} S_T(t) \\ S_I(t) \end{pmatrix} + \begin{pmatrix} R_T(t) \\ R_I(t) \end{pmatrix} \tag{12}$$

Here, $F_T(t)$ and $F_I(t)$ denotes the trend term. $S_T(t)$ and $S_I(t)$ denotes periodic term. $R_T(t)$ and $R_I(t)$ denotes random term.

Trend term is modelled by bivariate regression model:

$$\begin{pmatrix} F_T(t) \\ F_I(t) \end{pmatrix} = \begin{pmatrix} f_T(t,I) \\ f_I(t,T) \end{pmatrix} \tag{13}$$

Here, $f_T(t,I)$ and $f_I(t,T)$ denotes binary regression function, which can be a linear function, logarithmic function, exponential function or power function, etc.

Periodic term is modelled by binary hidden periodicity model:

$$\begin{pmatrix} S_T(t) \\ S_I(t) \end{pmatrix} = \sum_i \begin{pmatrix} \left(a_{Ti}\cos(\omega_{Ti}t) + b_{Ti}\sin(\omega_{Ti}t)\right) \\ \left(a_{Ii}\cos(\omega_{Ii}t) + b_{Ii}\sin(\omega_{Ii}t)\right) \end{pmatrix} \tag{14}$$

Here, a_{Ti}, a_{Ii}, b_{Ti} and b_{Ii} denote amplitude of periodic term. ω_{Ti} and ω_{Ii} denotes angular frequency.

Random term is modelled by binary autoregressive model:

$$\begin{pmatrix} R_T(t) \\ R_I(t) \end{pmatrix} = \sum_j \begin{pmatrix} \varphi_{(T,T)j} & \varphi_{(T,I)j} \\ \varphi_{(I,T)j} & \varphi_{(I,I)j} \end{pmatrix} \begin{pmatrix} R_T(t-j) \\ R_I(t-j) \end{pmatrix} + \begin{pmatrix} \varepsilon_T(t) \\ \varepsilon_I(t) \end{pmatrix} \tag{15}$$

Here, $\phi_{(T,T)j}$, $\phi_{(T,I)j}$, $\phi_{(I,T)j}$, and $\phi_{(I,I)j}$ denote autoregressive coefficient and $\varepsilon_T(t)$ and $\varepsilon_I(t)$ denote white noise of random term.

For considering the correlation between the two influencing factors, parameters in the three models are solved simultaneously. The bivariate time-series model of the two influencing factors is constructed as follows:

$$\begin{aligned} \begin{pmatrix} T(t) \\ I(t) \end{pmatrix} &= \begin{pmatrix} F_T(t) \\ F_I(t) \end{pmatrix} + \begin{pmatrix} S_T(t) \\ S_I(t) \end{pmatrix} + \begin{pmatrix} R_T(t) \\ R_I(t) \end{pmatrix} \\ &= \begin{pmatrix} F_T(t) \\ F_I(t) \end{pmatrix} + \sum_i \begin{pmatrix} \left(a_{Ti}\cos(\omega_{Ti}t) + b_{Ti}\sin(\omega_{Ti}t)\right) \\ \left(a_{Ii}\cos(\omega_{Ii}t) + b_{Ii}\sin(\omega_{Ii}t)\right) \end{pmatrix} \\ &\quad + \sum_j \begin{pmatrix} \varphi_{(T,T)j} & \varphi_{(T,I)j} \\ \varphi_{(I,T)j} & \varphi_{(I,I)j} \end{pmatrix} \begin{pmatrix} T(t-j) \\ I(t-j) \end{pmatrix} + \begin{pmatrix} \varepsilon_T(t) \\ \varepsilon_I(t) \end{pmatrix} \end{aligned} \tag{16}$$

Water temperature $T(t)$ and illumination $I(t)$ is predicted based on Eq. (16). Time-varying algae growth rate and death rate are predicted by putting water temperature and illumination forecasting into Eq. (11).

3.3. Nonlinear time-varying dynamic analysis based on bifurcation theory and bloom forecasting

The equilibrium points of algae growth system with time-varying growth rate $G(t)$ and death rate $D(t)$ are obtained as follows:

$$\begin{cases} c_{a1}^{*}=0 \\ N_1^{*}=\dfrac{N_0}{d_N} \end{cases} \quad \begin{cases} c_{a2}^{*}(t)=\dfrac{N_0 G(t)-d_N D(t)}{g_N G(t) D(t)} \\ N_2^{*}(t)=\dfrac{D(t)}{G(t)} \end{cases} \tag{17}$$

The Jacobin matrix is also changed to

$$\text{Jacobin}=\begin{pmatrix} -d_N-g_N G(t)c_a^{*} & -g_N G(t)N^{*} \\ G(t)c_a^{*} & -D(t)+G(t)N^{*} \end{pmatrix} \tag{18}$$

Similarly, the first equilibrium point is meaningless to study. Matrix eigenvalue of the second equilibrium points ($c_{a2}^{*}(t)$, $N_2^{*}(t)$) is as follows:

$$\lambda_{1,2}(t)=-\frac{G(t)N_0}{2D(t)} \pm \frac{1}{2}\sqrt{\left(\frac{G(t)N_0}{D(t)}\right)^2-4\left(N_0 G(t)-d_N D(t)\right)} \tag{19}$$

When the system oscillates, algae grow explosively. The condition of system oscillation or the condition of algae explosive growth is

$$\left(\frac{G(t)N_0}{D(t)}\right)^2-4\left(N_0 G(t)-d_N D(t)\right)<0 \tag{20}$$

It can be seen that not only the system equilibrium point changes with time but also the stability of the equilibrium point and the condition of the system oscillation are also changed with time for the time-varying system. Hence, conditions of algae explosive growth are changed with time.

When time-varying water temperature ($T(t)$) and illumination ($I(t)$) are predicted by time-series model (Eq. (16)) and satisfy Eq. (17), algae explosive growth conditions are reached and there is a risk of algae bloom outbreak. Then, when chlorophyll a concentration is predicted to reaches the peak value or the prescribed threshold value based on Eqs. (10) and (11), algae bloom outbreaks. In this way, algae bloom forecasting is realized.

4. Instance verification

In order to validate dynamic analysis of algae growth multi-factor time-varying system and forecasting method of algae bloom in this paper, chlorophyll a concentration, nutrients (TN) concentration, water temperature and illumination were monitored by the algae bloom multi-sensor monitor system from 2010 to 2012 in Taihu River Basin of Jiangsu Province.

Structural diagram of the algae bloom multi-sensor monitor system is as shown in Fig. 2, which includes three sections: multi-sensor on-site data sampling module, GPRS network communication module and monitor center module.

A total of 784 d of data were monitored by multi-sensor once every 2 d. In order to facilitate the analysis, the original monitoring data of chlorophyll a concentration are standardized and the abnormal points are removed. It is shown in Fig. 3.

First, the parameters of Eq. (1) are calibrated by the calibration method which is proposed in Section 2.2 of this paper. This result of parameter calibration is as shown in Table 1. The fitting result of chlorophyll a concentration based on parameter calibration of Eq. (1) is shown in Fig. 3. Monitoring data of nutrient (TN) concentration, water temperature and illumination are used for fitting. Fig. 3 shows that the parameter calibration result is ideal.

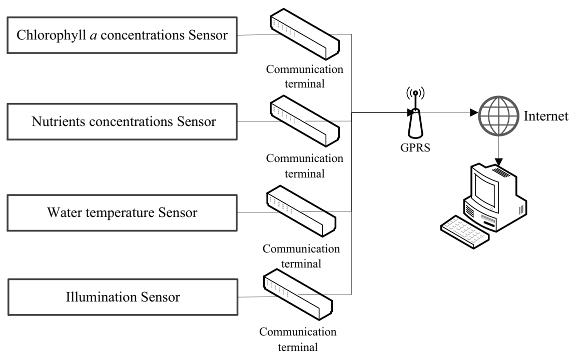

Fig. 2. Structural diagram of algae bloom multi-sensor monitor system.

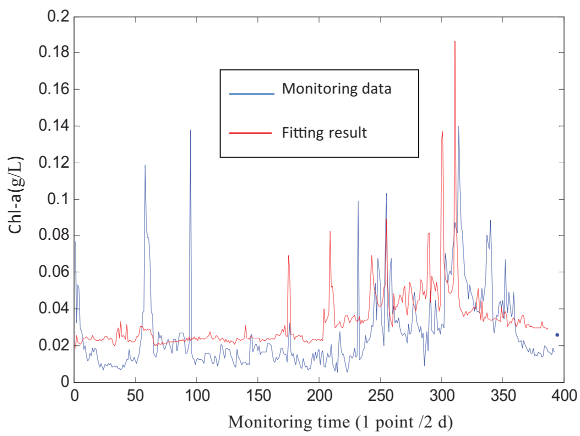

Fig. 3. Monitoring data of chlorophyll a concentration and the fitting data by parameter calibration.

Table 1
Parameters of algae growth dynamics model

Parameter	Range	Calibration result
Maximum growth rate of algae g_{max}	0.01–10	0.088
Illumination saturated concentration k_I	0.01–10	0.36
Maximum death rate of algae d_{max}	0.01–5	0.537
Initial value of nutrient concentration N_0	0–15	0.192
Absorption rate of algae to nutrients g_N	0.01–10	4.2011
Nutrient loss rate d_N	0.01–5	0.02

Second, according to equilibrium point stability of Eq. (1), the critical bifurcation behaviour of chlorophyll a concentration changing with algae growth rate is analyzed. For example the death rate of algae is 0.5, which is shown in Fig. 4. In Fig. 4, the blue line and red curves are equilibrium points 1 and 2 of chlorophyll a concentration, solid line is the stable solution, and dashed line is unstable solution.

It can be seen that system equilibrium state is different at different algae growth rate. When the growth rate is 0.0524, the critical bifurcation occurs in the system.

In order to compare with the time-varying system, time invariant system dynamic behaviour is analyzed first. When algae growth rate (G) is 0.3, 0.1, 0.07, 0.05, system phase trajectory is as shown in Fig. 5 and time history of chlorophyll a concentration and nutrient concentration is as shown in Figs. 6 and 7.

Figs. 5–7 show that when nutrients accumulate to a certain degree, and water temperature, illumination and other factors meet certain conditions, algae (chlorophyll a concentration) began to grow explosively. Then system oscillates.

Large amount of nutrients is consumed by algae growth. When nutrient is consummated not enough to maintain algae growth, chlorophyll a concentration also begin to reduce. Hence outbreak mechanism of algae bloom is explained by Eq. (1), which accords with the actual situation to a certain extent.

When algae growth rate is constant at any time, system oscillation amplitude decrease monotonically with time and it finally approaches the stable equilibrium point at the algae growth rate. However, with the decrease of algae growth rate, duration of system oscillation is prolonged, and amplitude and steady state of the system are decreased.

When algae growth rate is reduced to 0.0526, Eq. (7) cannot be satisfied. There is no oscillation in the system, which tends to stabilize the equilibrium point. In other words, there is no algae bloom, and vice versa. Hence, the behaviour of algae bloom is closely related to algae growth rate.

Considering the actual situation in Taihu River Basin, the dynamic behaviour of the time-varying system is analyzed. For the effects of water temperature and illumination changing with time, growth rate and death rate of algae also changed with time. Hence, oscillation process

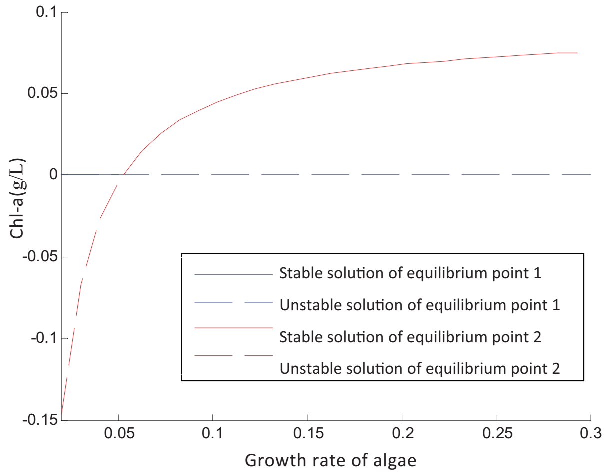

Fig. 4. Bifurcation diagram of chlorophyll a concentration changing with algae growth rate.

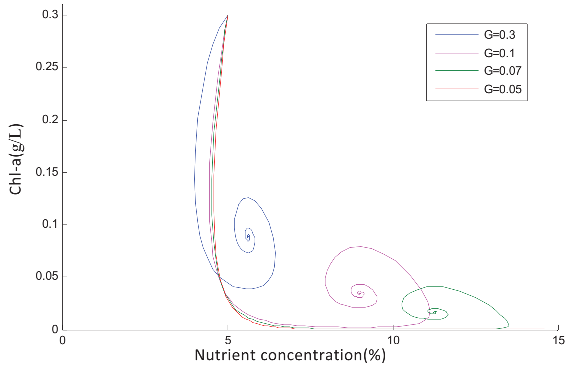

Fig. 5. System phase trajectories at G = 0.3, 0.1, 0.07, 0.05.

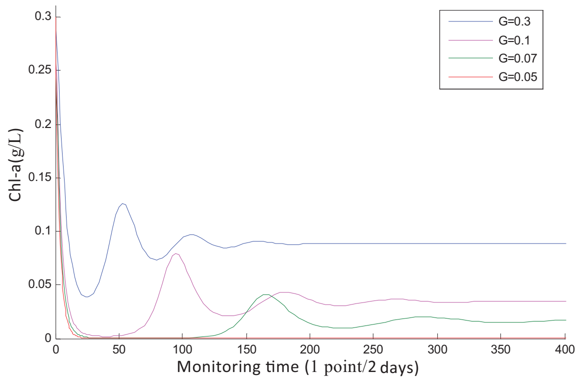

Fig. 6. Time history of chlorophyll a concentration at G = 0.3, 0.1, 0.07, 0.05.

of the time-varying system is more complicated. Based on time-series modelling method proposed in Section 3.2 of this paper, time-series forecasting results of water temperature and illumination are shown as the red curve in Figs. 8 and 9.

Time-series forecasting values of water temperature and illumination are substituted into Eq. (5), which includes growth rate and death rate of algae with time-varying

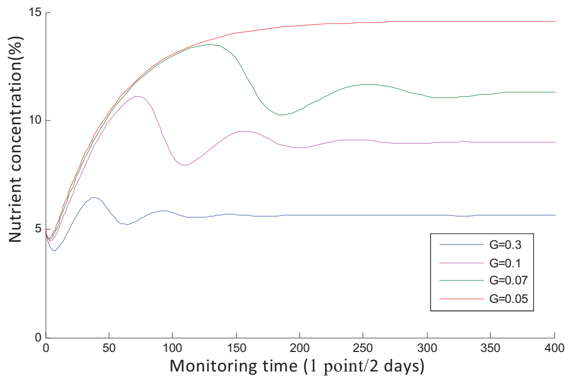

Fig. 7. Time history of nutrient concentration at G = 0.3, 0.1, 0.07, 0.05.

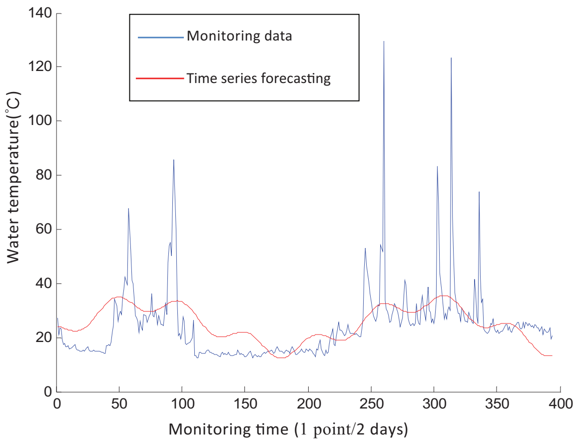

Fig. 8. Water temperature time-series forecasting.

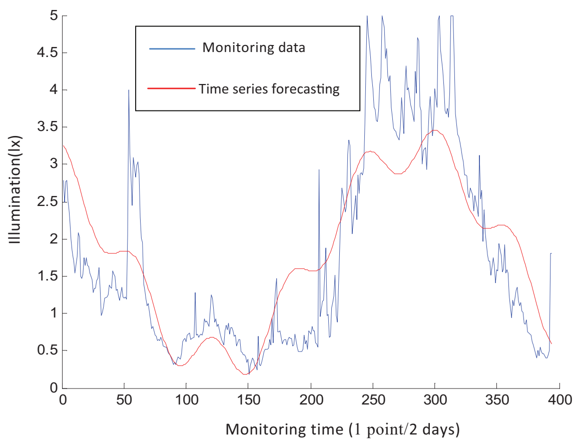

Fig. 9. Illumination time-series forecasting.

parameters. Phase trajectory diagram of algae growth time-varying system is as shown in Fig. 10, and the time history of chlorophyll a concentration and nutrient concentration is as shown in Figs. 11 and 12.

Obviously, based on the water temperature and illumination time-series forecasting, there is a significant deviation at phase trajectory and time history of the time-varying system

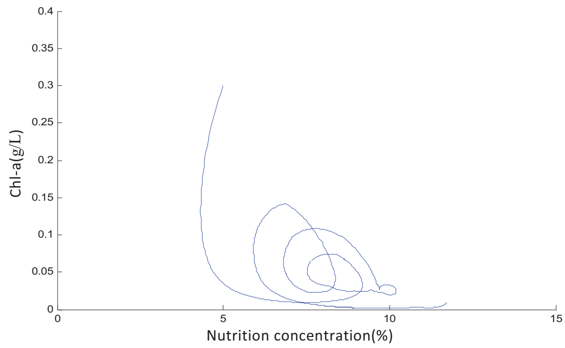

Fig. 10. Phase trajectory of time-varying system.

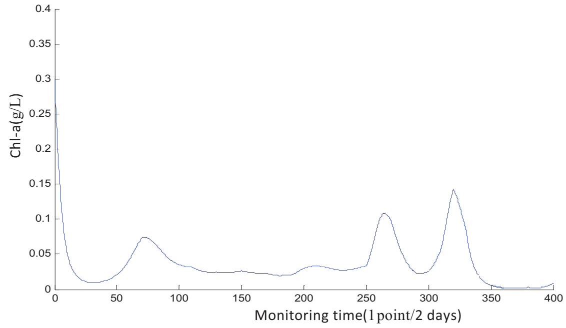

Fig. 11. Time history of chlorophyll a concentration in time-varying system.

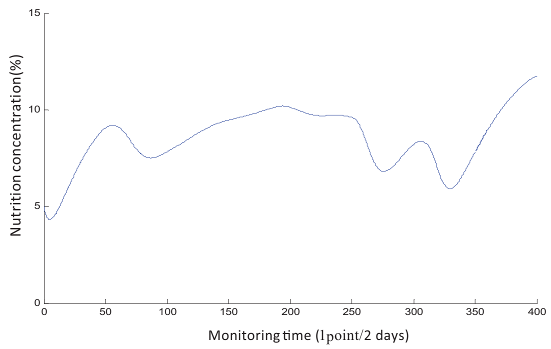

Fig. 12. Time history of nutrient concentration in time-varying system.

compared with the time invariant system. For the influence of time-varying parameters, system equilibrium point changes with time, system oscillation process is no longer close to a fixed point, and amplitude of oscillation is no longer following the law of monotonic decay with time. Hence, system stability is not fixed but changing at any time. Time history (forecasting) of chlorophyll a concentration in algae growth is compared with the monitoring data .which is as shown in Fig. 13.

Evidently, time-varying parameter model can explain why the outbreak of algae bloom is different in every year, and there are characteristics of trend, periodicity and randomness in algae bloom. Compared with the time invariant system model, algae growth process described by

Table 2
Forecasting results of time-varying parameters and algae bloom

Peak value	Growth rate	Death rate	Calculated value of Eq. (14)	Predicted value of c_a	Monitoring value of c_a
1	0.1275	1.1343	–0.0067	0.074	0.118
2	0.1722	1.3870	–0.0207	0.108	0.103
3	0.1664	1.3378	–0.0202	0.141	0.140

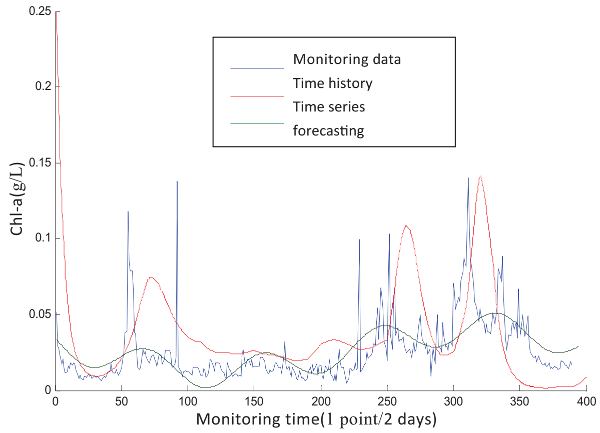

Fig. 13. Comparison of time history of chlorophyll a concentration and monitoring data.

time-varying system model is more consistent with the actual situation obviously.

Algae bloom forecasting results are validated at three peaks of chlorophyll a concentration. It is as shown in Table 2.

Obviously, at three peaks of chlorophyll a concentration, the predicted values of the growth rate and death rate are all satisfied with the condition that Eq. (14) is calculated as a negative value. In other words, they all reached the conditions of algae bloom, and the peak value of chlorophyll a concentration forecasting is basically consistent with the peak value of the actual monitoring value.

5. Conclusions

In this paper, based on the multi-sensor real-time information of chlorophyll a concentration, nutrient concentration, water temperature and illumination, algae growth dynamic characteristics are analyzed based on algae growth dynamic model which contains algae feeding and nutrient circulation model, and genetic algorithms and numerical algorithm is used to optimize the parameters of the model. On this basis, algae growth dynamics model with multi-factor time-varying parameters which are algae growth rate and death rate is put forward to describe that the growth rate and death rate of algae affected by water temperature, illumination and other factors variation with time in algae growth time-varying system. By multi-variate time-series modelling of water temperature, illumination factors, algae growth time-varying system dynamic analysis and algae bloom forecasting are realized. An example of multi-sensor monitoring data of Taihu River Basin in Jiangsu validates that the dynamic characteristics of algae bloom described by multi-factor time-varying system model is more accordant with the actual conditions than the time invariant system model, and the model forecasting results are more accurate than the current algae bloom forecasting methods.

Acknowledgements

This work was financially supported by National Natural Science Foundation of China (61703008, 61673002), Major Project of Beijing Municipal Education Commission science and technology development plans (KZ201510011011), Support Project of High-level Teachers in Beijing Municipal Universities in the Period of 13th Five-year Plan (CIT&TCD201804014). Those supports are gratefully acknowledged.

References

[1] J.D. Brookes, C.C. Carey, Resilience to Blooms, Science, 334 (2011) 46–47.
[2] M. Qu, D.D. Lefebvre, Y. Wang, Y. Qu, D. Zhu, W. Ren, Algal blooms: proactive strategy, Science, 346 (2014) 175–176.
[3] B. Qin, The changing environment of Lake Taihu and its ecosystem responses, J. Freshwater Ecol., 30 (2015) 1–3.
[4] L.G. Vilas, E. Spyrakos, J.M.T. Palenzuela, Y. Pazos, Support vector machine-based method for predicting pseudo-*nitzschia* spp. blooms in coastal waters (Galician *rias*, NW Spain), Progr. Oceanogr., 124 (2014) 66–77.
[5] S. Bruder, M.B. Sebens, L. Tedesco, E. Soyeux, Use of fuzzy logic models for prediction of taste and odor compounds in algal bloom-affected inland water bodies, Environ. Monit. Assess., 186 (2014) 1525–1545.
[6] H. Wang, X. Yan, H. Chen, C. Chen, M. Guo, Chlorophyll-A predicting model based on dynamic neural network, Appl. Artif. Intell., 29 (2015) 962–978.
[7] I. Lou, Z. Xie, W.K. Ung, K.M. Mok, Freshwater algal bloom prediction by extreme learning machine in Macau storage reservoirs, Neural Comput. Appl., 21 (2016) 19–26.
[8] J. Deng, F. Chen, X. Liu, J. Peng, W. Hu, Horizontal migration of algal patches associated with algal blooms in an eutrophic shallow lake, Ecol. Eng., 87 (2016) 185–193.
[9] D.R. Obenour, A.D. Gronewold, C.A. Stow, D. Scavia, Using a Bayesian hierarchical model to improve Lake Erie algae bloom forecasts, Water Resour. Res., 50 (2014) 7847–7860.
[10] Y. Kim, H.S. Shin, J.D. Plummer, A wavelet-based autoregressive fuzzy model for forecasting algal blooms, Environ. Model. Software, 62 (2014) 1–10.
[11] L. Wang, Z. Liu, C. Wu, W. Hua, X. Zhang, Water bloom prediction and factor analysis based on multidimensional time series analysis, CIESC J., 64 (2013) 4649–4655.
[12] S.E. Jorgensen, H. Mejer, M. Friis, Examination of a lake model, Ecol. Model., 4 (1978) 253–278.
[13] M. Scheffer, S. Carpenter, J.A. Foley, C. Folke, B. Walker, Catastrophic shifts in ecosystems, Nature, 413 (2001) 591–596.
[14] C.S. Chen, R. Beardsley, P.J.S. Franks, A 3-D prognostic numerical model study of the Georges Bank ecosystem, Deep Sea Res. Part II, 48 (2001) 419–456.

[15] F. Kong, R. Ma, J. Gao, X. Wu, The theory and practice of prevention, forecast and warning on algae bloom in Lake Taihu, J. Lake Sci., 21 (2009) 314–328.

[16] X. Wang, L. Tang, Z. Liu, L. Cun, J. Xu, X. Zhao, Formation mechanism of cyanobacteria bloom in urban lake reservoir, CIESC J., 63 (2012) 1492–1497.

[17] Z. Liu, Q. Wu, X. Wang, L. Cui, X. Lian, Algae growth modeling based on optimization theory and application to water-bloom prediction, CIESC J., 59 (2008) 1869–1873.

[18] Q. Li, W. Hu, S. Zhai, Integrative Indicator for assessing the alert levels of algal bloom in lakes: lake Taihu as a case study, Environ. Manage., 57 (2016) 237–250.

[19] L. Wang, X. Wang, J. Xu, H. Zhang, J. Yao, X. Jin, C. Liu, Y. Shi, Time-varying nonlinear modeling and analysis of algal bloom dynamics, Nonlinear Dyn., 84 (2016) 371–378.

[20] Y. Tanaka, H. Mano, Functional traits of herbivores and food chain efficiency in a simple aquatic community model, Ecol. Model., 237 (2012) 88–100.

[21] X. Wang, J. Yao, Y. Shi, T. Su, L. Wang, J. Xu, Research on hybrid mechanism modeling of algal bloom formation in urban lakes and reservoirs, Ecol. Model., 332 (2016) 67–73.

[22] L. Wang, X. Wang, X. Jin, J. Xu, H. Zhang, J. Yu, Q. Sun, C. Gao, L. Wang, Analysis of algae growth mechanism and water bloom prediction under the effect of multi-affecting factor, Saudi J. Biol. Sci., 24 (2017) 556–562.

[23] L. Wang, B. Wan, Research on the physics-of-the failure model for corrosion damage accumulation under a multi-level stress profile based on the acceleration factor, Bulg. Chem. Commun., 49 (2017) 204–209.