Linear programming determination of blend ratios for desalinated water produced by multistage flash and reverse osmosis in a hybrid plant

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ABSTRACT

When operating hybrid desalination plants combining multistage flash (MSF) and reverse osmosis (RO) processes, it is crucial to determine the blend ratios of water produced by these two processes. These blend ratios must take economic and environmental aspects of the hybrid plant's sustainability into consideration, and must enable the plant to meet constraint conditions with respect to water demand, energy consumption savings, and saline content. We mathematically resolve this issue by formulating it as a linear programming problem and by computing that problem’s solutions. Permissible solutions occur as an area in a triangle in the MSF–RO blend ratio diagram, and the most desirable solutions occur at (1) the intersection point between the water demand limit line and salinity limit line and at (2) the intersection point between the water demand limit line and the energy savings limit line.

Keywords: Hybrid plant; Multistage flash distillation; Reverse osmosis; Blend ratio; Linear programming

1. Introduction

Multistage flash (MSF) and reverse osmosis (RO) processes are used by many desalination plants. MSF tends to emit larger amounts of carbon dioxide than RO but can produce a higher quality of water. This RO deficiency is due to the small amount of salt usually remaining in water processed through an RO membrane [1]. As an alternative to either individual method, some studies have suggested a combination of MSF and RO processes (e.g., 2–5). The hybrid form of MSF and RO, hereafter hybrid MSF–RO, compensates for the disadvantages in each method, that is, mitigating energy consumption and CO₂ emissions while eliminating dissolved salts in distilled water. Thus, it is crucial for hybrid MSF–RO to be established as a technology to meet increasing global demand and mitigate the increasing scarcity of fresh water.

The involvement of these two desalination methods (MSF and RO) in the hybrid plant raises two questions: (1) what should be the blend ratio of MSF and RO? (2) By what means and criteria should this ratio be determined? This study offers a theoretical method yielding an answer to these questions that considers environmental and economic aspects of sustainability in hybrid MSF–RO plant operation. In the method proposed by this study, an indicator that quantifies both the economic benefits and environmental impact of plant operation is employed to evaluate the sustainability of hybrid MSF–RO plant operation inclusively. A smaller indicator corresponds to a more desirable blend ratio, meaning a
more sustainable plant. Thus, the blend ratio is determined so that the indicator value is minimized.

As in many optimization problems, merely minimizing the indicator value gives a trivial solution, that is, 0% MSF and 100% RO. To enable the practical specification of blend ratios, other constraint conditions are needed. In this study, three constraint conditions are considered: (1) a lower threshold for water production amount that meets demand; (2) a carbon dioxide emission upper threshold, required as one of the 17 goals defined in sustainable development goals; and (3) an upper threshold for saline content in the produced water.

The present study considered hybrid desalination in the United Arab Emirates (UAE) because it has the highest average population of desalinated water consumers in the Persian Gulf region. The following sections describe a method of computing an inclusive indicator, and show that minimizing the indicator under the added constraint conditions can be represented in the form of linear programming. This exposition is followed by some examples of computed results and a discussion.

2. Formulation as a linear programming problem

2.1. Method for calculating inclusive indicator of hybrid MSF–RO system sustainability

The Inclusive Impact Index light (Triple-I light, denoted as III) is used to evaluate the magnitude of the economic and environmental impacts of human activities on society [6]. Its use for evaluating an MSF plant’s sustainability was reported in a study by Tokui et al. [7]. In the present study, the method used by Tokui et al. [7] is extended to a hybrid MSF–RO plant. III, with a unit of gha/year, is calculated as

\[
III = (EF - BC) + \Sigma (COST - BENEFIT)
\]  

(1)

where gha stands for global hectare and represents the activity’s ecological footprint and the environment’s biocapacity [8]. EF is the ecological footprint (gha/year), quantifying the impact of human activities, and is measured in terms of the area of biologically productive land and the amount of water used [9]. Biocapacity, represented by BC (gha/year), is the ecosystem’s capacity to absorb waste materials generated by humans. A biocapacity deficit occurs when the ecological footprint of a population’s activities exceeds the available area’s biocapacity [10]. The annual cost of conducting a human activity is represented as COST, whereas BENEFIT is the annual financial benefit of the activity in US$/year.

The factor \( \Sigma = \sum_{\text{region}} \frac{EF_{\text{region}}}{\sum \text{GDP}_{\text{region}}} \) converts the units in Eq. (1) from US$/year to gha/year and measures the environmental quality of the industry in a region. The quantity \( \sum EF_{\text{region}} \) denotes the summation of the ecological footprints in a region (gha), and \( \sum \text{GDP}_{\text{region}} \) denotes the summation of the gross domestic product (GDP) in a region (US$).

The four values (EF, BC, COST, and BENEFIT) need to be quantified in advance for calculating III, using Eq. (1). The following sub-subsections explain how these quantifications are carried out.

2.1.1. Ecological footprint

The ecological footprint of a desalination plant is expressed as follows:

\[
EF = \frac{\Delta EF \times \epsilon_f}{A_r}
\]  

(2)

\[
\Delta EF = E_r \times P_r \times u \times \frac{365}{1000}
\]  

(3)

where \( \Delta EF \) (tons of CO₂ per year) is the amount of CO₂ emitted per year, \( \epsilon_f \) (gha/ha) is the forest equivalent factor, and \( A_r \) (tons of CO₂/ha) denotes the amount of CO₂ absorbed. The parameter \( E_r \) (kWh/m²) represents the energy consumption per unit volume of water produced involving thermal and electrical energy for MSF and electrical energy for RO. The daily water production of the desalination plant is denoted by \( P_r \) (m³/d), while \( u \) (kg of CO₂ per kW·h) represents the CO₂ emission intensity [11].

2.1.2. Biocapacity

The biocapacity is calculated as follows:

\[
BC = \frac{\Delta BC \times \epsilon_f}{A_r}
\]  

(4)

\[
\Delta BC = \Delta C \times S_{\text{farm}} \times 3.67
\]  

(5)

where the factor 3.67 is the conversion factor from carbon mass to CO₂ mass, \( \Delta BC \) (tons of CO₂ per year) denotes the quantity of CO₂ absorbed by soil, and \( \Delta C \) (tons of C per ha per year) represents the soil’s annual carbon absorption. A modified Rothamsted carbon model [12] was used to calculate the soil’s carbon absorption by determining the climatic conditions, the soil management condition, and the type of soil at a site. The parameter \( S_{\text{farm}} \) (ha) in Eq. (5) is the area of farmland occupied by the desalination plant, computed as follows:

\[
S_{\text{farm}} = S_{\text{region}} \times r_D \times r_5
\]  

(6)

where \( r_{D} \) denotes the agricultural land area in a region. The dimensionless quantities \( r_D \) and \( r_5 \) are defined as follows:

\[
r_D = \frac{\text{Agricultural water demand}}{\text{Total water demand}}
\]  

(7)

\[
r_5 = \frac{P}{P_t}
\]  

(8)

where \( P \) denotes the annual water supply from seawater desalination in the target region (UAE in this study), and \( P_t \) denotes the annual total water supply in the target region.
2.1.3. Cost and benefit

The cost and benefit terms are calculated as

\[
\text{COST} = \Delta\text{COST} \times P \times 365
\]

(9)

\[
\text{BENEFIT} = \Delta B_{\text{region}} \times P \times 365
\]

(10)

where \(\Delta\text{COST}\) represents the cost of producing a unit volume of water (US$/m^3), and 365 is the conversion factor from a daily to an annual timescale. \(\Delta B_{\text{region}}\) represents the price of produced water (US$/m^3) in the target region (UAE in this study).

2.1.4. Parameter values

The values of the parameters used in various equations are presented in Table 1. In the hybrid MSF–RO desalination plant at Fujairah, UAE, 62.5% of the total fresh water is produced via the MSF process, while 37.5% is produced via RO [23]. The present study used percentages (blend ratios) close to these (Table 2). These blend ratios were used to calculate the standard value of III, for the hybrid MSF–RO plant. The extent to which changes in the ratios affect the plant’s sustainability was then examined through linear programming.

2.2. Linear programming formulation

In this subsection, a method for calculating blend ratios based on linear programming is described [24]. The desalination plant examined in this study is required to minimize a sustainability evaluation index (III) regarded as an objective function under constraints. The constraints considered in the present study include water production that meets demand, reduced energy consumption in plant operations, and minimized salt content in produced water.

In order to simplify the mathematical expressions to general forms applicable to various situations, III, is written in the following dimensionless form:

\[
III' = e' \alpha + \beta - \gamma,
\]

(11)

where \(\alpha\) and \(\beta\) denote the respective blend ratios of MSF and RO. The constants in Eq. (11) are defined as follows:

\[
III' = \frac{III}{\left(P E_{\text{MSF}} \right) / \left(P E_{\text{RO}} \right)}
\]

(12)

where the definitions of \(P\) and \(Q\) are

\[
\begin{align*}
P &= P_{\text{H}} \times 365 \times \frac{e_f}{1,000 \times A_v} \\
Q &= \sum GDP_{\text{region}, \text{US$}} \times 10^{11}
\end{align*}
\]

(13)

The parameters \(E_{\text{MSF}}\) and \(E_{\text{RO}}\) are the respective energy consumption values (Table 2) of MSF and RO processes. The energy consumption ratio is a key parameter that is defined as follows:

\[
e = \frac{E_{\text{MSF}}}{E_{\text{RO}}}
\]

(14)

which in general remains above unity.

The constraints mentioned above are expressed mathematically below. The water demand constraint is

\[
\alpha + \beta \geq 1
\]

(15)

whereas the energy-saving constraint is

\[
\alpha + \frac{1}{e} \beta \leq e_{\text{max}}
\]

(16)

Table 1

List of notations and values of parameters and references therein

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{r})</td>
<td>kW h/m³ 23.41 (MSF)</td>
<td>[13]</td>
</tr>
<tr>
<td>(P_{r})</td>
<td>m³/d 3.00 (RO)</td>
<td>[14]</td>
</tr>
<tr>
<td>(u)</td>
<td>kg-CO₂/kW h 0.74</td>
<td>[15]</td>
</tr>
<tr>
<td>(e_r)</td>
<td>gha/ha 1.34</td>
<td>[16]</td>
</tr>
<tr>
<td>(A_{e})</td>
<td>t-CO₂/ha 5.68</td>
<td>[17]</td>
</tr>
<tr>
<td>(\Delta C)</td>
<td>t-C/ha/y 5.00 × 10⁻²</td>
<td>[18]</td>
</tr>
<tr>
<td>(S_{\text{region}})</td>
<td>ha 260,732</td>
<td>[19]</td>
</tr>
<tr>
<td>(r_{D})</td>
<td>0.8284</td>
<td>[20]</td>
</tr>
<tr>
<td>(P_{\text{w}})</td>
<td>m³/year 0.950 × 10⁹</td>
<td>[21]</td>
</tr>
<tr>
<td>(P_{\text{s}})</td>
<td>m³/year 1.348 × 10⁹</td>
<td>[22]</td>
</tr>
<tr>
<td>(\Delta\text{COST})</td>
<td>US$/m³ 1.125</td>
<td>[23]</td>
</tr>
<tr>
<td>(\Delta B_{\text{region}})</td>
<td>US$/m³ 2.83</td>
<td>[24]</td>
</tr>
<tr>
<td>(\sum\text{EF}_{\text{region}})</td>
<td>gha 2.8279 × 10⁷</td>
<td>[25]</td>
</tr>
<tr>
<td>(\sum\text{GDP}_{\text{region}, \text{US$}})</td>
<td>3.90427 × 10¹²</td>
<td>[26]</td>
</tr>
</tbody>
</table>

Letters “t-” and “-kg” in Unit column are abbreviations of ton and kilogram, respectively.

Table 2

Percentages of the total fresh water produced via MSF and RO processes in the present study’s hypothetical hybrid MSF–RO plant

<table>
<thead>
<tr>
<th>Type of desalination process</th>
<th>Produced water (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSF</td>
<td>63</td>
</tr>
<tr>
<td>RO</td>
<td>37</td>
</tr>
</tbody>
</table>
and the salinity elimination constraint is

$$\frac{\beta}{\alpha + \beta} \leq \eta_{\text{max}}$$  \hspace{1cm} (17)

where $e_{\text{max}}$ denotes the dimensionless maximum-energy consumption, normalized by $E_{1,\text{MSF}}$, which must be below unity for energy saving. $\eta_{\text{max}}$ represents the maximum permissible volume ratio of RO for achieving salinity lower than a threshold. In the present study, $e_{\text{max}}$ was set to 0.9 based on the assumption that the plant operator reduces energy consumption to 90% of $E_{1,\text{MSF}}$, and $\eta_{\text{max}}$ was set to 0.3.

Since Eqs. (11) and (15)–(17) are first-order equations with respect to $\alpha$ and $\beta$, these are drawn as lines on the $\alpha\beta$-plane. The solutions to this linear programming problem can be determined graphically, without complex calculations.

3. Results and discussion

3.1. Standard value of the Triple-I light index

Triple-I light sustainability index (III) was computed for the hybrid MSF–RO plant, as well as for MSF-only and RO-only plants, hereafter standalone MSF and RO (Fig. 1). The index for the hybrid MSF–RO was 37% below (i.e., better than) that of the standalone MSF.

The value of III for the hybrid MSF–RO (Fig. 1a) is controlled primarily by the value of EF–BC (the first term in Eq. (1)). The economic term (the second term in Eq. (1)) for the hybrid MSF–RO is negative, and its absolute value is 14% of that of the first term. For the standalone MSF (Fig. 1b), the value of III also depends on the first term, and its second term is also negative, with an absolute value 7% of that of the first term. The standalone RO (Fig. 1c) produced a positive value for the first term and a negative value for the second term, which were almost equal in magnitude, yielding a small III value.

Comparing these indices revealed that hybridization offers more economical and environmentally sustainable desalination than standalone MSF. The lower index of the hybrid plant is attributed primarily to the small ecological footprint of RO. Increasing the number of RO plants in the Persian Gulf region is, therefore, expected to be an effective way of sustaining the desalination industry. Presently, in the Persian Gulf region, MSF plants dominate the desalination market, producing over 70% of total desalinated water [25]. Therefore, hybridizing the MSF method with RO offers a wise short-term alternative to standalone MSF during the ramp-up to standalone RO predominance.

3.2. Sensitivity of III to uncertainty in parameters

These calculations' sensitivity is examined by varying the parameters whose values are uncertain to determine the results' resilience. III is quite sensitive to changes in energy consumption ($E_1$; Fig. 2) because it is a governing parameter in Eq. (1). III decreases linearly as $E_1$ decreases, and the high sensitivity suggests that reducing energy consumption is vital for sustainable desalination. III is negative for $E_1$ values below 2.3 kW h/m$^3$, indicating that the desalination process is sustainable. Nevertheless, energy consumption of 2.3 kW h/m$^3$ is below that of standalone RO (3.0 kW h/m$^3$) (see vertical dashed line indicated by “R” in Fig. 2). Hence, using the standard value of $E_1$ for the hybrid MSF–RO plant cannot result in sustainable operation.
The conversion factor in Eq. (1) depends on the population, GDP, and GDP projection. Variations in the conversion factor are estimated using the UAE’s GDP projection (Fig. 3a). Although III varies in response to variation in the conversion factor (Fig. 3b), the deviation from the average is 2.2% at maximum (Fig. 3c), showing that III is insensitive to the conversion factor.

The conversion factor also depends on the country hosting the hypothetical plant because the conversion factor is a function of the ecological footprint and GDP. Conversion factors are computed using regionally representative data from Saudi Arabia, Qatar, Kuwait, Yemen, and Oman (Fig. 4a). The data for Yemen showed the highest conversion factor, with a value 2.9 times that of the UAE; this difference is attributed to Yemen’s large population. The lowest conversion factor value was for data from Qatar, about 0.7 times that of the UAE; this difference is attributed to its lowest population among the six countries. The conversion factor for Qatar yields the largest III, and is 5% above that of the UAE. These differences in conversion factors influence the magnitude of III (Fig. 4b). Note that in calculating III, other parameters were held constant.

A negative value of the COST–BENEFIT factor in (1) indicates that overall benefits of the plant operation are exceeding costs. This implementation produces negative correlations between conversion factor (Fig. 4a) and III (Fig. 4b). In other words, the strongly positive (resp. closest to zero) conversion factor for Yemen (resp. Qatar) corresponds to the smallest (resp. largest) III for Yemen (resp. Qatar). The small differences in III alter the results only slightly from the standard calculation.

3.3. Sensitivity of III, to changes in RO percentage

The examination of blend ratios, thus far, has relied on the static values from Table 2. In the present section, the blend ratios are allowed to vary to assess the response of III. The III index for hybrid MSF–RO decreases linearly with the increasing RO percentage in overall water production (Fig. 5). This means that any increased use of the RO process contributes to enhancing the plant operation’s sustainability. This response of III, demonstrates a higher sensitivity to the RO percentage than to other parameters. This difference further supports the notion that any increase in the RO percentage effectively improves the hybrid plant operation’s sustainability.

It should be noted, however, that the water produced through the RO process contains a relatively large quantity of dissolved salts [1]. Therefore, any plan to augment the RO percentage must consider the salt residue. Since water processed through MSF can dilute the blended water, hybridization is a viable alternative for Persian Gulf countries until the RO process is sufficiently modernized [2,3].

3.4. Determination of blend ratios using the (α, β) diagram

The two inequalities in Eqs. (15) and (16) are plotted in Fig. 6a to examine the water demand and energy-saving constraints. In order to satisfy these constraints, the point (α, β) must fall within the shaded area. The salinity elimination constraint is excluded here for simplicity but will be introduced below. The minimum value of III, occurs at the point (0.000, 1.000), corresponding to the standalone RO

Fig. 3. (a) Conversion factors estimated using population and GDP of the UAE over ten years. (b) III, indices computed using the conversion factors plotted in (a). (c) Same as (b) but with a longer range on the vertical axis.
that produces the required amount of water (open circle in Fig. 6a). To make the hybrid plant effective, the objective of minimizing III necessitates flexibility for the hybrid plant's effectiveness. Increasing III, from the minimum causes a rightward shift in the III line; eventually, the vertical coordinate (β) of the intersection with the maximum-energy consumption line attains unity at (0.772, 1.000). This point corresponds to the hybrid MSF–RO producing excess water (closed circle in Fig. 6a). The maximum-energy line also intersects with the water demand line at (0.885, 0.115). This point meets the water demand, but the energy consumption reaches the maximum value permitted, and represents the hybrid plant's most desirable solution (gray circle in Fig. 6a).

The salinity elimination constraint was next incorporated instead of that for energy saving (Fig. 6b). The solution for the minimum III value occurs at the point (0.700, 0.300), which corresponds to the intersection between the water demand and salinity elimination lines (open circle in Fig. 6b). If the operator allows water production to exceed demand, III increases until the intersection reaches the point (1.000, 0.429) shown by the closed circle in Fig. 6b. Meanwhile, if the operator considers the saline content alone, the solution will be at the point (1.000, 0.000), which is the standalone MSF.

The simultaneous treatment of the water demand, energy savings, and salinity elimination constraints is illustrated in Fig. 6c. The solution with the intersection between the water demand limit and the energy-saving limit lines is equivalent to that in Fig. 6a (gray circle in Fig. 6c). The solution with a minimum III value is equivalent to that represented by the open circle in Fig. 6b. A new intersection emerges from the intersection of the energy-saving limit and salinity elimination limit lines; this intersection is equivalent to the solution

![Fig. 4](image-url)  
Fig. 4. (a) Conversion factors estimated using population, GDP, and the ecological footprint for each country indicated. (b) III indices computed using the conversion factors plotted in (a).

![Fig. 5](image-url)  
Fig. 5. Relationship between the III index and the contribution (in percentage terms) of RO to total water production. The dashed line indicates the standard value. The dotted line indicates the zero value of III.

![Fig. 6](image-url)  
Fig. 6. Blend ratio diagrams. The water demand limit (designated as “W”), energy-saving limit (“E”), and salinity elimination limit (“S”) constraints are drawn as lines. The shaded areas are the solutions jointly permissible under constraints (a) W and E, (b) W and S, and (c) W, E, and S. The open and closed circles are the solutions corresponding to the minimum and maximum III index (solid lines), respectively. The gray circles in (a) and (c) represent the intersections between constraints W and E.
with a maximum III value (closed circle in Fig. 6c). A point \((\alpha, \beta)\) that satisfies the three inequalities (inside the shaded triangle, including its sides) depends on the plant’s operational conditions. If inclusive sustainability is regarded as paramount, the blend ratio at the open circle in Fig. 6c with the minimum III, value is chosen. On the other hand, if salinity content alone is paramount, the solution at the gray circle in Fig. 6c is chosen.

Although the three constraints are essential in deciding on the operation of a hybrid MSF–RO plant, other constraints exert their own influence in practical situations. This requires consideration of variables other than \((\alpha, \beta)\) and computations to obtain solutions, which will be addressed in future work.

4. Conclusion

This study presented a theoretical method for selecting the blend ratios of water produced by MSF and RO processes in a hybrid MSF–RO plant. Hybridization reduced CO₂ emissions while retaining water quality by combining the advantages of the MSF and RO processes. This hybrid MSF–RO showed a positive sustainability index that was 37% below (i.e., better than) that for standalone MSF, whereas the standalone RO (RO use alone) was shown to be far more sustainable yet, producing a negative index (a better sustainability). Compared with standalone MSF, the hybrid MSF–RO process provided better desalination from an economic and environmental perspective, but it remained less sustainable than standalone RO because of CO₂ emissions from the MSF desalination process. The hybrid MSF–RO’s suboptimal sustainability is attributed to its relatively high ecological footprint, which is caused by high energy consumption during the MSF desalination process.

Permissible solutions of the linear programming problem considering the three constraints occur as a triangular area in the MSF–RO blend ratio diagram. The most desirable solutions occur at the vertices, particularly the intersection points between the water demand limit and salinity limit lines. If salinity elimination is of paramount importance, the chosen solution occurs at another vertex, that is, the intersection point between the water demand limit and energy-saving limit lines.

References