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# Probabilistic solution to rainwater harvesting system and its impact on climate change

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**ABSTRACT**

The rainwater harvesting system (RHS) is one of the effective low impact development facilities for urban water supply and water management. This study focuses on the probabilistic characteristics of rainfall and proposes a probability model to quantify RHS water supply reliability for urban water demand management and stormwater interception efficiency of RHS for urban water cycle management. Using the master key Fokker–Planck equation, the governing equation of the RHS, composed of a stochastic ordinary differential equation, is transformed into a deterministic partial differential equation with the probability distribution function (PDF) of the normalized water depth of the rainwater tank as the state variable. Using the steady-state PDF of the normalized water depth of RHS, the sensitivity of RHS performance to various parameters is analyzed. Finally, various climate change scenarios are applied to investigate the ability of RHS to offset climate change adverse effects.

*Keywords:* Climate change; Rainwater harvesting system; Reliability; Stormwater

## 1. Introduction

Rapid industrial development and urbanization in Korea have been taking place since the 1960s, and urbanization has been intensified, leading to an increase in impervious area and population density. In this way, the urbanization of the urban area has changed the water cycle system compared to the past [1,2]. Urbanized lands with high impermeability will have a large influx of nonpoint pollutants during rainfall events, affecting surface water and groundwater [3]. As a result, flood damage increases, water resources are difficult to secure, river water quality deteriorates, and groundwater depletion is increasing [4].

In order to solve the problems caused by the water cycle distortion due to urbanization and to manage the water resources sustainably, the urban water cycle management paradigm of low impact development (LID) technique is introduced in USA, Europe, Canada, and Australia, and it is proceeding [5–9]. LID technique is a method to preserve the characteristics of the existing area by infiltrating, filtering, and storing the rainwater to the ground without the direct discharge of the rainwater so as to be similar to the water cycle system in the natural state. It is an eco-friendly rainwater management technique that can sustain natural ecosystems and biological resources including rivers. As such, LID is currently being applied with great interest in Korea. It is actively implemented to urban water cycle management, non-point sources pollutant management, and stormwater management.

Among various LID application technologies, the rainwater harvesting system (RHS) is becoming an increasingly important alternative to water supply in water-scarce areas [10–12]. In addition to the water supply business, there is an advantage that it can be effectively applied to water resources management by reducing or reusing stormwater

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that flows directly into urban impervious surfaces [13,14]. RHS should be carefully installed to preserve the natural water cycle without compromising the functionality of a city. RHS is considered to be a sustainable and efficient means of managing urban water resources [15–17].

Among the cases of RHS that are currently installed worldwide, in Japan, RHS has been used to achieve effects such as flood protection, water conservation, river pollution prevention, and construction pipe system cost reduction in addition to water supply in the city since 1985 [18]. In Australia, RHS has been installed for many years in arid inland areas. Recently, due to drought and climate change, RHS has become an important alternative source of freshwater, and the installation of RHS is increasing [19]. In the United States, attempts have been made to actively use rainwater in California for the first time, and the use of rainwater is increasing in island areas such as Guam [20]. Meanwhile, in Germany, a rainwater management infrastructure has been established to prevent flooding in cities, and a rainwater storage facility for groundwater reclamation is also installed and managed. Unlike other countries, most German cities use groundwater as their source of water, making it one of the most active countries for rainwater use [21].

As the interest in reuse of water is rapidly increasing, there is an increasing tendency to promote the reuse of water and to utilize water resources efficiently, and therefore, many related studies have been conducted [22–24]. Ghisi [25] assessed the actual water availability, estimated the potential for potable water savings, and discussed water availability indicators that demonstrate the benefits of using rainwater. In addition, stochastic rainfall models for rainwater use assessment in South Africa with low water access rates have been developed [26]. Basinger et al. [27] introduced the Storage and Reliability Estimation Tool (SARET), which evaluates the reliability of RHS. Guo and Guo [28] proposed a probability model to quantify the water supply reliability and stormwater capture ratio of RHS, and tried to adjust the size of RHS using the probability model and to evaluate its performance [29]. The RHS is evaluated using various models, and studies such as RHS design, capacity, installation efficiency and RHS optimal design capacity are being actively conducted [30–32].

In Korea, there is a statute for the installation of RHS and related researches are being carried out. Choi et al. [16] established detailed procedures for the design of rainwater use facilities using SARET to assess the reliability of RHS and to quantify the reduction efficiency of stormwater and annual tap water use. In addition, Keem et al. [14] presented a method for estimating model parameters based on the data available in Korea to increase the domestic applicability of the reliability assessment model proposed by Choi et al. [16] and analyzed the annual average and seasonal reliability of RHS. Hydrological evaluation of RHS has also been carried out through long-term continuous runoff analysis [33,34]. Based on the economic assessment of the introduction of RHS, studies such as estimating the optimal design capacity have also been conducted [35–39]. However, researches that analyze the behavior of RHS probabilistically are rare, and comprehensive and systematic studies are still insufficient compared to studies of developed countries in the RHS field.

Therefore, this study focuses on the probabilistic characteristics of rainfall and suggests a probability model considering the inflow and loss of RHS. Using the derived probability model, the reliability for water supply and the stormwater capture efficiency for water resource management are quantified. The formula for estimating the water supply reliability and the stormwater interception ratio with respect to RHS parameters is also proposed. Using the proposed model, the future water supply reliability of RHS and the capability of stormwater capture under various climate change scenarios are analyzed to investigate the applicability of RHS to offset the adverse effect of climate change.

## 2. Material and methods

### 2.1. Data

Rainfall observation data were obtained from the Korea Meteorological Administration Automated Synoptic Observing System (ASOS) data. The daily ASOS meteorological data are available from the KMA website (http://data.kma.go.kr). Rainfall data from six major Korean sites (Busan, Daegu, Daejeon, Gwangju, Incheon, and Seoul) were used. The data period is 40 y from 1979 to 2018.

In this study, dynamically down-scaled present and future climate data (KOR-11) were used with a horizontal resolution of 12.5 km in the East Asia region including the Korean Peninsula. Future climate change scenarios in KOR11 were applied to representative concentration pathways (RCP) 4.5 and 8.5, and two global climate models (GCMs) including MPI-ESM-LR (Max Plank Institute Earth System Model-Low Resolution) and HadGEM2-AO (Hadley Center Global Environmental Model version 2 coupled with the atmosphere-ocean) and four regional climate models (RCMs) (Mesoscale Model version 5 (MM5), regional climate model version 4 (RegCM4), regional spectral model (RSM), weather research and forecasting (WRF)) were used. Therefore, a total of 16 future ensembles were used.

### 2.2. Stochastic model for dynamic water balance in RHS

The characteristics of rainfall play an important role in RHS, and reflecting the water remaining in the existing RHS storage and the newly inflow water will be key to model dynamic water balance in RHS. In this study, a stochastic model for dynamic water balance in RHS was derived based on a simple storage equation as follows:

$$\frac{ds}{dt} = \eta(s,t) = \frac{1}{W_c}\left[ I(R,s) - L(s) \right] \quad (1)$$

where $W_c$ is the capacity of the RHS storage (mm), $I$ (mm/d) is the amount of rainwater supplied to the RHS storage from the rainfall $R$, and $s$ is the ratio of the water remaining in the RHS storage, that is, the normalized water depth. The loss rate $L$ (mm/d) is the function related to the amount of demand used for water supply.

The exponential probability distribution function (PDF) for daily rainfall $R$ (mm/d) is shown in Eq. (2) and Fig. 1, respectively.

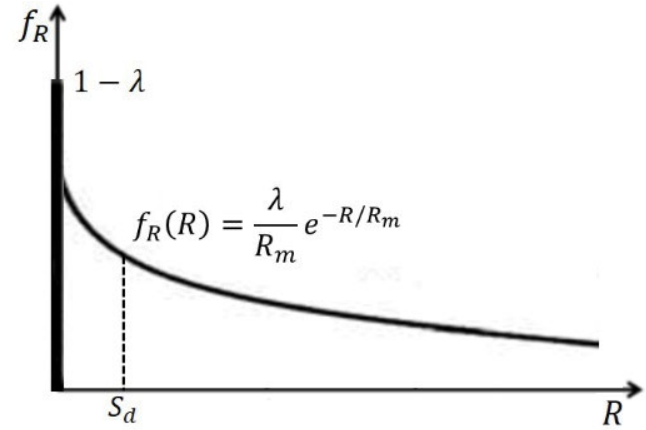


Fig. 1. The probability distribution function of rainfall depth.

$$f_R(R) = \frac{\lambda}{R_m} e^{-R/R_m} \tag{2}$$

where $\lambda$ is the probability that rainfall will occur on a day (that is, the probability of rainfall occurrence in a day), and the probability that rainfall will not occur on a day ($P_{R_0}$) is $1-\lambda$. In Fig. 1, $R_m$ (mm/d) means the mean value of the rainfall depth on the rainy day, and $S_d$ is the incipient loss [40].

The contribution drainage area (mainly roof) is assumed to be impermeable, and the outflow $Q$ (mm/d) refers to the stormwater depth occurring in the contributing area of RHS. If the outflow is smaller than the surface depression depth $S_d$ (mm/d), the outflow does not occur, and the outflow occurs only when outflow is larger than $S_d$, which is expressed by Eq. (3). Note that not all of the generated $Q$ enters RHS storage but only the amount that can be received by the capacity of RHS enters and the remaining is overflowed [41].

$$\begin{aligned} Q &= 0, && \text{for} \quad R \le S_d \\ &= \phi^{-1}(R - S_d), && \text{for} \quad R > S_d \end{aligned} \tag{3}$$

where $\phi$ means the ratio ($A_R/A_d$) of the bottom area of the RHS storage ($A_R$) to the catchment area ($A_d$), and the PDF of the outflow $Q$ can be expressed by Eq. (4) through the PDF of the rainfall $R$.

$$f_Q(Q) = \frac{\lambda}{R_m} e^{-(Q+S_d)/R_m} \tag{4}$$

In this case, when the probability that the outflow will occur on a day is defined by $\lambda'$, the probability that the outflow does not occur, that is, the probability $(1-\lambda')$ that $Q = 0$ is equal to the sum of the probability of rainlessness and the probability that the rainfall does not exceed the incipient loss $S_d$, and can be expressed as follows:

$$\begin{aligned} P_Q(0) &= \text{Prob}[Q = 0] \\ &= \text{Prob}[R = 0] + \text{Prob}[R \le S_d] \\ &= 1 - \lambda' \end{aligned} \tag{5}$$

where the probability of outflow $\lambda'$ is $\lambda e^{-S_d/R_m}$.

If an outflow occurs, an inflow into RHS occurs, in which the inflow is allowed only as much as the remaining space of RHS. Therefore, it is important to consider the amount of water currently remaining in RHS. If the free space remaining in the RHS can accommodate $Q$, the inflow $I$ entering RHS will be equal to $Q$, but if the remaining space cannot accommodate all $Q$, only the remaining space is allowed. This can be expressed as follows:

$$\begin{aligned} I &= 0, && \text{for} \quad R < S_d \\ &= \phi^{-1}(R - S_d), && \text{for } \phi^{-1}(R - S_d) < F(s) \\ &= F(s), && \text{for } \phi^{-1}(R - S_d) > F(s) \end{aligned} \tag{6}$$

where the remaining space of RHS is denoted by $F(s)$ and can be expressed as follows:

$$F(s) = (1-s)\frac{W_c}{\Delta t} \tag{7}$$

Note that the probability $P[I = 0]$ that there is no inflow to RHS on a day, such as the probability that the outflow in Eq. (5) will not occur, is as follows:

$$P[I = 0] = P[R = 0] + P[R \le S_d] = 1 - \lambda' \tag{8}$$

The probability $P_F$ that $F(s)$ will inflow to RHS on a day, that is, the probability of $I = F(s)$, can be expressed as follows:

$$\begin{aligned} P[I = \mathrm{F}(s)] &= P_{F(s)} \\ &= P[S_d + \phi F(s) < R] \\ &= \lambda' G(s) \end{aligned} \tag{9}$$

where $G(s)$ is expressed as follows:

$$G(s) = e^{-\phi F(s)/R_m} \tag{10}$$

The PDF for the remaining inflows except for both extreme cases ($I = 0$ and $I = F(s)$) can be derived as follows using the same relation as $f_I\, dI = f_R\, dR$:

$$\begin{aligned} f_I &= f_R(R) \cdot \frac{dR}{dI} \\ &= \frac{\lambda'}{r_m} e^{-I/r_m} \end{aligned} \tag{11}$$

where $r_m$ (mm/d) is introduced to simplify the equation as follows:

$$r_m = \frac{R_m}{\phi} \tag{12}$$

As a result, the PDF for inflow to RHS can be presented as shown in Fig. 2.

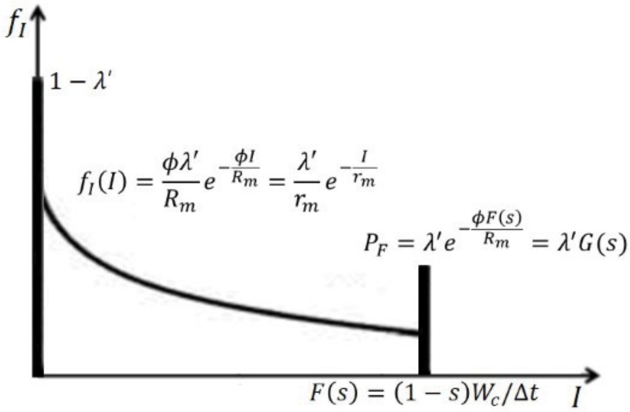


Fig. 2. The probability distribution function of inflow.

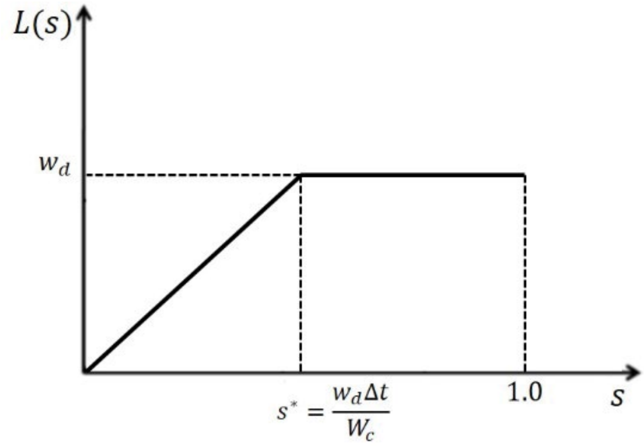


Fig. 3. Loss of function.

Rainwater stored in RHS can be used for water supply, and the amount of water used is the loss function $L$ of RHS. The loss function $L$ is defined in two cases (Fig. 3):

$$L(s) = \frac{w_d}{s^*}s = \frac{W_c}{\Delta t}s, \quad \text{for} \quad 0 < s < s^* \qquad (13)$$
$$= w_d, \quad \text{for} \quad s^* < s \le 1$$

where $w_d$ is the predefined water supply-demand (mm/d). If enough rainwater is stored in RHS, a predefined $w_d$ is supplied, but otherwise only the stored water is supplied. Hence, the threshold value $s$* of rainwater stored in RHS can be expressed as follows:

$$s^* = \frac{w_d \Delta t}{W_c} \qquad (14)$$

Based on Kavvas [42], the governing Eq. (1) of RHS can be transformed into the master key Fokker–Planck equation as follows:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial s}\left[\left(\langle\eta\rangle - \int_0^t \operatorname{cov}\left[\eta_t ; \frac{\partial \eta}{\partial s}|_{t-\tau}\right] d\tau\right) p\right] + \frac{\partial}{\partial s}\left[\left(\int_0^t \operatorname{cov}\left[\eta_t ; \eta_{t-\tau}\right] d\tau\right) \frac{\partial p}{\partial s}\right] \qquad (15)$$

where $p(s,t)$ is the state variable of Eq. (15) and is the PDF of the normalized water depth of RHS. The Eq. (15) can be fully expressed as follows:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial s}\left[\left(\frac{\lambda' r_m}{W_c}(1-G) - \frac{s}{\Delta t} - \frac{\theta_1 \lambda' G}{2W_c \Delta t}\left(\lambda' r_m (1-G) - F\right)\right) p(s,t)\right] + \frac{\partial}{\partial s}\left[\left(\frac{\theta \lambda' r_m^2}{W_c^2}\left(1 - G - \frac{FG}{r_m}\right) - \frac{\theta(\lambda')^2 r_m^2}{2W_c^2}(1-G)^2\right) \times \frac{\partial p(s,t)}{\partial s}\right], \text{ for } s < s^*$$
$$= -\frac{\partial}{\partial s}\left[\left(\frac{\lambda' r_m}{W_c}(1-G) - \frac{w_d}{W_c} - \frac{\theta_1 \lambda' G}{2W_c \Delta t}\left(\lambda' r_m (1-G) - F\right)\right) p(s,t)\right] + \frac{\partial}{\partial s}\left[\left(\frac{\theta \lambda' r_m^2}{W_c^2}\left(1 - G - \frac{FG}{r_m}\right) - \frac{\theta(\lambda')^2 r_m^2}{2W_c^2}(1-G)^2\right) \times \frac{\partial p(s,t)}{\partial s}\right], \text{ for } s \geq s^* \qquad (16)$$

where $\theta_1$ is the scale of fluctuation of daily $I_t$ and $\partial I/\partial s$ time series, and $\theta$ is the scale of fluctuation of daily $\eta_t$ time series, respectively.

$$\theta_1 = 2\int_0^\infty \rho_1(\tau) d\tau = |\rho_1(0)| \Delta t \qquad (17)$$

$$\theta = 2\int_0^\infty \rho_1(\tau) d\tau \qquad (18)$$

where $\rho_1(\tau)$ is the cross-correlation function of daily $I_t$ and $\partial I/\partial s$ time series at lag-$\tau$, and $\rho(\tau)$ is the auto-correlation function of daily $\eta_t$ time series at lag-$\tau$, respectively.

In order to analyze numerically the stochastic model derived above, Eqs. (15) or (16) can be expressed more simply as follows:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial s}\left[Ap\right] + \frac{\partial}{\partial s}\left[D\frac{\partial p}{\partial s}\right] \qquad (19)$$

where $A(s)$ and $D(s)$ are called respectively the advection and dispersion coefficients, as the form of Eq. (19) closely resembles the advection-dispersion equation. Eq. (19) is basically a continuity equation and the state variable of Eq. (19) is the probability density.

### 2.3. Steady-state PDF for a normalized water depth of RHS

According to Chang and Cooper [43], one can finally obtain the steady-state PDF of the normalized water depth in RHS under stochastic rainfall forcing and RHS and catchment parameters as follows:

$$p(s) = N_0 \exp\left[\int_0^s \frac{A(\xi)}{D(\xi)} d\xi\right] \qquad (20)$$

where $N_0$ can be expressed as a constant of integration or a normalization constant as follows:

$$N_0 = \int_0^1 p(s) ds = 1 \qquad (21)$$

Note that the steady-state PDF of the normalized water depth in RHS is expressed as a function of rainfall characteristics such as rainfall frequency, average rainfall in a rainy day, and scale of fluctuation of daily rainfall time series, and RHS properties such as storage capacity, water demand, RHS area, contributing area, and incipient loss.

The state variable $p(s,t)$ of Eq. (19) is strictly a probability density. However, the normalized water depth in RHS has a probability mass at $s$ = 0 and 1. Therefore, it is necessary to consider the probability mass separately. We define $p_0$ as the probability mass that $s$ is zero today, no matter how much water is left in RHS the previous day. Since the occurrence probability of the outflow $Q$ is $\lambda'$, the outflow $Q$ occurs on average once every $1/\lambda'$–$d$. Assuming that the occurrence of the runoff event follows the Poisson distribution, the PDF of the time $T$(d) between runoff events can be described as follows:

$$f\left(T\right)=\lambda' e^{-\lambda'/T} \tag{22}$$

If the time between runoff events is greater than $T_a = W_c/w_d$, then the amount of rainwater remaining in RHS is unconditionally zero. Hence, the probability of mass $p_0$ can be written as follows:

$$p_0 = P[T > T_a] = e^{-\lambda' T_a} \tag{23}$$

Similarly, we can define $p_1$ as the probability mass that $s$ is 1 today, no matter how much water is left in RHS the previous day. The probability mass $p_1$ can be estimated to be the probability of rainfall exceeding the critical rainfall depth that satisfies $W_c + w_d\,\Delta t = \phi^{-1}(R_c - S_d)\Delta t$. The critical rainfall depth is as follows:

$$R_c = S_d + \frac{\phi}{\Delta t}\left(W_c + w_d \Delta t\right) \tag{24}$$

Hence, the probability of mass $p_1$ can be written as follows:

$$p_1 = P\left[R > R_c\right] = \lambda e^{-R_c/R_m} \tag{25}$$

After determining above all parameters, the steady-state PDF of the normalized water depth in RHS can be obtained numerically.

### 2.4. Water supply reliability and stormwater interception ratio

What we most want to know from the RHS installation is how much water we can supply and how much stormwater can be captured. Using the steady-state PDF of the normalized water depth in RHS, the reliability of how much water can be supplied and how much stormwater can be captured can be derived. The water supply reliability $R_e$ can be expressed as follows:

$$R_e = \frac{E\left[L\right]}{E\left[D\right]} = \frac{\int_0^1 L\left(s\right)p\left(s\right)ds}{w_d} \tag{26}$$

where $E[D]$ is the total amount of water desired to be supplied from RHS, and $E[L]$ is the total amount of actual water supplied from RHS.

Meanwhile, the stormwater interception ratio $R_r$ is can be written as follows:

$$R_r = \frac{E\left[L\right]}{E\left[Q\right]} = \frac{\int_0^1 L\left(s\right)p\left(s\right)ds}{\lambda \dfrac{R_m}{\phi} e^{-S_d/R_m}} \tag{27}$$

where $E[Q]$ means the amount of outflow from the contributing area, and stormwater interception ratio can be expressed as a function of supply and outflow.

## 3. Results

### 3.1. Stochastic model verification

In this section, the results of numerical models and probability models are compared with each other to examine the adequacy of probability models. Water supply reliability and stormwater interception ratio were calculated every 5 y using rainfall data from March to November (Figs. 4 and 5). The water supply reliability shows a relatively similar result, although the two models are not perfectly consistent with each other. Stormwater interception ratios are in good agreement with the results of the two models.

To examine the adequacy of the probabilistic model in detail, we used the data from the Busan site to explore the agreement between the two model results for the change in the RHS parameters. Fig. 6 shows the comparison of water supply reliability and stormwater interception ratio for various values of RHS storage capacity, demand, and catchment area. Parameters other than the parameters that changed for the probability model verification were fixed at the default values.

In Fig. 6, the dotted line represents the numerical model and the solid line represents the probability model. The blue line shows the result of water supply reliability and the black line shows the result of the stormwater interception ratio. As the capacity of storage capacity increases, more rainfall can flow into RHS. As a result, the reliability of the supply of demand is gradually increased. As storage capacity increases, stormwater interception ratios also increase with water supply reliability because more rainwater is converted to supply (Fig. 6a). Increasing water demand with fixed storage capacity reduces water supply reliability but improves stormwater interception ratios due to the conversion of rainwater supply (Fig. 6b). Increasing the catchment area leads to an increase in RHS inflow, which improves water supply reliability, whereas increasing RHS inflow adversely affects stormwater interception ratios (Fig. 6c). The results of Fig. 6 show that the proposed probabilistic model reproduces the numerical model results very well for various situations.

### 3.2. Further analysis of stochastic RHS model

Fig. 7 shows the cumulative distribution functions (CDFs) of normalized water depth in RHS for various parameters. As the capacity of the RHS decreases, the rainwater in

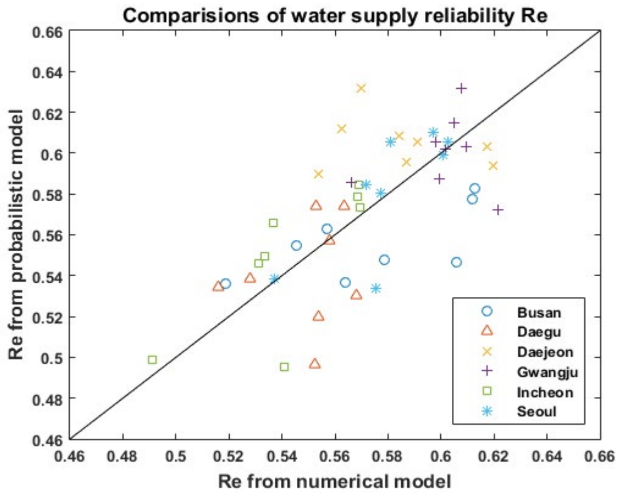


Fig. 4. Comparison of water supply reliability $R_e$ in six sites.

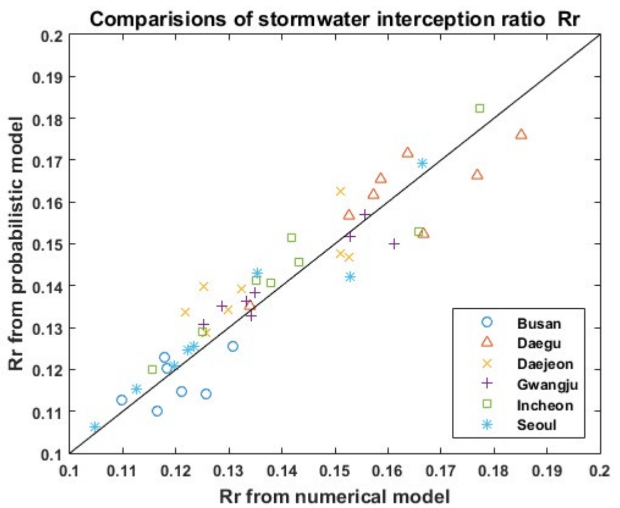


Fig. 5. Comparisons of stormwater interception ratio $R_r$ in six sites.

the storage depletes faster, leading to the formation of CDFs as shown in Fig. 7a. If the required water demand increases, the probability of depletion of the remaining water in the RHS will increase, resulting in CDFs shaped like Fig. 7b. In light of the similarity of CDF change patterns in Figs. 7a and b, the capacity of RHS and the demand for water from RHS have a similar effect on the condition of normalized water depth in RHS. However, the change in CDF shape of the normalized water depth in RHS with respect to the change of the catchment area is different from the two cases. It can be seen that the change of catchment area affects the probability that the stored rainwater is sufficient more than the probability of running out of stored rainwater (Fig. 7c).

Fig. 8 shows the average of the normalized water depth in RHS and the probability when the normalized water depth is 0 over the range of various parameters. As the RHS capacity increases, more rainwater can be stored, so the average of normalized water depth in RHS gradually increases, whereas the probability of zeroing normalized water depth in RHS gradually decreases (Fig. 8a). As the water demand increases, the stored rainwater is depleted at a faster rate, so the average of the normalized water depth in RHS decreases, and the probability of normalized water depth in RHS being zero moves in the increasing direction (Fig. 8b). As the catchment area increases, the inflow of RHS increases, so the average of normalized water depth in RHS increases gradually, and the probability when normalized water depth in RHS is zero decreases (Fig. 8c). However, it is worth noting that the catchment area has a relatively small effect on the behavior of normalized water depth in RHS compared to the capacity and water demand of RHS. Therefore, it can be seen that the normalized water depth in RHS is mainly determined by the RHS capacity and water demand rather than the catchment area.

Figs. 9 and 10 show the water supply reliability and stormwater interception ratio for various ranges of parameters calculated using the proposed probabilistic model. Fig. 9 shows the sensitivity of water supply reliability to RHS parameters. Increasing RHS capacity, decreasing water demand, and increasing catchment area can increase the reliability of water supply from RHS. Fig. 10 shows the sensitivity of the stormwater interception ratio to RHS parameters. Increasing RHS capacity, increasing water demand and decreasing catchment areas have been shown to increase the efficiency of intercepting stormwater. However, the catchment area was found to be less sensitive to the performance of the RHS compared to the other two parameters, and it can be recognized that the capacity and water demand of the RHS mainly determine the performance of the RHS. In general, when RHS is applied to an existing building, the catchment area is often determined prior to the design of the RHS. Therefore, it would be reasonable to determine the capacity and water demand of the RHS to reflect the required water supply reliability and the required stormwater interception ratio. Alternatively, after determining the demand and capacity of the RHS corresponding to the required reliability, it may be possible to evaluate the ability to capture stormwater.

## 4. Applications

### 4.1. Design formula of water supply reliability and stormwater interception ratio

In this section, using the rainfall characteristics of the Busan site, a design formula for water supply reliability ($R_e$) and stormwater interception ratio ($R_r$) was derived using a combination of three RHS parameters ($W_c$, $w_d$, and $\phi$) as follows:

$$R_e = 0.00027286W_c - 0.0014282w_d - 3.5366\phi + 0.6215 \tag{28}$$

$$R_r = 0.00007924W_c + 0.0002707w_d + 27.177\phi - 0.1621 \tag{29}$$

The design formula has the form of a multiple regression model, and the regression coefficients are estimated using the least-squares method to best reproduce the outputs of the model proposed in this study. In order to evaluate the

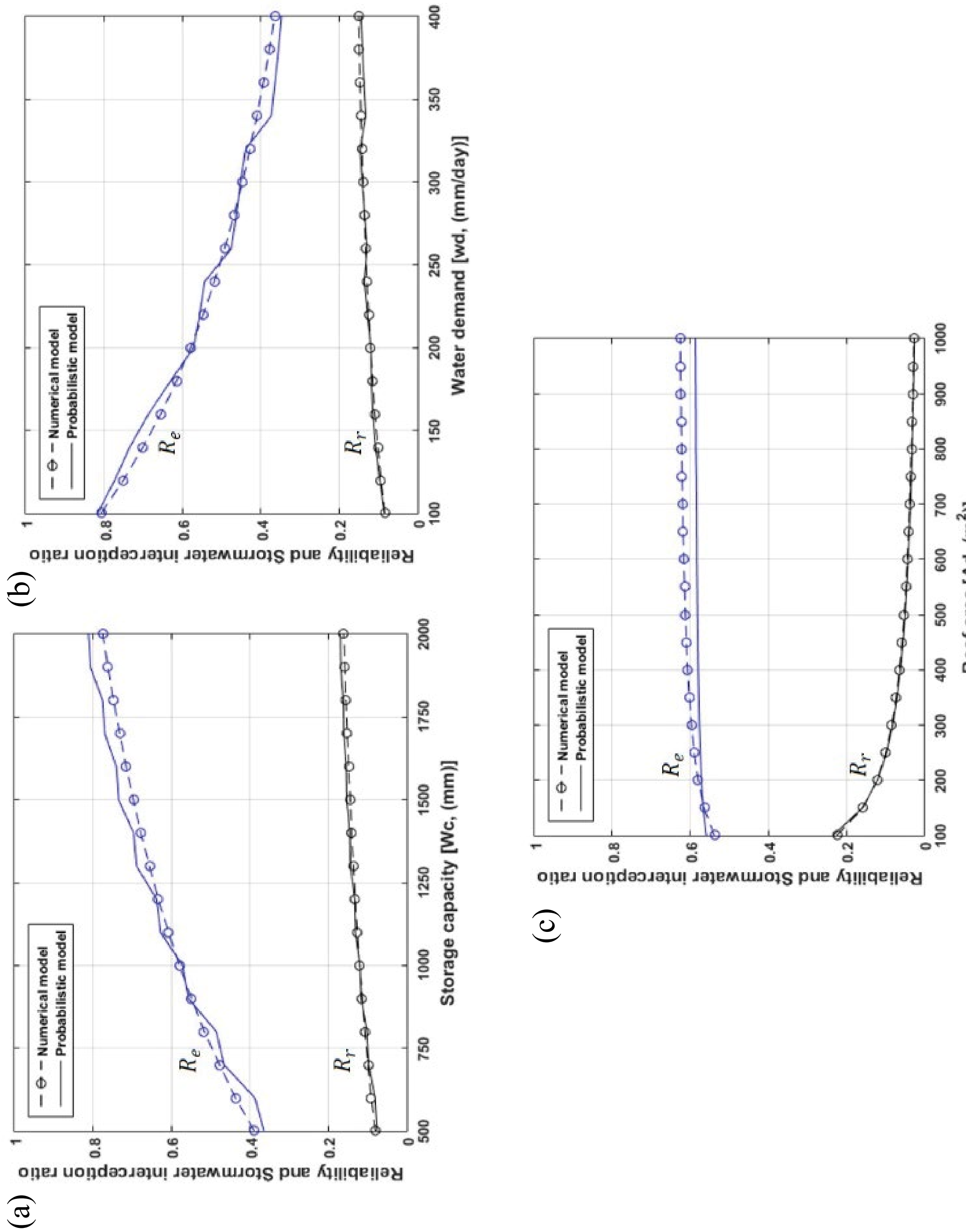


Fig. 6. Water supply reliability and stormwater interception ratio of RHS with varying (a) storage capacities ($W_c$), (b) water demands ($w_d$), and (c) roof areas ($A_d$).

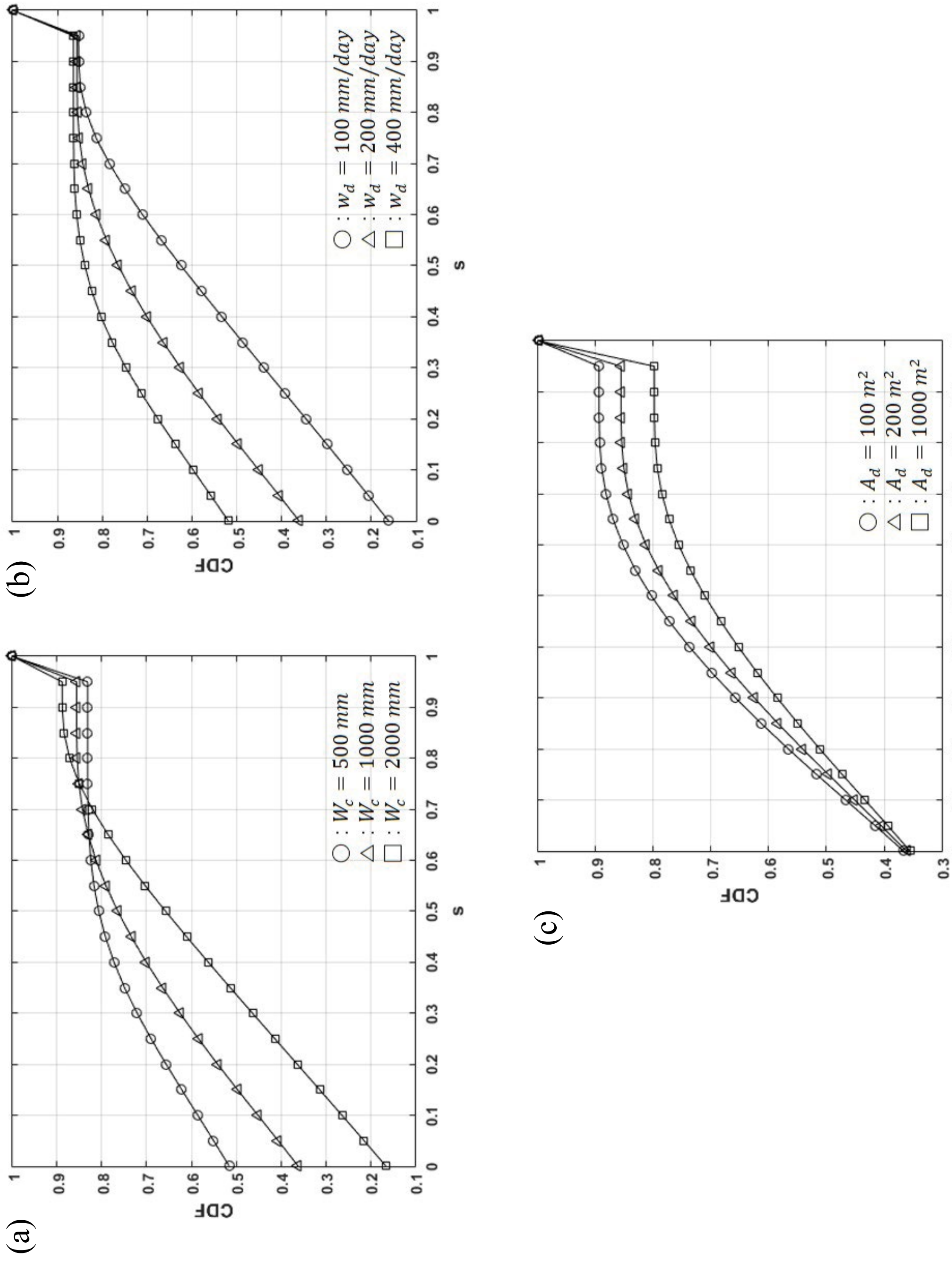


Fig. 7. CDF of $s$ with varying (a) storage capacities ($W_c$), (b) water demands ($w_d$), and (c) roof areas ($A_d$).

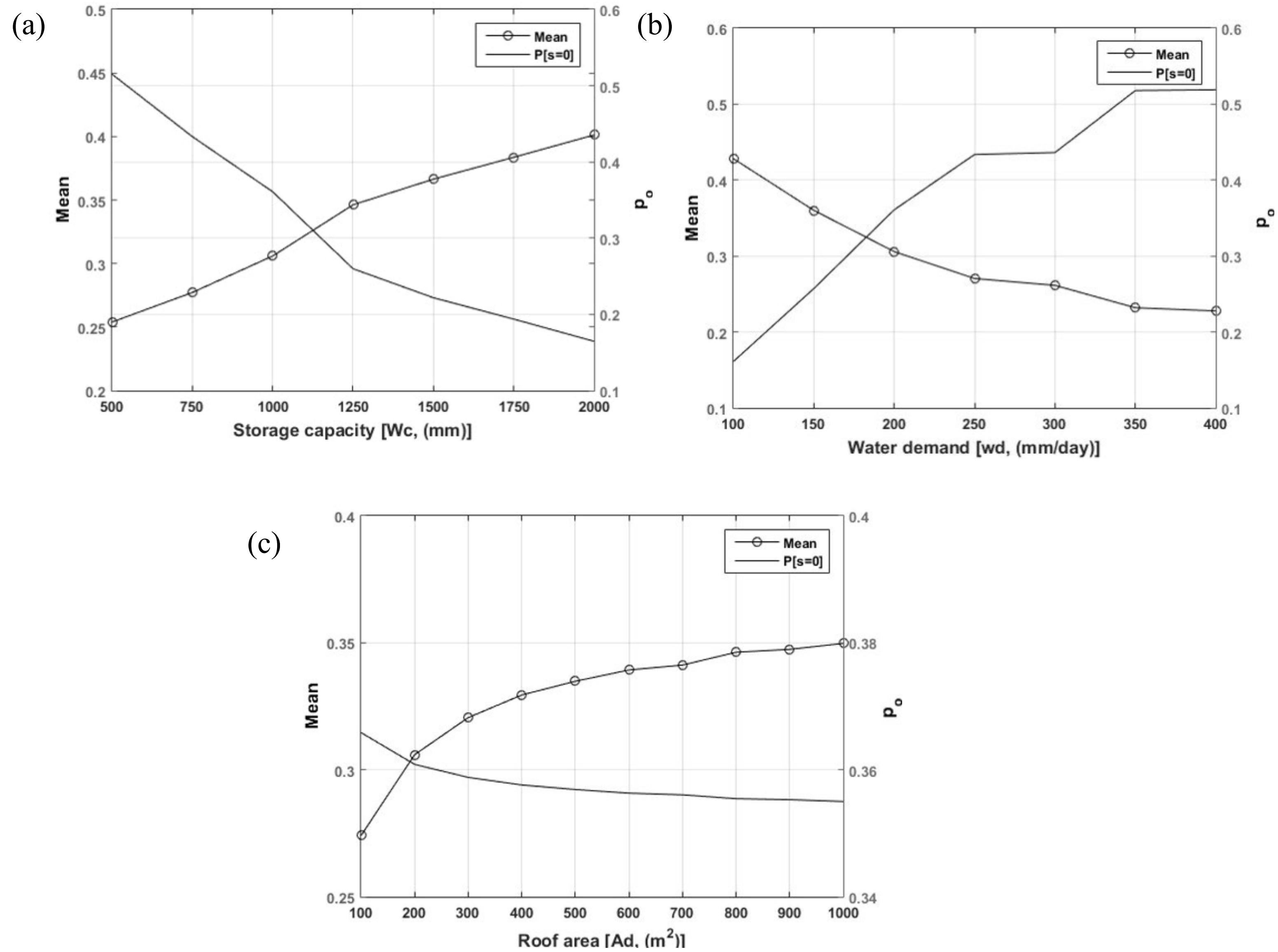


Fig. 8. Mean of $s$ and $p_0$ with varying (a) storage capacities ($W_c$), (b) water demands ($w_d$), and (c) roof areas ($A_d$).

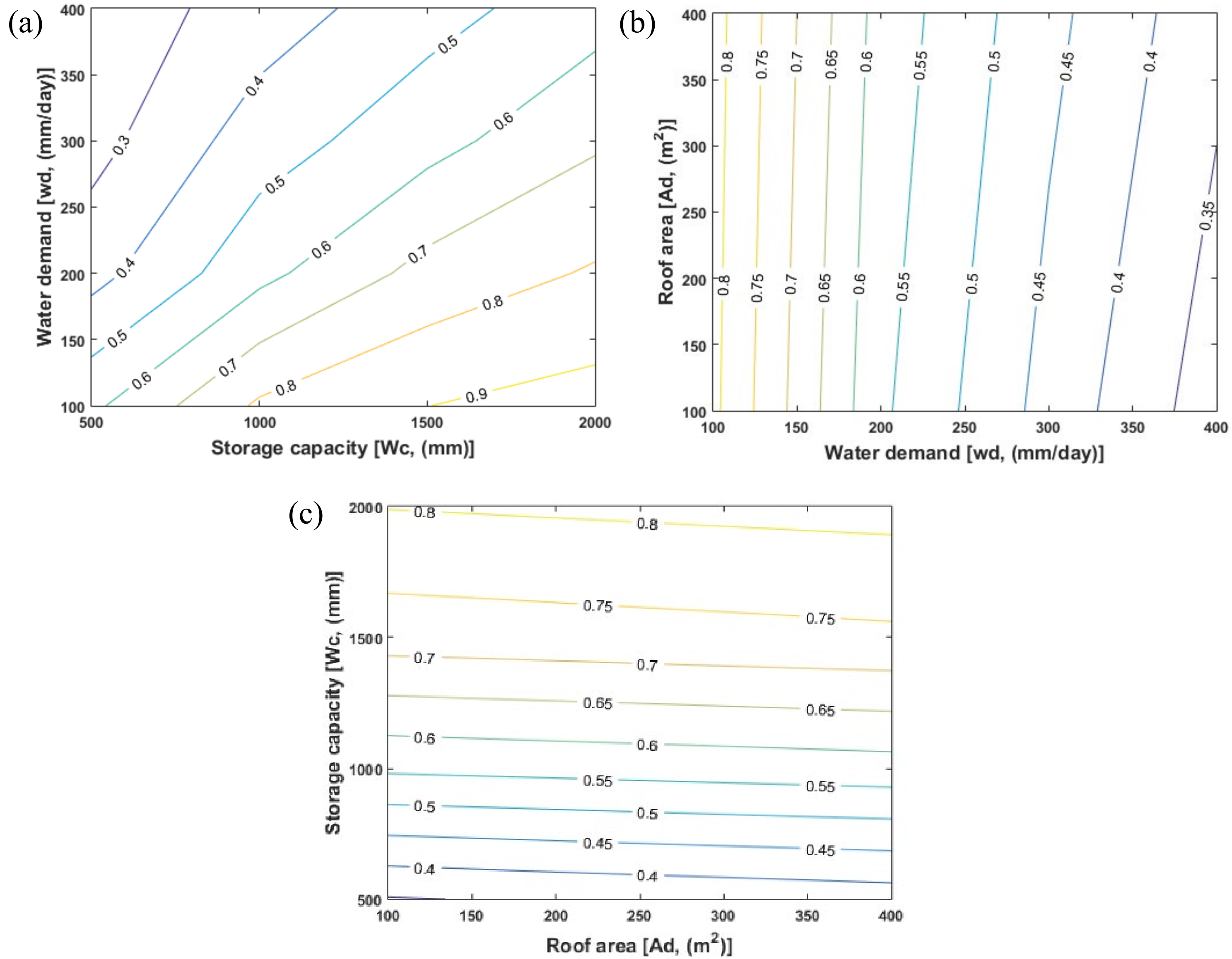


Fig. 9. Water supply reliability $R_e$ varying with storage capacities, water demands and roof areas (a) storage capacity ($W_c$) – water demand ($w_d$), (b) water demand ($w_d$) – roof area ($A_d$), and (c) roof area ($A_d$) – storage capacity ($W_c$).

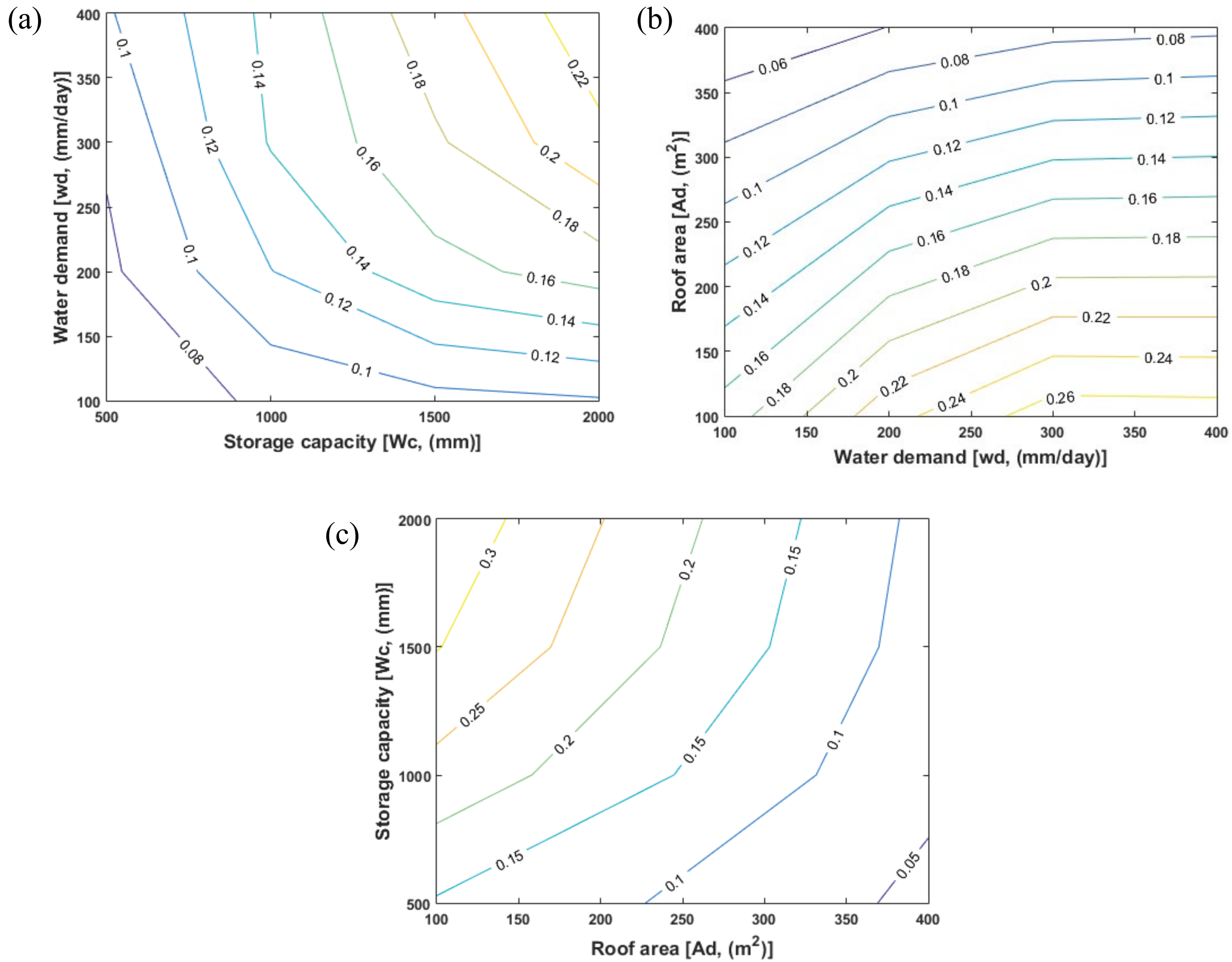


Fig. 10. Stormwater interception ratio $R_r$ varying with storage capacities, water demands and roof areas (a) storage capacity ($W_c$) – water demand ($w_d$), (b) water demand ($w_d$) – roof area ($A_d$), and (c) roof area ($A_d$) – storage capacity ($W_c$).

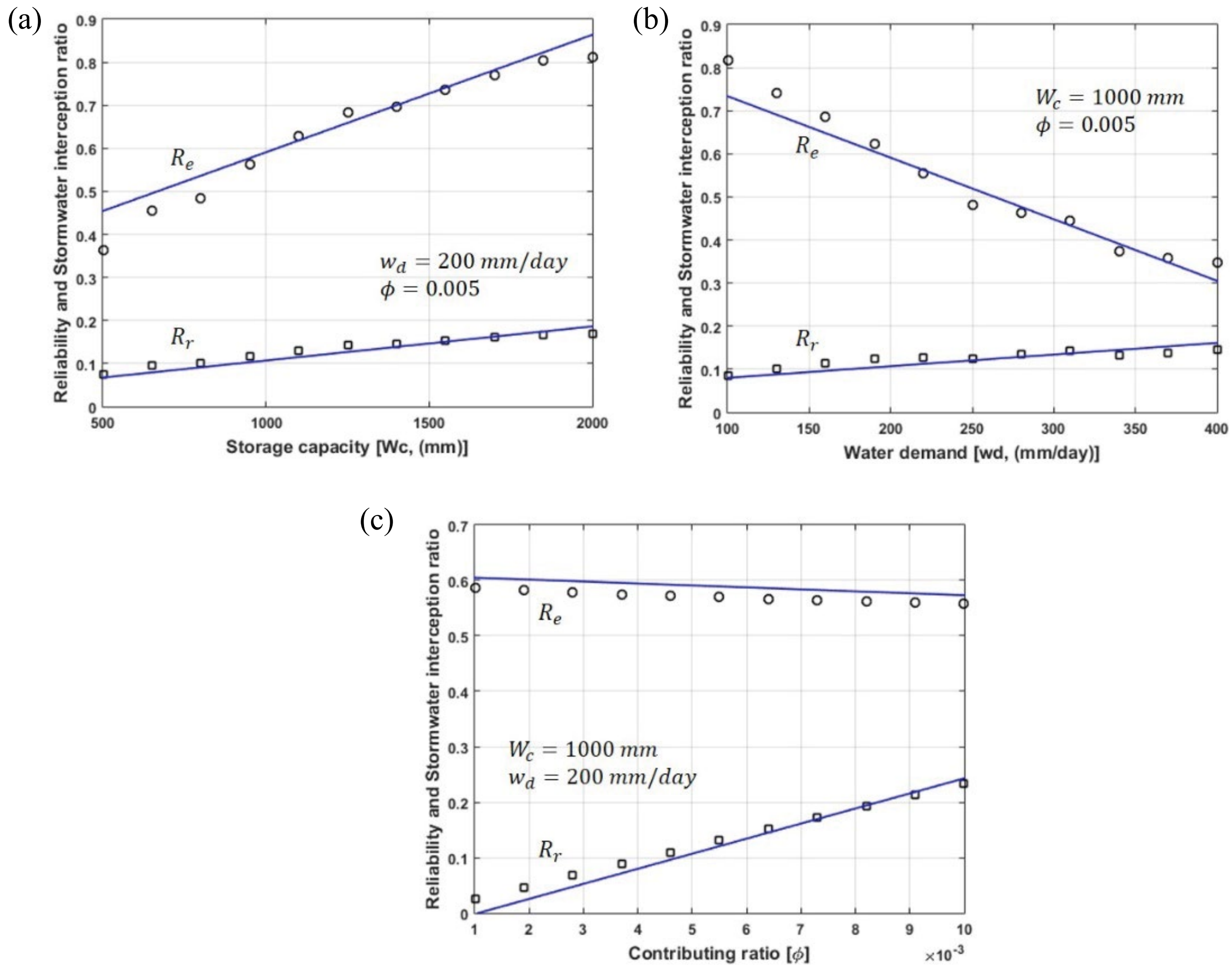


Fig. 11. Comparisons of water supply reliability and stormwater interception ratio derived from the stochastic model and the design formula with varying (a) storage capacities ($W_c$), (b) water demands ($w_d$), and (c) contributing ratios (ϕ).

proposed design formulas, the design values obtained from the probabilistic model are compared with those of the corresponding design formulas (Table 1). In addition, the design values of the probabilistic model and the design formula for the range of various parameters are compared and shown in Fig. 11.

Comparing $R_e$ and $R_r$ derived from the probabilistic model and the design formula, respectively, it is confirmed that $R^2$ is more than 0.9 and root mean square error (RMSE) is satisfactorily small. The water supply reliability and stormwater interception ratio derived from the probabilistic model in Fig. 11 are indicated by O and □, respectively, and the corresponding values derived from the design formulas are indicated by solid blue lines. In terms of the agreement between the probabilistic model and the design formula for the various parameter ranges, the reliability of the design formula seems to be sufficiently secured. Therefore, it is expected that the proposed design formulas can be used to estimate the reliability of water supply and rainfall-runoff in practical use, and can be fully utilized in RHS design.

### *4.2. RHS mitigates the adverse effects of climate change*

Climate model data have serious biases from observed data, and there are more biases in the results of rainfall simulations, especially since the simulation reliability for extreme weather events is relatively low [44]. In order to use these data, the bias between the observed data and the model data should be corrected. In this study, the quantile mapping technique, which has been used in many studies and can be applied relatively easily, was used [45–55].

Before analyzing changes in stormwater due to climate change, climate model data were used to analyze how much annual precipitation changes in the future (Fig. 12). In Fig. 12, 'present' is the range of ensemble of annual precipitation simulated under present (1981–2010) climate condition, while 'RCP 4.5' and 'RCP 8.5' represent the range of ensemble of annual precipitation simulated under the future (2021–2050) climate conditions. Annual precipitation is likely to increase at Busan, Daegu, Daejeon, and Gwangju sites, while future annual precipitation is likely to decrease at Incheon and Seoul sites. It is projected that future annual precipitation under the RCP 8.5 scenario will increase more than future annual precipitation under the RCP 4.5 scenario, however, the uncertainty in the RCP 8.5 scenario is greater than that in the RCP 4.5 scenario.

The parameters applied when analyzing the annual mean stormwater change due to climate change are $W_c$ = 1,000 mm, $w_d$ = 200 mm/d, $\phi$ = 0.005. The annual average stormwater depths generated during the present and future periods without RHS and the annual average stormwater depths generated during the future period with RHS were estimated, respectively. These comparisons have shown how RHS can reduce stormwater increased by climate change (Fig. 13). Looking at the Busan, Daegu, Daejeon, and Gwangju sites where the stormwater depth is likely to increase, we can find that the increased stormwater depth can be reduced almost by RHS. These results reveal that RHS can offset the adverse effects of climate change on stormwater management in cities.

RHS has the ability to supply water stored in RHS as well as the ability to reduce stormwater increased by climate change. Fig. 14 shows the future water supply reliability achieved by RHS. Although site-specific, it can be found that the planned water supply plan from the RHS can be satisfied with about 50% confidence. Therefore, the introduction of RHS will provide additional benefits of securing available water resources as well as counteracting the adverse effects of climate change, such as stormwater reduction.

Table 1
$R^2$ and RMSE between design values from the stochastic model and the design formula

| | Water supply reliability ($R_e$) | Stormwater interception ratio ($R_r$) |
|---|---|---|
| $R^2$ | 0.9659 | 0.9104 |
| RMSE | 0.0328 | 0.0262 |

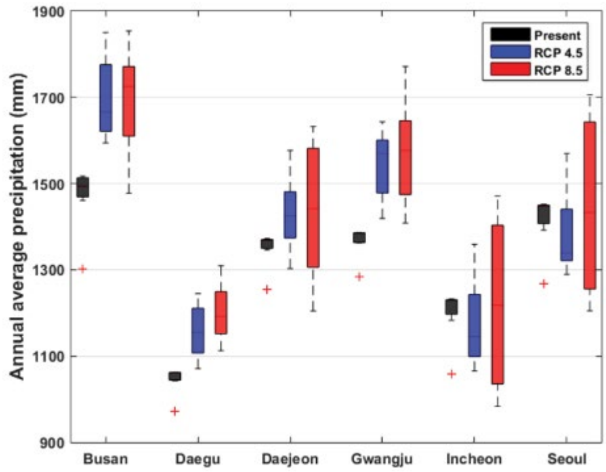


Fig. 12. Annual average precipitation from present and future climate data at six sites.

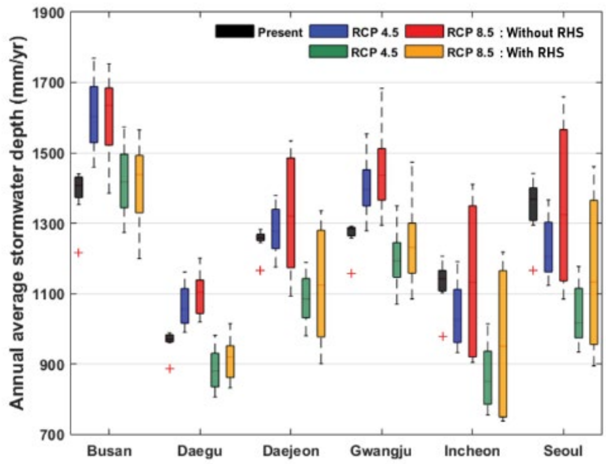


Fig. 13. Annual average stormwater depth of present and future climate data at six sites.

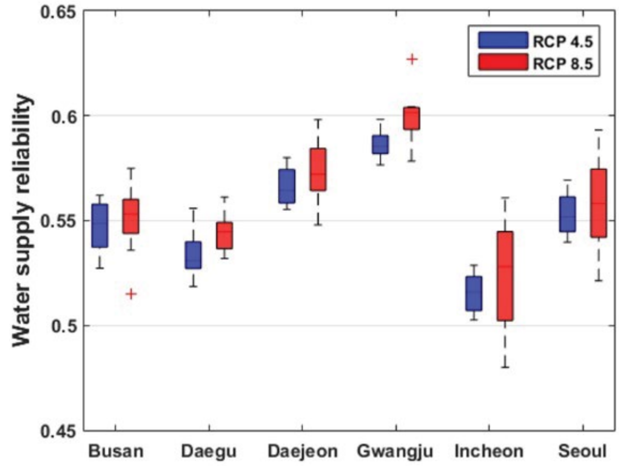


Fig. 14. Water supply reliability of future climate data at six sites.

## 5. Conclusions

RHS is generally designed to collect and store rainwater falling on catchment surfaces (e.g. rooftops or other impervious areas) for home or urban multipurpose use. The performance of these RHSs is nonlinear with various factors such as climate, watershed characteristics, and RHS design specifications, so design optimization is necessary to balance cost and performance. The most commonly used indicators for RHS performance assessment are the water supply reliability from RHS and the stormwater interception efficiency expected from RHS. In this study, we focused on the stochastic characteristics of rainfall and presented a probabilistic model for quantifying stormwater interception efficiency for stormwater management and reliability for water supply.

To verify the results of the derived probabilistic model, the results were compared with the numerical results. As a result of the comparison, it is confirmed that the numerical results and the results of the derived probabilistic model are in good agreement. We also analyzed the probabilistic behavior of RHS for changes in RHS characteristic parameters (capacity, water demand, and catchment area). In the same rainfall event, increasing RHS capacity, decreasing water demand, and increasing catchment areas were found to increase RHS water supply reliability. In addition, increased RHS capacity increased water demand, and reduced catchment areas were found to increase stormwater interception efficiency. In general, however, when RHS is actually applied to existing buildings, water demand and catchment areas are often determined depending on the surrounding conditions. Therefore, the capacity of RHS will be the most important determinant of RHS behavior. In addition, design formulas for estimating water supply reliability and stormwater interception efficiency for characteristic parameters were derived so that they could be used in future RHS planning.

In this study, using various climate model data and the proposed probability model, we investigated how much RHS can reduce future stormwater and how much water supply reliability can be obtained from RHS at six major sites in Korea. The RHS installation could play a role in responding to the possibility of future stormwater growth and additionally secure available water resources.

Using the probabilistic model proposed in this study, RHS water supply reliability and stormwater interception efficiency can be easily implemented in relatively simple computer codes. Therefore, the results of this study are expected to be useful tools for evaluating RHS performance, determining RHS capacity, and analyzing the role of RHS as a means of adaptation to climate change. Overall, this probabilistic approach has proven to be feasible and reliable in modeling the long-term balance of RHS. The proposed solution needs to emphasize that the daily precipitation series is valid for RHS located in areas that can be approximated by an exponential distribution.

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