The mathematical model of ecological carrying capacity evaluation in the water source area of Haikou tidal flat habitat

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ABSTRACT

Water environment carrying capacity is the link between human activities and water environment system, determining the coordination between social economy and water environment system. The traditional evaluation methods cannot normalize the evaluation index and control the skewness coefficient of the index, resulting in the large root mean square error of the evaluation results. Therefore, this paper proposes a mathematical model for evaluating the ecological carrying capacity of Haikou tidal flat habitat. Firstly, the evaluation index of ecological carrying capacity was selected from system elasticity, resource carrying capacity and carrying pressure, and then the indexes were regularized. Secondly, fuzzy mathematics method was adopted to construct the ecological carrying capacity evaluation mathematical model of Haikou tidal flat biological habitat water source area by the selected indicators. Eventually, the ecological carrying capacity evaluation was achieved. The results show that the indexes evaluated by this method have low skewness coefficient and low root mean square error (<0.08).

Keywords: Haikou tidal flat; Biological habitat; Water source area; Ecological Carrying Capacity; Evaluation model

1. Introduction

Haikou tidal flat is located at the junction of land and sea, which is the habitat of migratory birds and benthos, and also the land reserve resources in coastal areas [1]. Haikou tidal flat is very important to social economy and ecological environment, therefore, it has been widely concerned [2]. Recently, the reduction of incoming sediment and the rise of sea level in the basin have resulted in the slow deposition of tidal flats in many estuaries in the world, even from siltation to scouring [3,4]. Therefore, the protection of tidal flat wetland has attracted more attention, and it is imperative to evaluate the ecological carrying capacity of Haikou tidal flat habitat water source area [5].

Zhang et al. [6] proposed an evaluation method of ecological carrying capacity of water source area based on multiple driving. According to the water resources, water environment and social economy, the method selected 16 indicators, such as water resources utilization rate, sewage treatment rate, water consumption per ten thousand yuan GDP, etc., and established the evaluation model of water ecological carrying capacity based on particle swarm gravity search algorithm projection pursuit with multiple driving function. However, this method ignored the regularization of indicators, causing the inability to control the skewness coefficient of indicators. Huang and Wei [7] reported an evaluation method of ecological carrying capacity based on subjective and objective evaluation. This method combined the objective evaluation method with the subjective evaluation method to find the evaluation system from the natural, economic and social aspects, thus evaluating the ecological carrying capacity. However, the evaluation indexes selected by this method were abnormal and the root mean square error was large. Jiao et al. [8] proposed an evaluation method of ecological carrying capacity of water source area based on ESEF. In this method, water support, water quality limitation and water ecological stability were...
considered. Based on the ESEF, the water ecological carrying capacity was evaluated by the union method and the average value method. However, there is a big difference between the index skewness coefficient and zero, leading to the disaccord between the evaluation results and the actual ecological carrying capacity [9–12].

In this regard, a mathematical model for evaluating ecological carrying capacity of Haikou tidal flat habitat water source area was constructed in this paper. The main contents are as follows:

• The evaluation index of ecological carrying capacity was selected.
• The evaluation index of ecological carrying capacity was normalized.
• According to the fuzzy mathematics method, the ecological carrying capacity evaluation mathematical model of Haikou tidal flat habitat water source area was constructed.
• Experimental results and discussion. The effectiveness of the method in this word was verified from the index skewness coefficient and root mean square error.
• Conclusions.

2. Construction and regularization of ecological carrying capacity evaluation index system

2.1. Index selection

2.1.1. Evaluation index system of elasticity

Target layer: the first-level evaluation takes the elastic force of system as the evaluation index. Its purpose is to measure the potential carrying capacity of different regional ecosystems [13–15].

Criteria layer: the main factors affecting the ecosystem elasticity include geomorphology, climate, soil, vegetation, hydrology and human factors. Therefore, six factors are selected as the evaluation indexes of criteria layer.

Index layer: according to the characteristics and carrying capacity of each index in criterion layer, eight evaluation indexes are selected. The specific groups are shown in Fig. 1.

2.1.2. Evaluation index system of resource carrying capacity

The two-tier evaluation is based on the carrying capacity of single factor of resource and environment, and takes the carrying capacity of resource and environment as the target. It can be divided into target layer, criterion layer and index layer [16,17].

Target level: the target is the resource-environment carrying capacity. In practical application, this method can be used to compare the difference of carrying capacity in different regions, or evaluate the dynamic change of the carrying capacity in different development stages of the same region.

Criterion layer: water resource, cultivated land resource, forestry resource and animal husbandry resource are selected as evaluation factors.

Index layer: according to the characteristic of the factor in criterion layer and the data, each factor reflects the situation through specific indexes. The specific composition of factor is shown in Fig. 2.

2.1.3. Evaluation index system of carrying pressure index

The index system is composed of target layer, criterion layer and index layer.

![First-order index system](image)

Fig. 1. First-order index system.
Target layer: the three-tier evaluation takes the carrying pressure as the target. Its purpose is to reflect the relationship between the objective carrying capacity and the pressure of the load-carrying object, so the target layer takes the load-carrying degree as the measurement index [18].

Criterion layer: in the eco-economic system, people are the ultimate aim and the sustainable development object. Therefore, the population pressure degree is taken as the evaluation criterion. The pressure degree of population on resources is selected as the index.

Index layer: the resource elements are taken as specific evaluation indexes. The pressure degree of water resources, cultivated land resources, forestry resources, animal husbandry and tourism resources are taken as evaluation indexes. The specific composition of factor is shown in Fig. 3.

2.2. Index regularization

From definite index vectors, we can see that there is obvious difference in the order of magnitude among the indexes and the difference in the dimensions among index values, so it is necessary to deal with these indexes [19–22].

The range normalization method is used to process the evaluation index, or normalize the range of the i-th evaluation value. That is to transform $X_{ij}$ into:

$$
\mu(X_{ij}) = \frac{X_{ij} - \min X_{ij}}{\max X_{ij} - \min X_{ij}}
$$

(1)

After the transformation, the original data can be effectively compressed within [0,1], and the discrete degree has

![Fig. 2. Two-tier evaluation index system.](image)

![Fig. 3. Three-tier evaluation index system.](image)
consistency. Based on formula (1), the membership model of each index is built by piecewise function.

\[ \mu(x) = \begin{cases} 
0, x \leq \min x \\
\frac{x - \min x}{\max x - \min x}, \min x < x < \max x \\
1, x \geq \max x 
\end{cases} \quad (2) \]

In practical problems, in addition to calculating the values of \( \max x \) and \( \min x \) by survey data, empirical values, national standard or target values and reference values of the index can also be used to replace the index [23–25].

2.2.1. Topography and landform

Topography and landform restrict the development of habitat in Haikou tidal flat. The land surface relief RDLS is used to measure the influence of landform factors on the habitat in Haikou tidal flat. The land surface relief RDLS is expressed as:

\[ \text{RDLS} = \frac{\text{max}(h) - \text{min}(h)}{\text{max}(H) - \text{min}(H)} \times \left(1 - \frac{P(A)}{A}\right) \quad (3) \]

In the formula, \( \text{max}(h) \) is the highest altitude of a region. \( \text{max}(H) \) is the highest altitude in China. \( \text{min}(h) \) is the lowest altitude of a region. \( \text{min}(H) \) is the lowest altitude in China. \( P(A) \) is the area occupied by the flat land in a region. \( A \) is the total land area in a region. The higher the RDLS value, the bigger the “stress” or “force” of the terrain on the habitat, the weaker the environment.

The membership model of relief amplitude is expressed as follows:

\[ \mu(x) = \begin{cases} 
1, x = 0 \\
1 - \frac{x - 0}{1 - 0}, 0 < x < 1 \\
0, x = 1 
\end{cases} \quad (4) \]

2.2.2. Climatic factors

Frost-free period and dryness index are used to measure the influence of climate factors on ecosystem. Dryness is an index to reflect the degree of regional dryness and wetness. It also reflects the water conservation and agricultural production type. The frost-free period is an important index of heat resource in agricultural climate, which can reflect the crop growth rate. The longer the frost-free period, the longer the growth period of crop. The heat resources are also rich [26]. The formula of calculating the dryness index is shown as follows:

\[ K = 0.16 \sum T \geq 10^\circ C \frac{R}{R} \quad (5) \]

In the formula, \( K \) is the dry degree. \( \sum T \geq 10^\circ C \) is the accumulated temperature which is not less than 10°C. \( R \) is the average annual rainfall. The membership model of frost-free period was determined by the thermal preparation.

\[ \mu(x) = \begin{cases} 
0, x \leq 120 \\
\frac{x - 120}{330 - 120}, 120 < x < 330 \\
1, x \geq 330 
\end{cases} \quad (6) \]

The membership model of aridity index is expressed as follows:

\[ \mu(x) = \begin{cases} 
1, x \leq 1 \\
1 - \frac{x - 1}{4 - 1}, 1 < x < 4 \\
0, x \geq 4 
\end{cases} \quad (7) \]

2.2.3. Soil factors

The empirical formula of soil erosion modulus is adopted.

\[ \begin{align*}
Y &= 11182e^{-0.042x} \\
R &= -0.988
\end{align*} \quad (8) \]

In the formula, \( Y \) is soil erosion modulus. \( v \) is the vegetation coverage ratio. \( R \) is the correlation coefficient.

The membership model of soil erosion modulus is:

\[ \mu(x) = \begin{cases} 
1, x \leq 200 \\
1 - \frac{x - 200}{15000 - 200}, 200 < x < 1500 \\
0, x \geq 15000 
\end{cases} \quad (9) \]

2.2.4. Vegetation factors

The vegetation coverage ratio reflects the quality of ecological environment in a certain region, and plays an important role in the anti-interference ability and regulation buffer capacity of the regional ecosystem [27]. Vegetation coverage rate \( z_m \) is selected as the representative index of vegetation factor to measure the system elasticity.

\[ z_m = \frac{l_m}{z_m} \times 100\% \quad (10) \]

In the formula, \( l_m \) is the area of forest and grassland. \( z_m \) is the total land area.

The membership model of vegetation coverage is shown as follows:

\[ \mu(x) = \begin{cases} 
0, x = 0 \\
\frac{x - 0}{90\% - 0\%}, 0 < x < 90\% \\
1, x \geq 90\% 
\end{cases} \quad (11) \]
2.2.5. Hydrologic factors

Surface water and groundwater are important water supply sources in northern China, which play a very important role in ensuring and promoting social and economic development.

The membership model of surface runoff index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 0.2 \\
\frac{x - 0.2}{10.7 - 0.2} & 0.2 < x < 10.7 \\
1 & x \geq 10.7 
\end{cases} \quad (12)$$

The membership model of groundwater index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 3.15 \\
\frac{x - 3.15}{27.18 - 3.15} & 3.15 < x < 27.18 \\
1 & x \geq 27.18 
\end{cases} \quad (13)$$

2.2.6. Human development factors

The per capita income of farmers and herdsmen is used as an index to measure the elasticity of system. The expression of membership model is shown as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 1083 \\
\frac{x - 1083}{4330 - 1083} & 1083 < x < 4330 \\
1 & x \geq 4330 
\end{cases} \quad (14)$$

2.2.7. Animal husbandry resources

Per-capita grassland area index and unit area of grassland occupied by sheep are used to reflect the carrying capacity of animal husbandry resources.

The membership model of per-capita grassland area index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 1.89 \\
\frac{x - 1.89}{4.43 - 1.89} & 1.89 < x < 4.43 \\
1 & x \geq 4.43 
\end{cases} \quad (15)$$

The membership model of grassland area occupied by sheep is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 0.52 \\
\frac{x - 0.52}{2.78 - 0.52} & 0.52 < x < 2.78 \\
1 & x \geq 2.78 
\end{cases} \quad (16)$$

2.2.8. Water resources

The per capita total water resource index and water resource utilization rate are used as the evaluation indexes of water resource carrying capacity.

The membership model of per capita total water resources index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 1000 \\
\frac{x - 1000}{8800 - 1000} & 1000 < x < 8800 \\
1 & x \geq 8800 
\end{cases} \quad (17)$$

The expression of membership model of water resources utilization rate is shown as follows:

$$\mu(x) = \begin{cases} 
1 & x \leq 1000 \\
1 - \frac{x - 1.5\%}{40\% - 1.5\%} & 1.5\% < x < 40\% \\
0 & x \geq 40\% 
\end{cases} \quad (18)$$

2.2.9. Forest resource

Forest coverage rate, per capita forest area index, forest productivity and proportion of forest land to forestry land are used to reflect the carrying status of forestry resource.

The expression of membership model of forest coverage rate is shown as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 16.55\% \\
\frac{x - 16.55\%}{30\% - 16.55\%} & 16.55\% < x < 30\% \\
1 & x \geq 30\% 
\end{cases} \quad (19)$$

The expression of membership model of per capita forest area is shown as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 0.002 \\
\frac{x - 0.002}{0.610 - 0.002} & 0.002 < x < 0.610 \\
1 & x \geq 0.610 
\end{cases} \quad (20)$$

The expression of membership model of forest productivity is shown as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 7.68 \\
\frac{x - 7.68}{284.86 - 7.68} & 7.68 < x < 284.86 \\
1 & x \geq 284.86 
\end{cases} \quad (21)$$

The expression of membership model of the proportion of forest land to forestry land is shown as follows:
can establish the actual index matrix to the weight values provided by the ecological carrying capacity of water source area applied to the grading assessment for several evaluation criteria. Mathematics provides great convenience for this.

The per capita cultivated land area index and crop yield index are used to reflect the carrying capacity of cultivated land resources. The membership model of per capita cultivated land area index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 0.15 \\
\frac{x - 0.05}{0.12 - 0.05} & 0.05 < x < 0.12 \\
1 & x \geq 0.12
\end{cases} \tag{23}$$

The membership model of crop yield index is expressed as follows:

$$\mu(x) = \begin{cases} 
0 & x \leq 315 \\
\frac{x - 3155}{7044 - 3155} & 3155 < x < 7044 \\
1 & x \geq 7044
\end{cases} \tag{24}$$

3. Evaluation method of ecological carrying capacity in water source area

In the process of ecological carrying capacity evaluation of water source area, the complexity and multidimensionality of the ecosystem and the inherent fuzziness of the relationship between various characteristics lead to great difficulties for comprehensive evaluation. The fuzzy mathematics provides great convenience for this.

The process that the fuzzy recognition method is applied to the grading assessment for several evaluation indexes of ecological carrying capacity of water source area includes five steps.

3.1. Establish the actual index matrix of sample

Let’s set X as a set of n evaluation unit samples. According to the weight values provided by m evaluation factors, we can establish the actual index matrix $X_{\text{act}}$ of sample for different units:

$$X_{\text{act}} = \begin{bmatrix} 
x_{11} & x_{12} & \ldots & x_{1n} \\
x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \ldots & x_{mn}
\end{bmatrix} = (x_{ij}) \tag{25}$$

3.2. Establish the standard matrix of evaluation index

According to the evaluation criteria, the subordinate relationship of $m$th evaluation index corresponding to C evaluation criteria is evaluated, and the standard matrix $Y_{\text{std}}$ of index is established:

$$Y_{\text{std}} = \begin{bmatrix} 
y_{11} & y_{12} & \ldots & y_{1c} \\
y_{21} & x_{22} & \ldots & y_{2c} \\
\vdots & \vdots & \ddots & \vdots \\
y_{n1} & y_{n2} & \ldots & y_{nc}
\end{bmatrix} = (y_{ij}) \tag{26}$$

3.3. Establish the relative membership matrix of evaluation index

The fuzziness of membership level of evaluation index is described by the relative membership degree. The relative membership degree of level-i standard of the index of the ith assessment factor to the evaluation target is 0. The relative membership degree of level-C standard to the evaluation target is 1. Thus, the relative membership degree of level-$h$ standard of the index $i$ between the first level and the $h$th level is:

$$S_{hi} = \frac{y_{ih} - y_{ij}}{y_{ij} - y_{ij}} \tag{27}$$

When $x_{ij} < y_{ij}$, the relative membership degree $r_{ij}$ of the index of actual evaluation unit $i$ for evaluation target is 0. When $x_{ij} \geq y_{ij}$, the relative membership degree $r_{ij}$ is 1. Then:

$$r_{ij} = \frac{x_{ij} - y_{ij}}{y_{ij} - y_{ij}} \tag{28}$$

The above formula is used to transform matrix $X_{\text{act}}$ and matrix $Y_{\text{std}}$ into the relative membership matrix $R_{\text{rel}}$ of actual index and the relative membership matrix $S_{\text{rel}}$ of evaluation index:

$$R_{\text{rel}} = \begin{bmatrix} 
r_{11} & r_{12} & \ldots & r_{1n} \\
r_{21} & r_{22} & \ldots & r_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \ldots & r_{mn}
\end{bmatrix} = (r_{ij}) \tag{29}$$

$$S_{\text{rel}} = \begin{bmatrix} 
s_{11} & s_{12} & \ldots & s_{1c} \\
s_{21} & s_{22} & \ldots & s_{2c} \\
\vdots & \vdots & \ddots & \vdots \\
s_{n1} & s_{n2} & \ldots & s_{nc}
\end{bmatrix} = (s_{ij}) \tag{30}$$

3.4. Relative membership matrix of the evaluation object relative to the evaluation level is built

The matrix $R_{\text{rel}}$ is normalized, and then the normalized matrix $w_{ij}$ is obtained. According to the Euclidean distance
parameter, the derivation formula of the relative optimal membership degree of the sample \( j \) to the loss degree in level \( h \) is:

\[
\begin{align*}
    u_{jh} = & \begin{cases} 
    1 & h = a_{\min} = a_{\max} \\
    1 - \frac{\sum_{i=1}^{n} (w_i | r_i - s_{ih})^p}{\sum_{i=1}^{n} (w_i | r_i - s_{ih})^p} & a_{\min} < h < a_{\max} \\
    0 & h < a_{\min} \text{ or } h > a_{\max}
    \end{cases} 
\end{align*}
\] (31)

where \( a_{\min} \) and \( a_{\max} \) are the minimum category and the maximum category falling into the adjacent interval of relative membership values of standard \( s_{ih} \).

The relative membership matrix \( u_{jh} \) of “ecological carrying capacity” of the sample \( j \) belonging to level \( h \) is calculated.

\[
U_{hx} = \begin{bmatrix}
    u_{h1} & u_{h2} & \cdots & u_{hx} \\
    u_{h1} & u_{h2} & \cdots & u_{hx} \\
    \vdots & \vdots & \ddots & \vdots \\
    u_{h1} & u_{h2} & \cdots & u_{hx}
\end{bmatrix}
\] (32)

The following constraint conditions should be satisfied.

\[
\sum_{h=1}^{n} u_{jh} - 1 = 0, \forall j
\] (33)

3.5. A mathematical model \( H_j \) is built to evaluate the ecological carrying capacity of the water source area of Haikou tidal flat habitat

\[
H_j = \sum_{h=1}^{n} hu_{jh}
\] (34)

where \( H_j \) is the grade characteristic value of each unit sample. That is the comprehensive evaluation result.

4. Experiments and discussions

In order to verify the effectiveness of this method, we need to test the model designed in this paper. The experimental environment is as follows:

- The host configuration of management side: CPU: 1.86 ghz; memory: 2 GB; hard disk: 250 g; operating system: Windows XP Professional SP3; Web application server: Apache tomcat6.0.
- The host configuration of managed end: CPU: 1.6 GHz; memory: 1 GB; hard disk: 160 g; operating system: Windows XP Professional SP3; database: SQL Server 2000.

The methods in this paper (method 1), evaluation method of ecological carrying capacity of water source area based on multiple driving (method 2), evaluation method based on subjective and objective evaluation (method 3) and evaluation method based on ESEF (method 4) are used to test the ecological carrying capacity of water source area. The skewness coefficient is used as test index. When the index is in normal state, the skewness coefficient is close to zero, indicating that the index is normal. Fig. 4 shows that the index skewness coefficient results for the four methods.

As shown in Fig. 4, clearly, during many iterations, the index skewness coefficient of method 1 is close to zero, showing that the index of this method conforms to normal distribution and there is no abnormality, indicating that the index selected by this method has excellent effectiveness. This is because the evaluation indexes are selected from three levels of system elasticity, resource carrying capacity and bearing pressure, and the indexes are regularized to control the skewness coefficient of the indexes. However, there is a difference between the index skewness coefficient of method 2, method 3 and method 4 and zero, indicating that the effectiveness of the indicators selected by the three methods is poor.

In order to further verify the effectiveness of the proposed method, root mean square error is selected as the test index to test method 1, method 2, method 3 and method 4.

Fig. 4. The skewness coefficient for different methods.

Fig. 5. The root mean square error for different methods.
The root mean square error less than 0.08 indicates that the evaluation results are more in line with the ecological carrying capacity of the Haikou tidal flat habitat water source area. The test results of the four methods are shown in Fig. 5.

As shown in Fig. 5, the root mean square error of method 1 is less than 0.08 during many iterations, however, the root mean square error of the other three methods is far greater than 0.08. This is because method 1 uses the regularized indicators to construct the ecological carrying capacity evaluation model of Haikou tidal flat biological habitat water source area, and the evaluation results are close to the actual situation, thus reducing the root mean square error of the evaluation results and improving the evaluation accuracy of the method.

5. Conclusions

The water environment carrying capacity determines the coordination between social economy and water environment system. It is very significant to the comprehensive development and development scale of a country or region. Currently, the indexes selected by traditional evaluation methods are abnormal, which leads to large root mean square error of evaluation results. Therefore, this paper puts forward a mathematical model to evaluate the ecological carrying capacity of the Haikou tidal flat habitat. Firstly, normalize the selected evaluation indicators. Secondly, reasonable control index skewness coefficient to build the ecological carrying capacity evaluation model. Finally, the root mean square error of the evaluation results is reduced, providing a guarantee for the protection and development of the Haikou tidal flat habitat water source area.

References


