Modeling of fluoride removal by nanofiltration: coupled film theory model with Nernst–Planck equation and artificial neural network

F.Z. Addar\textsuperscript{a}, B. Fahid\textsuperscript{b}, M. Belfaquir\textsuperscript{a}, M. Tahaikt\textsuperscript{a}, A. Elmidaoui\textsuperscript{a}, M. Takya\textsuperscript{a, c, *}

\textsuperscript{a}Laboratory of Advanced Materials and Process Engineering, Faculty of Sciences, Ibn Tofail University, P.O. Box 1246, Kenitra – Morocco, emails: elmidaoui@uit.ac.ma (A. Elmidaoui), mohamed.taky@uit.ac.ma/takymogamed@gmail.com (M. Taky), Fatima.zahra@uit.ac.ma (F.Z. Addar), Mustapha.belfaquir@uit.ac.ma (M. Belfaquir), mustapha.tahaikt@uit.ac.ma (M. Tahaikt)

\textsuperscript{b}Laboratory of Analysis, Geometry and Applications, Superior School of Technology, Ibn Tofail University, P.O. Box 1246, Kenitra – Morocco, email: brahim.fahid@uit.ac.ma (B. Fahid)

\textsuperscript{c}International Water Research Institute, Mohammed VI Polytechnic University, Lot 660, Hay Moulay Rachid, Ben Guerir, 43150 – Morocco

Received 6 January 2022; Accepted 14 March 2022

\textbf{ABSTRACT}

Fluoride ions are necessary and beneficial for human beings, however high levels of fluoride in groundwater can be toxic and harmful to human health and water resources, causing serious problems on teeth and bones. This situation is widespread in many parts of the world, including Morocco. According to World Health Organization (WHO), the maximum allowable concentration of fluoride in drinking water is 1.5 mg/L. The objective of this work is to understand the mechanism of transfer of fluoride ions in nanofiltration (NF) membranes. Two methodologies have been used, one mathematical, Nernst–Planck equation combined with film theory (NP-FT), and the other statistical artificial neural network (ANN). This study has been realized with experimental data obtained from three NF membranes (TR60, NF270 and NF90) that have been used for the removal of fluoride from NaF-doped groundwater with different initial concentrations of 5, 10, 15 and 20 mg/L. In a first part, the NP-FT model that is proposed as a contribution to the modeling of the concentration polarization phenomenon and to study the influence of the initial concentration \((C_i)\) of fluoride on the reflection coefficient \((\sigma)\), the permeability of the solute \((P_s)\) and the thickness of the boundary layer \((\delta)\) has been made. In a second part, the two models ANN and NP-FT are used to predict the fluoride rejection as a function of the permeate flux. In addition, a comparison is made in terms of coefficient of determination \(R^2\) between the two models. The results obtained showed that the influence of concentration bias is more important for NF270 than for TR60 and NF90, and high reflection coefficients, almost equal to unity, were obtained for NF90, indicating that convective transport is almost completely hindered. Similarly, reflection coefficients of about 0.97 and 0.89 were found for TR60 and NF270, respectively, suggesting that both mechanisms are present, but that diffusion is more pronounced for NF270 than for TR60. Thus, the thickness of the boundary layer and permeability change inversely to one another. It is about \(10^{-5}\) m for NF270 and \(10^{-6}\) m for TR60. On the other hand, the value obtained for the NF90 membrane is \(10^{-14}\) m which is very low. The obtained solute permeability follows the order: \(R_{60} > R_{270} > R_{90}\). Although the experimental data agree well with the data predicted by ANN and NP-FT, which indicates that both models are appropriate for the prediction and satisfactory \(R^2\). However, the ANN model reveals a slight superiority over the NP-FT model.

**Keywords:** Fluoride removal; Nanofiltration; Artificial neural networks; Modelling; Concentration polarization; Nernst–Planck equation; Film theory

* Corresponding author.

\textit{Presented at the Second International Symposium on Nanomaterials and Membrane Science for Water, Energy and Environment (SNMS-2021), June 1–2, 2022, Tangier, Morocco}

1944-3994/1944-3986 © 2022 Desalination Publications. All rights reserved.
1. Introduction

Population growth and water scarcity put pressure on available natural water resources. This has been confirmed by (WHO) and United Nations International Children's Emergency Fund (UNICEF) report that millions of people do not have a safe and enough supply of water for drinking and other everyday tasks [1,2]. Groundwater represents only 0.6% of total water resources and surface water are threatened by pollution from intense industrialization and urbanization activities [3].

The physical presence of many elements and compounds, both natural and anthropogenic, due to the interaction between soil and water, soil–gas interaction, contact of rocks with groundwater and time of residence in aquifers [4], especially fluorides, arsenic, nitrates, sulphates, iron, manganese, chloride, and aluminium, etc., can significantly affect the quality of water and cause adverse effects on health [5,6].

The issue of high fluoride concentration in groundwater resources has become one of the most serious toxicological and geo-environmental problems in several countries. India is the country most concerned by this problem of fluorides. During the last three decades, higher concentration of fluoride in drinking water and the resulting disease “fluorosis” have been significantly highlighted worldwide. The effect of fluoride in drinking water can be beneficial or deleterious to mankind. The determining parameters are the concentration and the duration of absorption. Thus, when it is present in a narrow concentration range, it plays an imperative role in bone mineralization and also acts as an antibacterial agent in the mouth. On the contrary, an excessive concentration (3–6 mg/L) of fluoride can have a negative effect on bones and teeth [6,7]. Absorption of excess fluoride (>10 mg/L) in the human body can cause dental, skeletal and non-skeletal “fluorosis”. Similarly, lack of fluoride in groundwater (less than 0.5 mg/L) can cause dental caries which is why the WHO has fixed a maximum concentration of fluoride in drinking water of 1.5 mg/L [8].

In Morocco, fluoride levels exceed acceptable standards in several regions, for instance, the Ben Guerir Plateau (Central Morocco). The water in this region generally exceeds the fluoride standards of 2 to 3 mg/L. This contamination is due to the deposition of phosphate [9,10].

A multitude of conventional and non-conventional methods have been implemented for water defluoridation, namely adsorption [11,12], ion-exchange [13], electrodialysis (ED) [14], precipitation/coagulation [15], electrocoagulation [16], and pressure-driven membrane-based technologies such as nanofiltration (NF) and reverse osmosis (RO) [17–19].

Many studies have shown that NF in groundwater defluoridation is cost-effective makes it economically and operationally attractive [20,21]. Pontié et al. [22] employed a polyamide NF45 membrane (FilmTec), they obtained a fluoride rejection of 91% at 0.02 mol/L of NaF in the feed. Nasr et al. [23] examined the commercial NF5 and NF9 membranes (Manufactured by Applied Membranes Inc, USA) to remove fluoride ions from Tunisian groundwater with fluoride concentration around 3 mg/L. NF9 exhibited a better rejection of (88%) than NF5 (57%) with fluoride concentrations in the permeate of 0.38 and 1.45 mg/L respectively. Mnif et al. [24] studied the defluoridation of aqueous solutions exceeding 1.5 mg/L using a thin film polyamide NF composite membrane called HL 2514 T, manufactured by GE-Osmonics (USA). The results showed that the fluoride rejection by the HL membrane was over 80% at a permeate fluoride concentration of 2 × 10−2 mol/L.

Since the separation mechanism of NF membranes is very complex, several models have been established to illustrate and predict the flux and retention of a variety of species under different operating conditions [25]. The most applied models are Kedem–Katchalsky [26,27], Spiegler–Kedem [26–28] and models based on the Nernst–Planck equation [29].

In a previous study, Addar et al. [9], used statistical model to optimize the fluoride removal process by three NF membranes and applied surface response methodology (RSM) based on a central composite design (CCD), minimizing the permeate concentration and maximizing the fluoride rejection and permeate flux. Optimal conditions
to obtain water that meets the standards set by WHO of the two membranes TR60, NF270 were determined: fluoride concentration in the feed lower than 7 and 5.5 mg/L and operating transmembrane pressure (TMP) of 10 and 13 bars respectively. Whereas, for NF90, the fluoride feed concentration has no influence on the fluoride rejection and the fluoride contents obtained in the permeate were well below the standards. In addition, Tahair et al. [18], investigated the influence of initial fluoride concentration on the transfer of fluoride by applying the Spiegler–Kedem and Kedem–Katchalsky models while neglecting electrical interactions and concentration polarization layer. The results showed that NF270 and TR60 involved two different mechanisms: diffusion and convection with the preponderance of diffusion. On the other hand, for NF90, the transfer mechanism is close to that of RO.

Other mathematical models can be used to improve the understanding of the fluoride transfer in NF membranes by taking into account the previously neglected phenomena such as film theory coupled to the Nernst–Planck equation which accounts for concentration polarization layer splicing. Concentration polarization is a reversible phenomenon that takes place when the rejected solutes accumulate on the membrane surface, creating a local concentration, much higher than that of the feed solution and which is very difficult to be experimentally determined. This phenomenon has negative effects on membrane performance, including reduced flux and increased membrane fouling [30]. As a consequence, there is a need to find an alternative way to predict NF process performance by exploiting the available data and extending it to unavailable data. The extended Nernst–Planck model coupled with film theory (NP-FT) and artificial neural networks (ANNs) is capable of modeling highly complex and nonlinear systems for NF membranes [25].

The mathematical model used is a coupled model of Nernst–Planck equation and the film theory proposed by Chabaane et al. [30]. It is a combination of the model based on the extended Nernst–Planck equation which is widely used in the literature to describe the transport of ionic species in membrane separation systems and the
film theory equation to describe the mechanism of solute transfer in the concentration polarization layer [30,31]. This model is characterized by three parameters: reflection coefficient \( \sigma \), solute permeability \( P_s \) and boundary layer thickness \( \delta \) [32].

ANNs have received a lot of attention, largely due to their wide range of applications and the ease with which they handle large numbers of applications and complex and highly nonlinear data. Once the ANN is trained with experimental data, it can be used in a purely predictive mode to calculate the dependent variable(s) for any value of the input variables. Process modeling is an area in which ANNs of various configurations and structures have been considered as effective tools [33–35]. On the other hand, ANNs provide a more attractive alternative to the classic “black box” models for dealing with complex phenomena. ANNs successfully apply to different types of membranes: microfiltration (MF), ultrafiltration (UF) and NF [25,35–39]. For instance, Bowen et al. [39] applied ANN to provide a means of modeling the performance of the NF process, thus used ANN to predict the rejection of single salts (NaCl, Na\(_2\)SO\(_4\), MgCl\(_2\) and MgSO\(_4\)) and mixtures of these salts at a spiral rolled NF membrane. They found good agreement between ANNs predictions and experimental data single salts and mixtures.

In this study, two methodologies, one statistical (ANN) and the other mathematical, extended NP-FT are used to predict and explain the variation of fluoride rejection as a function of permeate flux for three NF membranes (NF90, NF270 and TR60) considering four initial concentrations. The NP-FT model is used to determine the reflection coefficient \( \sigma \), the solute permeability \( P_s \), and the boundary layer thickness \( \delta \), including the concentration polarization phenomenon. While the ANN model is applied to provide a means of modeling the performance of the studied membranes. To compare the predictive power of the two models, the values of the determination coefficient \( R^2 \) are calculated for the validity. Finally, a comparison between the two models in the fitting of fluoride rejection data with the permeate flux is performed.

2. Experimental

2.1. Characteristics of the feed water

The experiments are conducted on groundwater doped by NaF at different concentrations. A very slight variation of pH and conductivity was observed. The analytical results of the feed water are presented in Table 1.

2.2. Unit pilot testing

The experiments are performed on an NF/RO pilot plant (E 3039) supplied by TIA Company ( Technologies Industrielles Appliquées, France) shown in Fig. 1. The applied TMP can vary in the range of 5–70 bar using manual valves.

The pilot plant is equipped with two identical pressure vessel operating in series. Each pressure vessel contains one element. The pressure loss is about 2 bar corresponding to 1 bar of each pressure vessel. The two spiral wound modules are equipped with two identical commercial membranes. The water to be treated is taken from the tank by a pump and introduced into the first fessel, the retentate is admitted into the second fessel and the two permeates are recovered and mixed.

The washing is carried out by a basic solution of sodium hydroxide NaOH at pH between 9 and 10 for 10 min, followed by a rinse with water, then a washing with a solution of sulfuric acid H\(_2\)SO\(_4\) at pH between 3 and 4 for 15 min.

The temperature is kept at 29°C using the heat exchanger. Samples of permeate are collected and water parameters are determined analytically following standard methods previously described [19,42]. The other parameters followed are:

- The permeate flux is given by the equation [43,44]:
  \[
  J_p = \frac{Q_p}{S} \left( \frac{L}{m^2 \cdot s} \right) - \frac{m^3}{m^2 \cdot s}
  \]

where \( S \) is the membrane surface area (m\(^2\)) and \( Q_p \) the flow rate of the permeate (L/h or m\(^3\)/s).

### Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Feed water</th>
<th>Moroccan guidelines [40]</th>
<th>WHO standards [41]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>29</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Turbidity, NTU</td>
<td>&lt;2</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>pH</td>
<td>7.41 ± 0.03</td>
<td>6–9.2</td>
<td>6.5–8.5</td>
</tr>
<tr>
<td>pHs</td>
<td>7.80 ± 0.05</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Electric conductivity, µS/cm</td>
<td>1,492 ± 26</td>
<td>2700</td>
<td>–</td>
</tr>
<tr>
<td>Hardness, mg/L CaCO(_3)</td>
<td>440 ± 8</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Alkalinity, mg/L CaCO(_3)</td>
<td>320 ± 10</td>
<td>200</td>
<td>–</td>
</tr>
<tr>
<td>Fluoride, mg/L</td>
<td>5–10–15–20 ± 0.6</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Sulphate, mg/L</td>
<td>116 ± 4</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Nitrate, mg/L</td>
<td>20 ± 0.5</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Chloride, mg/L</td>
<td>560 ± 10</td>
<td>750</td>
<td>250</td>
</tr>
<tr>
<td>Sodium, mg/L</td>
<td>246 ± 15</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
The recovery rate (Y) is defined as:

\[
Y(\%) = \frac{Q_p}{Q_0} \times 100
\]  

(2)

where \(Q_p\) is the permeate flow (L/h) and \(Q_0\) the feed flow (L/h).

Salt rejection (R) is defined as:

\[
R = \left(1 - \frac{C_p}{C_0}\right) \times 100
\]  

(3)

where \(C_p\) is the solute concentration in permeate (g/L) and \(C_0\) the solute concentration in the feed water (g/L).

2.3. Characteristics of the membranes

The two spiral wound modules are equipped with two identical commercial NF membranes. Table 2 gives the characteristics of the membranes used. After the run, the membranes are cleaned with alkaline and acidic cleaning solutions according to the manufacturer recommendations.

2.4. Application of ANN

The feed-forward, back propagation network (FFBPN) model is constructed in this study. In the initial phase, the model is trained using the available historical data. If the model does not meet expectations, it is then used to propagate the process until it reaches the best requirement using the Levenberg–Marquardt (LM) backpropagation algorithm [46]. The network is composed of three layers, namely the input, hidden and output layers. The proposed network has been used by the sigmoid function (logsig + tansig) in the hidden layer and the linear function (Purelin) in the output layer. The input layer receives the input signals from the other source [47]. On the other hand, ANN algorithms, programmed in the Matlab toolbox for neural networks, are used to predict variation of the fluoride rejection as a function of permeate flux for the three different NF membranes used in this work. The input data of the ANN network are the permeate flux while the output

![Fig. 1. Schematic diagram of the NF/RO pilot plant [9]. T: Tank; M: NF module; P: Permeate recirculation; R: Retentate recirculation; H: Heat exchanger; 1: High pressure pump; 2: Pressure sensor. 3: Pressure regulation valves.](image)

Table 2

<table>
<thead>
<tr>
<th>Characteristics of the membranes used</th>
<th>NF270**4040</th>
<th>NF90*4040</th>
<th>TR60*4040</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (m²)</td>
<td>7.6</td>
<td>7.6</td>
<td>6.8</td>
</tr>
<tr>
<td>Salt rejection (%)</td>
<td>&gt;97.0%</td>
<td>97%</td>
<td>–</td>
</tr>
<tr>
<td>(P_{\text{max}}) (bar)</td>
<td>41</td>
<td>41</td>
<td>10</td>
</tr>
<tr>
<td>Material</td>
<td>Polyamide</td>
<td>Polyamide</td>
<td>Polyamide</td>
</tr>
<tr>
<td>Contact angle (°) [45]</td>
<td>27</td>
<td>54</td>
<td>–</td>
</tr>
<tr>
<td>Zeta potential (mV) [45]</td>
<td>4.9</td>
<td>3.7</td>
<td>–</td>
</tr>
<tr>
<td>pH = 3</td>
<td>–25.6</td>
<td>–19.4</td>
<td>–</td>
</tr>
<tr>
<td>pH = 12</td>
<td>–25.6</td>
<td>–19.4</td>
<td>–</td>
</tr>
</tbody>
</table>

- Salt rejection based on the following test conditions 2,000 ppm MgSO₄, 77°F (25°C), and 15% recovery rate at the pressure 4.8 bar.
- Salt rejection based on the following test conditions 2,000 ppm NaCl, 77°F (25°C), and 15% recovery rate at the pressure 10 bar.
- Salt rejection based on the following test conditions 2,000 ppm NaCl, 77°F (25°C), and 15% recovery rate at the pressure 15.5 bar.
is the fluoride rejection which are available for four concentrations, 5, 10, 15, and 20 mg/L.

3. Theoretical background of modeling methods

3.1. Extended Nernst–Planck equation coupled with film theory

The model used in this article is described by Chaabane et al. [30] and based on the extended Nernst–Planck equation simplified by Dresner [48], which can be written as follows:

\[
J_j = -P_j \left( \frac{dC_j}{dx} + \frac{FZ_j C_j}{RT} \frac{dY_j}{dx} \right) + J_j C_j (1 - \sigma_j) \tag{4}
\]

Rearrangement of equation with the electroneutrality conditions:

\[
\sum Z_i C_i = 0
\]

\[
\sum J_i C_i = 0
\]

\[Z_i \text{ is the ion valence. Give the Kedem–Speigler equation:} \]

\[
J_j = -P_j \frac{dC_j}{dx} + J_j C_j (1 - \sigma_j) \tag{6}
\]

Eq. (6) is integrated over the membrane thickness \((0 < x < \Delta x)\) using the following boundary conditions:

For \(x = 0, C_{in} = C_j\) and for \(x = \Delta x, C_{in} = C_j'\).

\[
C_j = \left\{ \begin{array}{ll}
1 - \frac{\sigma_j}{1 - \sigma_j} & \text{for} \quad x = 0 \\
\exp \left[ \frac{J_j (1 - \sigma_j)}{P_j} \right] & \text{for} \quad x = \Delta x 
\end{array} \right. \tag{7}
\]

Taking into account concentration polarization phenomena which occurs in a boundary layer adjacent to the membrane/feed solution interface where the cross-flow velocity is essentially laminar and the backward transport of the retained solute is by diffusion. The thickness of the polarization layer \(\delta\) can vary from a few microns to hundreds of microns, depending on factors such as the nature of the feed solution, module design, cross-flow rate and TMP [49,50].

In the steady state, and according to the film theory [51–53], the convective transport of solute across the membrane is therefore equivalent to the diffusive back-transport of the solute plus that passing with the permeate \(J_j\), the following mass balance can be constructed:

\[
J_j C_j = J_j C_j' - D_j \frac{dc}{dx} \tag{8}
\]

Integration of Eq. (8) over the thickness of the boundary layer results in:

\[
\frac{C_j - C_j'}{C_j - C_j} = \exp \left( \frac{J_j}{Q} \right) \quad \text{with} \quad Q = \frac{D_j}{\delta} \tag{9}
\]

where \(Q\) is the mass transfer coefficient.

The observed retention \(R_{obs}\) of fluoride can be written as follows:

\[
R_{obs} = 1 - \frac{C_j}{C_j'} \tag{10}
\]

With Eqs. (7) and (9), \(R_{obs}\) can be given by:

\[
R_{real} = 1 - \frac{1}{1 + \frac{\sigma_j}{1 - \sigma_j} \left\{ \exp \left( \frac{-J_j \delta}{D_j} \right) - \exp \left( -\frac{J_j (1 - \sigma_j)}{P_j} - \frac{J_j \delta}{D_j} \right) \right\}} \tag{11}
\]

Fig. 2 shows this case, assuming no fouling layer is present on the membrane/supply interface [54].

3.2. ANN approach

ANNs provide a rich framework for modeling nonlinear phenomena and for solving multivariate regression problems [25]. The ANN is a processing system composed of neurons (nodes) and connections between them that can be used to map input and output data, where the factors are called inputs and the response is called target (experimental response) or output (predicted response) [55]. According to the theory of universal approximation, a functional relationship between an arbitrary number of input variables and an arbitrary number of output variables can be constructed by an ANN with only two layers (an input layer, a hidden layer and an output layer), whose transfer function of the hidden layer is nonlinear and whose transfer function of the output layer is purely linear. The transfer functions compute the output of a layer from its net input and contain log-sigmoid (logsig), tan-sigmoid (tansig) and pure linear (purlin) functions [56,57]. There are several ANN models such as the feed-forward model, the multilayer perceptron (MLP) and the radial basis function (RBF) [25].

Our study includes the use of FFBPN network due to the ability of this method to modeling any function. Fig. 3 shows the general structure of the FFBPN model. Moreover, learning rule FFBPN is used to adjust the weight values and threshold values of a system to obtain the minimum error [58]. In addition, the input neurons received the experimental data and the network provided its outputs (ANN simulation data). If the output of the ANN network is not equal to the experimentally measured outputs, our procedure calculates the mean square error between the two values and modified the ANN network weights to minimize it. The training data are normalized to the interval [0–1]. Sixty percent of the collected data are used for model training, while 30% of the data is equally divided for testing and validation, respectively. The model is then trained based on Eq. (11) until the total mean square error is (or becomes) minimal [58].

\[
x_i = \sum W_i x_i \tag{12}
\]
where $x_J$ is the variable’s new value, $x_i$ is the variable’s initial value and $W_{ji}$ is the neuron/variable’s weight link value. The activation function between the input and the hidden layer is (tansig) and log-sigmoid (logsig), as indicated by Eq. (12).

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{and} \quad f(x) = \frac{1}{1 + e^{-x}}$$

The purelin function is used as Eq. (13) between the hidden layer and the output layer:

$$f(x) = x$$

Finally, the reliability of the model is validated with a new (unknown) data set and the results are found to be satisfactory. The rationale for these steps is discussed in the results section.

4. Results and discussion

In previous study [9], RSM statistical method was used for optimizing the defluorination operation using three membranes (TR60, NF270 and NF90). Furthermore, two theoretical models Spiegler–Kedem and Spiegler–Katchalsky were used to explain the ion transfer across these membranes [18]. The objective of this work is to test the extended Nernst–Planck model coupled with film theory (NP-FT) that accounts for the polarization layer is used to determine the phenomenological parameters of the models, namely the reflection coefficient $\sigma$, the solute permeability $P_s$ and $\delta$, the thickness of the boundary layer. In addition, the ANN model is also used to predict the fluoride rejection as a function of permeate flux. Finally, a comparison between the two models will be highlighted.

4.1. Nernst–Planck modeling coupled with film theory

4.1.1. Effect of transmembrane pressure

The variation of the permeate fluoride content and the permeate flux (Fig. 4) as a function of TMP is studied in batch mode and for a groundwater from the Ben Guerir region doped with different fluoride concentrations, already published in our previous paper [9,18].

As shown in Fig. 4, the permeate flux increases almost linearly with TMP according to Darcy’s law [59]. This flow behavior is well established in the literature [9,10,18]. In this illustration, the permeate flux follows the following order: NF270 > TR60 > NF90. Namely that the permeability of solvent water found 5.58 × 10$^{-6}$; 1.15 × 10$^{-6}$; 2.49 × 10$^{-6}$ m$^3$/m$^2$s bar for NF270, NF90, TR60 respectively. For all three membranes studied, fluoride passage into the permeate increases with increasing initial feed fluoride content and decreases with increasing TMP, except for NF270, a slight increase is observed for TMP above 10 bar.

4.1.2. Parameters of the model

The applied model, NP-FT has been used to study the influence of the initial fluoride concentration of the feed on the reflection coefficient ($\sigma$), the solute permeability ($P_s$) and the boundary layer thickness ($\delta$) taking into account the concentration polarization phenomenon. For this aim, a program in Python language has been developed on the basis of the flowchart (Fig. 5) proposed by Chaabane et al. [30]. The results obtained from the model simulation are used to plot the curves in the Origin software depicted in Fig. 6. The experimental data obtained experimentally are indicated by solid symbols, while the solid lines represent the function of the model with the parameters found by the program. A summary of the calculated model parameters are collected in Table 3.

Fig. 6 shows, for the studied membranes, and for the range of initial fluoride concentrations, a good fit of fluoride rejection obtained experimentally and by the NP-FT model. The obtained $R^2$-squared values confirm this accordance.

The calculated reflection coefficients are almost equal to unity for NF90. For NF270, the values found exceed 0.97 which are higher than those found in the paper by Tahaikt et al. [18], where the concentration polarization was neglected. For TR60, $\sigma$ is around 0.89 which is almost the same value obtained previously by Tahaikt et al. [18]. This indicates that the influence of concentration polarization is more important for NF270 than for TR60 and NF90. The high reflection coefficients indicate that convective transport in the NF90 membrane is almost completely hindered. In addition, these results could be explained on the basis of...
the steric hindrance mechanism, since NF90 has relatively small pores, as shown by atomic force microscopy [25]. For TR60 and NF270, both diffusive and convective mechanisms are present, but the predominance of diffusion is more pronounced for NF270 than for TR60. On the other hand the boundary layer thickness for NF270 is 10 times larger than that of TR60 which means that NF270 retains more fluoride.

The results obtained show that the greater the thickness layer, the lower the permeability. δ is about 10⁻⁵ m for NF270 and 10⁻⁴ m for TR60. On the other hand, the obtained δ for NF90 is 10⁻¹⁴ m which is very low or even non-existent. This explains the order of the solute permeability: TR60 > NF270 > NF90. However, the values obtained for either δ and $P_s$ are very low.

The formation of the boundary layer is generally due to the accumulation of the retained solute on the membrane surface, which is a function of the flow of the permeate. The lower the concentration of fluoride on the surface of the membrane, the lower the thickness of the boundary layer. In reality, during NF, other phenomena are involved in the formation of the boundary layer, such as the dielectric exclusion effect, the electrical interaction between the charged membrane surface and the solvent as well as the screen effect, which occurs when the concentration of different ions increases, the counter ions and other retained co-ions form films (also called “screen”) near the surface of the membrane, which are able to reduce or eliminate the solute particles [18,30].

The plot of the evolution of $P_s$ and δ (Fig. 7) as a function of the initial fluoride ion concentration shows how these two parameters are influenced slightly by the initial fluoride content for TR60, NF270 and NF90. As shown in Fig. 7, the behavior of these two parameters is almost linear. The increase of the fluoride content in the raw water causes a slight decrease in $P_s$ for NF270 and TR60. On the other hand, for NF90, $P_s$ values are very low and a very slight occurs with increase of fluoride concentration is observed.

Table 3 gives the estimated parameters of the NP-FT model, namely the reflection coefficient ($\sigma$), solute permeability ($P_s$), and boundary layer thickness (δ) considering the concentration polarization phenomenon.
Fig. 5. Program flowchart considering concentration polarization phenomenon.

Table 3
Parameters, \( \sigma, P_s \), and \( \delta \) obtained by the coupled NP-FT model

<table>
<thead>
<tr>
<th>Membrane</th>
<th>( C_i ) (ppm)</th>
<th>( \delta ) (m)</th>
<th>( \sigma )</th>
<th>( P_s )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR60</td>
<td>5</td>
<td>1.6 ( \times 10^{-6} )</td>
<td>0.890</td>
<td>4.90 ( \times 10^{-6} )</td>
<td>0.900</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.6 ( \times 10^{-6} )</td>
<td>0.897</td>
<td>3.28 ( \times 10^{-6} )</td>
<td>0.900</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.4 ( \times 10^{-6} )</td>
<td>0.900</td>
<td>4 ( \times 10^{-6} )</td>
<td>0.800</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1.4 ( \times 10^{-6} )</td>
<td>0.800</td>
<td>2.84 ( \times 10^{-6} )</td>
<td>0.880</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.5 ( \times 10^{-5} )</td>
<td>0.980</td>
<td>2.24 ( \times 10^{-6} )</td>
<td>0.700</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>6.8 ( \times 10^{-5} )</td>
<td>0.970</td>
<td>6.68 ( \times 10^{-7} )</td>
<td>0.760</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>6.5 ( \times 10^{-5} )</td>
<td>0.970</td>
<td>6.14 ( \times 10^{-7} )</td>
<td>0.750</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>6 ( \times 10^{-5} )</td>
<td>0.980</td>
<td>4.43 ( \times 10^{-7} )</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>3.4 ( \times 10^{-14} )</td>
<td>0.991</td>
<td>5.74 ( \times 10^{-8} )</td>
<td>0.720</td>
</tr>
<tr>
<td>NF270</td>
<td>10</td>
<td>3.8 ( \times 10^{-14} )</td>
<td>0.993</td>
<td>4.76 ( \times 10^{-8} )</td>
<td>0.700</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.0 ( \times 10^{-14} )</td>
<td>0.994</td>
<td>6.03 ( \times 10^{-8} )</td>
<td>0.810</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>4.2 ( \times 10^{-14} )</td>
<td>0.994</td>
<td>6.84 ( \times 10^{-8} )</td>
<td>0.818</td>
</tr>
<tr>
<td>NF90</td>
<td>5</td>
<td>3.4 ( \times 10^{-14} )</td>
<td>0.991</td>
<td>5.74 ( \times 10^{-8} )</td>
<td>0.720</td>
</tr>
</tbody>
</table>
4.2. Simulation by ANN

4.2.1. ANN prediction of fluoride rejection as a function of permeate flux

The ANN prediction (Fig. 8) of fluoride rejection as a function of permeate flux for different initial concentrations and for the three NF membranes is performed. The experimental data used in the learning process are indicated by solid symbols, while the solid lines represent the best polynomial fit of the ANN prediction.

Fig. 8 shows the ANN predictions for the fluoride rejection as a function of permeate flux data for the three membranes using different initial fluoride concentrations. It is clear that there is good agreement between the ANN prediction and the experimental data for the three membranes with very little deviation (slight difference). The good prediction by ANN obtained can be explained by the fact that the sweatpants used in the learning phase are relatively few, and the use of other set-points, especially at the level of initial concentration of fluoride intermediate, will certainly improve the interpretive performance of ANN predictions as shown in previous work [25]. The ANN model perfectly describes the fluoride rejection data for TR60, NF270 and NF90 membranes.

4.2.2. Validation of the model ANN

The results obtained for the training, the test, the validation and the global $R^2$ for the training data set, for the three membranes are depicted in Fig. 9.

The summaries of the $R^2$ plots during the training, testing, and validation stages during the training process for the three membranes are shown in Fig. 9. In order to evaluate the ANN model, the model is presented with new fluoride rejection values that are not used during training. The fluoride rejection values estimated by the ANN
Fig. 7. Variation of permeability (a) and layer thickness (b) as a function of initial fluoride concentration for the three membranes.

Fig. 8. Experimental and adjustment data by (ANN), fluoride rejection as a function of permeate flux.
model is then compared to their corresponding actual values. An overall $R^2$ value of 1–0.9483–0.9982 is obtained for TR60, NF270 and NF90, respectively, showing a perfect correlation (output exactly equal to the target). On the other hand, for all curves, the points are located very close to the straight line and the best-fit equations for the training, validation, test, and ensemble subsets have mainly a range of slope between 0.97 and 1. These results indicate that the trained ANN model could accurately simulate the fluoride rejection as a function of permeate flux for the NF process dealing with fluoride removal.

4.3. Comparison of the ANN and NP-FT models

The experimental results predicted by both ANN and NP-FT models are compared in terms of coefficient of
determination ($R^2$). Table 4 shows the results obtained for the three membranes. According to the results reported in Table 4 and the deviation of the predicted response values from the experimental data for both models illustrated in Fig. 10, both models show good and satisfactory $R^2$. However, the ANN model reveals a slight superiority over the NP-FT model. This slight prediction accuracy can be attributed to the general ability of ANNs to approximate the nonlinearity of the system \cite{60,61}. In addition, ANNs have the advantage of flexibility and the ability to add new experimental data to build a more reliable ANN model \cite{62}, but, they have the disadvantage of providing little information about the membrane structures. In contrast the applied NP-FT model helps us to understand the transfer mechanism in the concentration polarization layer for the three membranes studied. In other words, both approaches are complementary to improve predictive modeling in the membrane system.

The scatter plot of the predicted values of the NP-FT and ANN models vs. the actual values for the three tested membranes is presented in Fig. 10.

5. Conclusion

In this work, three NF membranes are studied on NaF-doped groundwater at different fluoride ion concentrations of which the experimental data are correlated and analyzed using two models (NP-FT and ANN).

<table>
<thead>
<tr>
<th>Membrane</th>
<th>$C_i$ (ppm)</th>
<th>$R^2$</th>
<th>ANN</th>
<th>NP-FT model</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR60</td>
<td>5</td>
<td>0.999</td>
<td>0.900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.999</td>
<td>0.900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.999</td>
<td>0.880</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.819</td>
<td>0.700</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.927</td>
<td>0.760</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.884</td>
<td>0.750</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.845</td>
<td>0.770</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.989</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.999</td>
<td>0.700</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.967</td>
<td>0.810</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.999</td>
<td>0.818</td>
<td></td>
</tr>
</tbody>
</table>

Table 4

ANN and NP-FT models comparison

Fig. 10. Predicted values vs. actual values of ANN and NP-FT models for the three membranes.
According to the NP-FT model, the transfer mechanism of fluoride ions in NF membranes is studied and the obtained results show that $\sigma$ are almost around (1–0.97–0.89) for the three membranes (NF90, NF270 and TR60) respectively. These values indicate that the transport is diffusive in NF90, while for TR60 and NF270 both mechanisms are present, but the predominance of diffusion is more pronounced for NF270 than for TR60. On the other hand, the thickness of the boundary layer formed by the gradient of fluoride ions is small and is about $10^{-5}$ m for NF270 and $10^{-4}$ m for TR60. In addition, the value obtained for the NF90 membrane is $10^{-14}$ m which, is very small, even non-existent. The greater the thickness of the boundary layer, the lower the permeability, and the permeability obtained follows the order below:

\[ \text{TR60} > \text{NF270} > \text{NF90} \] (15)

The ANN model shows a good agreement between the ANN prediction and the experimental data for all the studied membranes with a very low deviation. In terms of comparison between the two models used, the ANN model presents a superiority in the prediction of fluoride rejection, but this model gives no information neither on the mode of transfer, neither on the phenomena which intervenes in this transfer. On the opposite, the NP-FT model gives a good prediction, in addition to the determination of the phenomenological parameters to know (the coefficient of reflection $\sigma$, the permeability of the solute $P_r$, and the thickness of the boundary layer $\delta$).

**Symbols**

$P_s^*$ — Permeability of ion, m$^2$/s

$P_r$ — Local solute permeability, m$^2$/s

$C_i$ — Concentration in the membrane surface, mg/L

$C_i^*$ — Concentration of ion, mg/L

$C_p$ — Permeate concentration, mg/L

$C_o$ — Feed concentration solution, mg/L

$D_i$ — Effective diffusion coefficient of solute, m$^2$/s

$F$ — Faraday constant, C/mol

$I_s$ — Solute flux, kg/m$^2$/m$^2$ s

$I_f$ — Volume flux, m$^3$/m$^3$ s

$Q$ — Mass transfer coefficient in the boundary layer, m$^3$/m$^2$ s

$R$ — Universal gas constant, J/K mol

$R_{obs}$ — Observed solute retention, %

$R_{real}$ — Real solute retention, %

$T$ — Absolute temperature, K

$x$ — Distance variable, m

$z_i$ — Ion valence number

**Greek**

$\sigma$ — Reflection coefficient

$\delta$ — Thickness of the boundary layer, m

$\psi$ — Electrostatic potential of the system, V

$\xi$ — Error

**References**


M. Jarzynska, M. Pietruszka, The application of the Kedem–Katchalsky equations to membrane transport of ethyl alcohol and glucose, Desalination, 280 (2011) 14–19.


L. Dresner, Some remarks on the integration of the extended Nernst–Planck equations in the hyperfiltration of multicomponent solutions, Desalination, 10 (1972) 27–46.


