Evaporation of CuO/H$_2$O nanofluid film liquid flow downward in a vertical parallel plate evaporator by mixed convection

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**A B S T R A C T**

The current work focuses on the numerical analysis of evaporation of (water/copper oxide) nanofilm flow downward in a parallel plate evaporator under mixed convection. Two parallel vertical plates make up the vertical channel. Copper oxide nanoparticles are added to the water-liquid film with a small volume fraction. The wetted plate is heated, while the other is dry and isothermal. The results detail the impact of copper oxide nanoparticle dispersion on the heat and mass transfer as well as the efficiency of the parallel plate evaporator. The results show that an increase in the volume fraction of copper oxide nanoparticles enhances heat and mass transfer as well as the efficiency of the parallel plate evaporator.

**Keywords:** Numerical investigation; Nanofilm; Nanoparticles; Parallel plate evaporator; Water film evaporation; Mixed convection; Heat and mass transfer; Water-copper oxide

1. Introduction

This literature review shows that the problem of the numerical study of the liquid film condensation from water-ammonia vapors mixture flowing downward along a parallel plate condenser has not been sufficiently studied. This paper aims to discuss the effects of the gas mixture and liquid mixture film’s parameters on the heat and mass transfer and the total cumulated condensation rate of the water-ammonia vapors mixture. Nanoparticle dispersion in liquids is used to improve heat transfer in many applications of heat exchangers. A part of those applications includes flowing liquid nanofilms. Thus, simultaneous heat and mass transfer between the liquid nanofilms and gas/vapor stream inside a heat exchanger is widely encountered. At this stage, the evaporation process significantly impacts performance and energy efficiency. Those processes involve conjugate heat transfer between nanofilms and gas with the change of phase due to evaporation inside a heat exchanger. Namburu et al. [1] showed that increasing the nanoparticles’ volume fraction induces an increase in pressure loss. Sefiane and Bennacer [2] experimentally analyzed the nanoparticles’ volume fraction impact on nanofluid viscosity. They showed that, during a pinning phase, an increase in nanoparticles’ volume fraction causes a decrease in drop evaporation. Gorjai et al. [3] analyzed the effect of introducing Al$_2$O$_3$ nanoparticles on heat transfer inside a three-dimensional annulus. The convective heat transfer coefficient of the nanofluid can be improved with a lower volume concentration of CuO nanoparticles, as shown by Lazarus et al. [4]. The effects of nanoparticles’ dispersion on droplet evaporation were presented by Chen et al. [5]. They demonstrated an enhancement of droplet evaporation when nanoparticles are added. A study on heat transfer caused by free convection of nanofluid flowing on a vertical plate has been affected by Siddiqa et al. [6]. They demonstrated that the nanoparticles’ dispersion improved the heat exchange. The free convective heat transfer within
a porous wavy cavity with a nanofluid was numerically examined by Sheremet et al. [7]. The evaporation of water droplets having a low concentration of nanoparticles was examined by Askounis et al. [8]. They showed that the droplet evaporation rate is unaffected by the dispersion of low-concentration nanoparticles. Perrin et al. [9] compared theoretical and experimental results of nanofluid liquid drop evaporation. A numerical study by mixed convection of the falling liquid film evaporation was presented by Yan [10]. An experimental analysis of the liquid film evaporation was conducted by Huang et al. [11]. They showed that an increase in the temperature and airflow rate accelerated evaporation. Wei et al. [12] simulated CuO-water nanofluid's flow and heat transfer in a tube using the single- and two-phase (mixture) models. Nasr and Alzahrani [13] presented a numerical study to simulate the evaporation of liquid nanofluids on the inside of heat exchangers by mixed convection. Najim et al. [14] studied water/alumina nanofluid film's evaporation by mixed convection inside a heated vertical channel. Nasr and Al-Ghamdi [15] studied the liquid nanofilms' condensation on the inside of heat exchangers by mixed convection. In their study, Yu and Choi [16] focused on the significance of interfacial layers in the improved thermal conductivity of nanofluids. They adopted the Maxwell equation, which describes the effective thermal conductivity of solid/liquid mixtures, to incorporate the impact of these structured nanolayers. Chiam et al. [17] investigated the thermal conductivity and viscosity of Al2O3 nanofluids at varying ratios of water and ethylene glycol. The current study expands upon this by analyzing the thermal conductivity and dynamic viscosity of Al2O3 nanoparticles dispersed in different volume ratios of water (W) and ethylene glycol (EG) mixtures. Chadva et al. [18] presented the impact of nanofluids on the heat transfer properties of a double-pipe heat exchanger. They showed that the improvement of heat transfer is primarily determined by factors such as the type, size, and concentration of nanoparticles present in the base fluid. Alabd et al. [19] conducted experiments to investigate the flow characteristics and forced convective heat transfer of a nano-fluid comprising water and various volume concentrations of Al2O3 nanoparticles. Molana [20] centers their research on utilizing nanofluids in tubular heat exchangers, such as shell and tube, double pipe, and spiral tube designs. Xuan and Roetzel [21] presented a heat transfer correlation of nanofluids. The research conducted by Choi et al. [22] highlights anomalous phenomena that expose the inherent limitations of traditional heat conduction models for suspensions containing solid and liquid components. Das et al. [23] observed that the enhancement characteristics of nanofluids become more pronounced at higher temperatures, which could make them a more attractive choice for high-energy density applications when compared to earlier studies that only investigated their behavior at room temperature. Keblinski et al. [24] demonstrated that it is crucial to consider the ballistic nature of heat transport in the nanoparticles to comprehend the thermal properties of nanofluids, instead of just diffusive transport. Wang and Mujumdar et al. [25] provides an overview of the theoretical and numerical investigations on nanofluids. They examine the factors that influence the thermal properties of nanofluids, such as particle size, shape, and concentration, as well as the effects of temperature and pressure. Choi and Eastman [26] introduce the concept of nanofluids and then present a theoretical model for predicting their thermal conductivity. The model considers the effects of particle size, concentration, thermal conductivity, and the interfacial layer between the particles and the fluid.

As regards the prior research, the numerical study of water nano film evaporation containing a copper oxide nanoparticle behavior in a parallel plate evaporator under mixed convection was not examined. However, the effect of thermophoresis and Brownian motion is considered for the evaporation of a liquid film flowing down on a vertical plate by mixed convection. This study's novelty that no published papers address the same problem of evaporation of a copper oxide nanofilm flowing down on a vertical plate evaporator while considering the effects of thermophoresis and Brownian motion. The primary objective of the current study is to inspect the impact of copper oxide nanoparticle dispersion in the liquid film on mass and heat transfer, as well as the efficiency of a parallel plate evaporator.

2. Analysis

In the current study, the evaporation of a water film flowing downward over a vertical channel with height $H$ and width $d$ under mixed convection is modeled numerically (Fig. 1a). The liquid film of width $\delta(x)$ is a water-based nanofluid liquid containing copper oxide nanoparticles. The wet plate was heated with $q_i$, heat flux, while the other plate was isothermal and dry at a temperature $T_w$. The nanofluid nanofilm film entered the plate with a mass flow rate $m_{air}$ an inlet liquid temperature $T_{liq}$, and an inlet volume fraction of nanoparticles $\phi_p$. The air entered the channel at an ambient pressure $p_i$, temperature $T_{air}$ constant inlet gas velocity $u_{air}$ and water vapor concentration $c_v$.

2.1. Assumptions

The simplifying hypotheses adopted for this study are:

- Vapor gas is perfect.
- The nanoparticles and the liquid are in thermal equilibrium.
- The boundary layer approximations are used.
- Transfers and flows are two-dimensional, laminar, and steady.

2.2. Equations for the liquid film

The continuity equation for the nanofluid can be written as:

$$\frac{\partial (\rho \cdot u)}{\partial x} + \frac{\partial (\rho \cdot v)}{\partial y} = 0$$

(1)

where $\rho$, $u$, and $v$ are the density, the velocity in the vertical direction, and the velocity in the horizontal direction of the nanofluid, respectively.
The x-momentum equation for the nanofluid region is:

\[
p_{nf}\left(\mu_{nf}\frac{\partial u_{nf}}{\partial x} + v_{nf}\frac{\partial u_{nf}}{\partial y}\right) = \rho_{nf}\frac{d p_{nf}}{dx} + \frac{\partial}{\partial y}\left(\mu_{nf}\frac{\partial u_{nf}}{\partial y}\right)
\] (2)

where \(g\), \(\rho_{nf}\), and \(\mu_{nf}\) are the gravitational acceleration, the nanofluid's pressure, and the nanofluid's dynamic viscosity, respectively.

The energy equation for the nanofluid is expressed in terms of temperature as:

\[
\rho_{nf}c_{nf}\left(\mu_{nf}\frac{\partial T_{nf}}{\partial x} + v_{nf}\frac{\partial T_{nf}}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda_{nf}\frac{\partial T_{nf}}{\partial y}\right) + \rho_{nf}c_{nf}D_b\left(\frac{\partial \phi}{\partial y}\right)^2
\] (3)

where \(c_{nf}\), \(c_{pm}\), \(D_b\), \(D_{nf}\), \(T_{nf}\), \(\lambda_{nf}\), \(\lambda_{p}\), \(\mu_{nf}\), and \(\rho_{nf}\) are specific heat for nanoparticles, specific heat for nanofluid, Brownian motion coefficient, thermal diffusion coefficient, the temperature of nanofluid, the thermal conductivity of the liquid, thermal conductivity of nanoparticles dynamics viscosity of the liquid, and liquid density, respectively.

The equation of transport of \(\phi\) is a simple advection–diffusion equation that has the following form.

\[
\frac{\partial u_{nf}\phi}{\partial x} + \frac{\partial v_{nf}\phi}{\partial y} = \frac{\partial}{\partial y}\left(D_{b}\frac{\partial \phi}{\partial y} + \frac{D_{nf}}{T_{nf}}\frac{\partial T_{nf}}{\partial y}\right)
\] (4)

The substitutive equations are:

\[
D_b = \frac{0.26\lambda_{p} \mu_{nf}}{(2\lambda_{L} + \lambda_{p})\rho_{L}}
\] (5)

\[
D_{nf} = \frac{k_{T}T}{3\mu_{nf}d_{p}}
\] (6)

where \(k_{T}\), \(d_{p}\), \(\beta\), \(\lambda_{L}\), \(\lambda_{p}\), \(\mu_{nf}\), and \(\rho_{nf}\) are Boltzmann's constant, nanoparticles diameter, the thermal expansion coefficient, thermal conductivity of the liquid, thermal conductivity of nanoparticles dynamics viscosity of the liquid, and liquid density, respectively.

The system of PDEs can be solved by applying the inlet condition for the nanofluid at the inlet boundary of the nanofluid region \((x = 0)\).

\[
T_{L}(0,y_{L}) = T_{0L}
\]

\[
m(0,y_{L}) = m_{0L}
\]

\[
\phi(0,y_{L}) = \phi_{L}
\] (7)

The no-slip boundary condition is applied for the velocity while \(q_{L}\) heat flux is applied to the wet plate \((y_{L} = 0)\).

\[
u_{nf}(x,0) = 0
\]

\[
u_{nf}(x,0) = 0
\]

\[
\frac{\partial T_{nf}}{\partial y} \bigg|_{y_{L}=0} = -q_{L}/\lambda_{nf}
\] (8)

\[
D_{b}\phi \bigg|_{y_{L}=0} = \frac{D_{nf} \frac{\partial T_{nf}}{T_{nf}} \bigg|_{y_{L}=0}}{\frac{\partial y}{\partial y} \bigg|_{y_{L}=0}}
\]
The boundary at the liquid-vapor interface will be discussed after discussing equations for the gas flow.

2.3. Equations for the gas flow

The continuity equation for the nanofluid can be written as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u \right) + \frac{\partial}{\partial y} \left( \rho v \right) = 0$$

(9)

where \( \rho, u, \) and \( v \) are the density of the gas, the velocity of the gas in the horizontal direction, and the velocity of the gas in the vertical direction, respectively.

The x-momentum equation for the nanofluid region is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - \rho g (T - T_w) - \beta \sigma (c - c_v) + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right)$$

(10)

where \( \sigma, c_v, \) and \( \beta \) are water vapor concentration, the gas's pressure, the gas's temperature, the dynamic viscosity of the gas, and the mass expansion coefficient, respectively.

The energy equation for the gas is expressed in terms of temperature as:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( \rho \frac{\partial c_p \frac{\partial T}{\partial y}}{\partial y} \right)$$

(11)

where \( c_p, c_v, \) and \( \lambda \) are specific heat at constant pressure for the mixture, specific heat for air, the specific heat of water vapor, and thermal conductivity of gas, respectively.

The continuity of transport of \( c \) is a simple advection–diffusion equation that has the following form:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = -\frac{1}{\rho} \frac{\partial}{\partial y} \left( D \frac{\partial c}{\partial y} \right)$$

(12)

The PDEs can be solved by applying the inlet condition for the nanofluid at the top boundary of the gas region \((x = 0)\).

$$u(0, y) = u_0$$

$$T(0, y) = T_0$$

$$c(0, y) = c_0$$

(13)

The no-slip boundary condition is applied for the velocity, while the Dirichlet boundary condition is applied to the temperature with \( T_w \) at the right wall \((y = 0)\). The Neumann boundary condition with zeros right-hand side (open boundary) is applied to the mass fraction for water vapor at \( y = d \).

$$u(x, d) = 0$$

$$v(x, d) = 0$$

$$T(x, d) = T_v$$

$$\frac{\partial c}{\partial y} \bigg|_{y=d} = 0$$

(14)

2.4. Conditions at the gas–liquid interface

The interface of the gas–liquid is located at \( y_L = \delta \) and \( y = 0 \). The continuities of the temperatures and velocities between the two regions are applied:

$$u_{sl} (x, y_L = \delta) = u(x, y = 0)$$

$$v(x, y = 0) = -\frac{D}{1 - c(x, 0)} \frac{\partial c}{\partial y}$$

$$T_{sl} (x, y_L = \delta) = T(x, y = 0)$$

$$D \frac{\partial c}{\partial y} \bigg|_{y_L = \delta} = \frac{D}{T_{sl}} \frac{\partial T}{\partial y} \bigg|_{y_L = \delta}$$

(15)

The continuities of shear stress give the following condition:

$$u_{sl} \frac{\partial u_{sl}}{\partial y} \bigg|_{y_L = \delta} = \frac{\partial u}{\partial y} \bigg|_{y = 0}$$

(16)

The energy equation at the interface gives:

$$\frac{\partial T_{sl}}{\partial y} \bigg|_{y_L = \delta} = -\frac{\partial T}{\partial y} \bigg|_{y = 0} - \frac{\rho L D}{1 - c(x, 0)} \frac{\partial c}{\partial y}$$

(17)

2.5. Dimensionless governing equations

The following transformations are introduced:

In the gas: \( \eta = (y - d(d - d), x = x/H) \).

In the liquid phase: \( \eta_L = y/d, x = x/H \).

2.5.1. For the liquid phase

2.5.1.1. Continuity equation

$$\frac{\partial \rho_{sl} u_{sl}}{\partial \zeta} - \frac{\eta_L}{\delta} \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} + \frac{\partial}{\partial \eta_L} \left( \frac{H \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L}}{\partial \eta_L} \right) = 0$$

(18)

2.5.1.2. x-momentum equation

$$\frac{\partial u_{sl}}{\partial \zeta} + \left( u_{sl} - u_{sl} \right) \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} + \frac{\partial}{\partial \eta_L} \left( \frac{H \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L}}{\partial \eta_L} \right) = -\frac{1}{\rho_L} \frac{\partial P}{\partial \zeta} - \frac{H}{\rho_L} \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} + gH$$

(19)

2.5.1.3. Energy equation

$$\frac{\partial T_{sl}}{\partial \zeta} + \left( u_{sl} - u_{sl} - \frac{1}{\delta} \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} + \frac{H}{\delta} \frac{\partial v_{sl}}{\partial \eta_L} \right) \frac{\partial T_{sl}}{\partial \eta_L}$$

$$= \frac{1}{\rho_{sl} C_{sl}} \left\{ \frac{H}{\delta} \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} \right\} + \frac{\rho_{sl} C_{sl} D_{sl} H}{\delta} \frac{\partial T_{sl}}{\partial \eta_L} - \frac{H}{\delta} \frac{\partial \rho_{sl} u_{sl}}{\partial \eta_L} \left( \frac{\partial T_{sl}}{\partial \eta_L} \right)^2$$

(20)
2.5.1.4. Nanoparticles concentration equation

\[ u_{\text{np}} \frac{\partial \phi}{\partial z} + \left( u_{\text{np}} \frac{\partial}{\partial z} - \frac{1}{\delta} \frac{\partial}{\partial \eta} - \frac{H}{\delta} \frac{\partial u_{\text{np}}}{\partial \eta} \right) \frac{\partial \phi}{\partial \eta} = \frac{1}{\rho_{\text{np}}} \left( \frac{\partial}{\partial \eta} \left( \rho_{\text{np}} D_{\phi} \frac{\partial \phi}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial u_{\text{np}} D_{\phi}}{\partial \eta} \right) - \frac{T_{\text{ad}}}{\eta_{\text{ad}}} \right) \]

(21)

2.5.2. For the gaseous phase

2.5.2.1. Continuity

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \]

(22)

2.5.2.2. x-momentum

\[ \frac{\partial u}{\partial t} + \frac{\partial (\rho u u)}{\partial x} + \frac{\partial (\rho v u)}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{\partial}{\partial \eta} \left( H \frac{\partial u}{\partial \eta} \right) - \frac{\partial}{\partial \eta} \left( \frac{H}{\rho} \frac{\partial v}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \mu \frac{\partial u}{\partial \eta} \right) \]

(23)

2.5.2.3. Energy

\[ \frac{\partial T}{\partial t} + \frac{\partial (\rho u T)}{\partial x} + \frac{\partial (\rho v T)}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial \eta} \left( \mu \frac{\partial T}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \frac{D_{c}}{\rho} \frac{\partial c}{\partial \eta} \right) \]

(24)

2.5.2.4. Species diffusion

\[ \frac{\partial c}{\partial t} + \frac{\partial (\rho u c)}{\partial x} + \frac{\partial (\rho v c)}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial \eta} \left( \mu \frac{\partial c}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \frac{D_{c}}{\rho} \frac{\partial c}{\partial \eta} \right) \]

(25)

2.5.2.5. Overall mass balance

\[ \int_{0}^{1} \rho \frac{d z}{d \eta} u(\xi, \eta) d \eta = \left[ \frac{d - \delta}{d \delta} \right] \rho_{0} u_{0} - H \int_{0}^{1} \rho v(\xi, \eta = 0) d \xi \]

(26)

2.5.3. Boundary conditions

For inlet conditions (at \( \xi = 0 \)):

\[ T(0, \eta) = T_{w}; \quad c(0, \eta) = c_{w}; \quad u(0, \eta) = u_{w}; \quad p(0, \eta) = p_{0} \]

(27)

\[ T_{m}(0, \eta_{L}) = T_{w}; \quad \int_{0}^{1} \rho_{\text{np}} \delta_{l} u_{\text{np}}(0, \eta_{L}) d \eta_{L} = m_{\text{np}}; \quad \varphi = \varphi_{0} \]

(28)

For dry plate (at \( h = 1 \)):

\[ u(\xi, 1) = v(\xi, 1) = 0; \quad \frac{\partial c}{\partial \eta} \bigg|_{\eta = 1} = 0; \quad T(\xi, 1) = T_{w} \]

(29)

For wet plate (at \( h = 0 \)):

\[ u_{\text{np}}(\xi, 0) = v_{\text{np}}(\xi, 0) = 0; \quad q_{w} = -\lambda_{w} \frac{1}{\delta} \frac{\partial T}{\partial \eta} \bigg|_{\eta_{L} = 0} \]

(30)

For gas–liquid interface (at \( h = 0 \) and \( \eta_{L} = 1 \)):

\[ u_{\text{np}}(\xi, \eta_{L}) = v_{\text{np}}(\xi, \eta_{L}) = 0; \quad T_{m}(\xi, \eta_{L}) = T_{w}(\xi, \eta_{L}) \]

(31)

\[ c(\xi, \eta = 0) = c_{w}(T(\xi, \eta = 0)) \]

(32)

where \( c_{w}(T(\xi, \eta = 0)) = \frac{M_{L}}{M_{S}} \frac{P_{w}}{P_{S}} \).

The continuities of shear stress give:

\[ \frac{1}{\rho} \frac{\partial u_{w}}{\partial \xi} \bigg|_{\eta = 0} = \frac{1}{\rho} \frac{\partial u_{w}}{\partial \eta} \bigg|_{\eta = 0} \]

(33)

The energy balance at the liquid–gas interface:

\[ \frac{1}{\rho} \frac{\partial T_{m}}{\partial \eta} \bigg|_{\eta_{L} = 0} = -\frac{1}{\rho} \frac{\partial T}{\partial \eta} \bigg|_{\eta_{L} = 0} - \frac{1}{\rho} \frac{\partial T}{\partial \eta} \bigg|_{\eta = 0} \]

(34)

We introduced the following quantities:

- The latent heat flux is given by:

\[ q_{L} = \frac{\rho_{L} D_{c}}{\rho_{\text{np}}} \frac{\partial c}{\partial \eta} \bigg|_{\eta = 0} \]

(35)

- The sensible heat flux is given by:

\[ q_{s} = -\lambda_{w} \frac{1}{\delta} \frac{\partial T}{\partial \eta} \bigg|_{\eta = 0} \]

(36)

We introduced the following quantities:

- The local condensation rate is:

\[ \varphi(\xi) = \frac{\rho_{L} D_{c}}{\rho_{\text{np}}} \frac{\partial c}{\partial \eta} \bigg|_{\eta = 0} \]

(37)
The cumulated condensation rate is:

$$
Mr = \int_0^\infty \dot{m}(\xi) d\xi
$$

The density and heat capacity of the nanofluid is given by:

$$
\rho_{nf} = \phi \rho_p + (1-\phi) \rho_L
$$

While the heat capacity of the nanofluid is given by:

$$
(c_p)_{nf} = \phi c_{p,p} + (1-\phi) c_{p,L}
$$

The Brinkman [28] and Maxwell [29] models are given by:

$$
\mu_{nf} = \frac{\mu_p}{(1-\phi)^2}
$$

$$
\lambda_{nf} = \lambda_p + 2\lambda_L - 2\phi(\lambda_p - \lambda_L)
$$

$$
\lambda_{nf} = \lambda_p + 2\lambda_L + \phi(\lambda_p - \lambda_L)
$$

The partial differential Eqs. (1)–(17) were solved using FORTRAN code with the implicit finite difference method. The governing partial differential equations were transformed into finite difference equations using a fully implicit marching scheme. The axial convection terms were approximated by the upstream difference and the transverse convection used in the present work, we presented a comparison between our results of the evaporating rate ($M_0$) and the interfacial ($T_i - T_a$) for different values of density heat flux for the case of binary film evaporation (water-ethylene glycol) flowing down on a vertical heated plate by mixed convection and those obtained by Cherif and Daif [31] (Fig. 2a and b). This comparison was performed for $q_2 = 0$, $T_\infty = 293$ K, $T_0 = 293$ K, $q_1 = 3,000$ W/m², $Re = 1,000$, $m_{nf} = 0.02$ kg/s, $c_{water,ethylene glycol} = 0.5$, and $d/H = 0.015$ m. Fig. 2a and b show a good agreement conformity between our results and those obtained by Cherif and Daif [31].

### 3. Results and discussion

The current work concentrates on the numerical analysis of evaporation of (water/copper oxide) nanofilms flowing downward in a parallel plate evaporator under mixed convection. The liquid film contained a low nanoparticle volume fraction of copper oxide. The film (wetted) plate was heated by uniform flux, while the other plate was kept at dry isothermal condition. In this work, we showed the nanoparticles’ volume fraction $\phi_{nf}$ impact on the heat and mass transfers and the water film evaporation.

This work mainly focuses on the impact of the volume fraction of copper oxide nanoparticles $\phi_{nf}$ on the heat and mass transfer during liquid film evaporation. Figs. 3a and 4a show that the volume fraction of copper oxide nanoparticles $\phi_{nf}$ increases with increased temperature. This result could be justified by the lower heat capacity and higher thermal conductivity of nanoparticles compared to the primary fluid (pure water). Figs. 3b and 4b show that the increase in the volume fraction of copper oxide nanoparticles $\phi_{nf}$ induced an increase in the vapor concentration in the gas region and at the interface liquid–gas. Fig. 4b shows that the concentration gradient was increased when we increased the volume fraction of copper oxide nanoparticles $\phi_{nf}$ causing an improvement in the water film evaporation (Fig. 5). It is shown from this figure that using the nanofluid advantages the water film evaporation without increasing the energy consumption by heating the wetted plate.

Fig. 6a and b show variations of the sensible and latent heat fluxes for different volume fractions of copper oxide nanoparticles $\phi_{nf}$. Fig. 6 shows that the dispersion of the copper oxide nanoparticles enhances the latent and sensible

### 2.6. Solution method

Table 1

<table>
<thead>
<tr>
<th>Thermo-physical properties</th>
<th>Water (base fluid)</th>
<th>Copper oxide</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho) \text{(kg/m}^3\text{)}</td>
<td>996.59</td>
<td>6,320</td>
</tr>
<tr>
<td>(c_p) \text{(J/kg-K)}</td>
<td>4,179</td>
<td>460</td>
</tr>
<tr>
<td>(\lambda) \text{(W/m}^2\text{-K)}</td>
<td>0.6071</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>(I \times (J + K))</th>
<th>101 \times (101 + 41)</th>
<th>101 \times (71 + 41)</th>
<th>71 \times (51 + 41)</th>
<th>51 \times (51 + 41)</th>
<th>51 \times (21 + 41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi = 0.25)</td>
<td>3.841</td>
<td>3.885</td>
<td>3.973</td>
<td>3.959</td>
<td>3.736</td>
</tr>
<tr>
<td>(\xi = 0.50)</td>
<td>6.651</td>
<td>6.620</td>
<td>6.572</td>
<td>6.593</td>
<td>6.367</td>
</tr>
<tr>
<td>(\xi = 0.75)</td>
<td>9.140</td>
<td>9.109</td>
<td>8.984</td>
<td>8.986</td>
<td>8.959</td>
</tr>
<tr>
<td>(\xi = 1.00)</td>
<td>11.573</td>
<td>11.543</td>
<td>11.442</td>
<td>11.425</td>
<td>11.195</td>
</tr>
</tbody>
</table>
Fig. 2. Variation of total evaporating (a) and interfacial temperature (b) along the channel; $c_{\text{Liq, ethylene glycol}} = c_{\text{Liq, water}} = 0.5$ (50% water-ethylene glycol mixture).

Fig. 3. Impact of nanoparticles volume fraction on the temperature (a) and the vapor concentration (b) at the channel exit: $T_0 = 293.15\,K$, $T_w = 293.15\,K$, $q_1 = 3,000\,W/m^2$, $T_w = 293.15\,K$, $c_0 = 0$, $u_0 = 1\,m/s$, $m_{\text{L iq}} = 0.015\,kg/m\cdot s$, $H = 1\,m$, $d = 0.015\,m$, and $p_0 = 1\,\text{atm}$.

Fig. 4. Variation of the temperature and the vapor concentration at the interface liquid–gas for various nanoparticles volume fraction: $T_0 = 293.15\,K$, $T_w = 293.15\,K$, $q_1 = 3,000\,W/m^2$, $T_w = 293.15\,K$, $c_0 = 0$, $u_0 = 1\,m/s$, $m_{\text{L iq}} = 0.015\,kg/m\cdot s$, $H = 1\,m$, $d = 0.015\,m$, and $p_0 = 1\,\text{atm}$. 
Fig. 5. Impact of nanoparticles volume fraction on the total evaporation rate: $T_{w0} = 293.15$ K, $T_{w} = 293.15$ K, $T_{0} = 293.15$ K, $q_{1} = 3,000$ W/m$^2$, $c_{0} = 0$, $u_{0} = 1$ m/s, $p_{0} = 1$ atm, $m_{0L} = 0.015$ kg/m·s, $H = 1$ m, and $d = 0.015$ m.

Fig. 6. Effect of nanoparticles volume fraction on sensible heat flux ($q_{s}$) and latent heat flux ($q_{L}$): $T_{w0} = 293.15$ K, $T_{w} = 293.15$ K, $T_{0} = 293.15$ K, $q_{1} = 3,000$ W/m$^2$, $c_{0} = 0$, $u_{0} = 1$ m/s, $p_{0} = 1$ atm, $m_{0L} = 0.015$ kg/m·s, $H = 1$ m, and $d = 0.015$ m.

Fig. 7. Evolution of the liquid velocity at the exit channel for different values of nanoparticles volume fraction: $T_{w0} = 293.15$ K, $T_{w} = 293.15$ K, $T_{0} = 293.15$ K, $q_{1} = 3,000$ W/m$^2$, $c_{0} = 0$, $u_{0} = 1$ m/s, $p_{0} = 1$ atm, $m_{0L} = 0.015$ kg/m·s, $H = 1$ m, and $d = 0.015$ m.

Fig. 8. Evolution of the cumulated evaporation rate for various heat flux: $T_{w0} = 293.15$ K, $T_{w} = 293.15$ K, $T_{0} = 293.15$ K, $c_{0} = 0$, $u_{0} = 1$ m/s, $m_{0L} = 0.015$ kg/m·s, $H = 1$ m, $d = 0.015$ m, $p_{0} = 1$ atm, and $q_{0} = 0.03$.

Fig. 9. Variation of the cumulated evaporation rate for various inlet gas temperatures: $T_{w0} = 293.15$ K, $q_{1} = 3,000$ W/m$^2$, $T_{w} = 293.15$ K, $c_{0} = 0$, $u_{0} = 1$ m/s, $p_{0} = 1$ atm, $m_{0L} = 0.015$ kg/m·s, $H = 1$ m, $d = 0.015$ m, and $q_{0} = 0.03$. 
heat transfer. Fig. 6a and b show that the latent and sensible heat fluxes increased with an increase in $\phi$. This result is due to the effect of copper oxide nanoparticles which increases thermal conductivity value and decreases heat capacity compared to pure liquid. Fig. 7 shows that an increase in $\phi$ decreases the liquid velocity at the bottom of the channel due to the high nanofluid viscosity, which decelerates the liquid film flow. It is shown in Fig. 8 that an elevation in the external heat flux $q_1$ advantages the liquid film evaporation since the imposed flux $q_1$ represented a source of energy and served to enhance film evaporation. Fig. 9 shows an increase in film evaporation with larger values of $T_0$. Fig. 10 shows that increasing inlet film temperature ameliorates film evaporation.

4. Conclusions

A numerical study is carried out to improve our understanding of heat and mass transfer during the evaporation of water/copper oxide nanofilm flowing down on a plate of a vertical channel. The wet plate was heated, while the other plate was dry and isothermal. Analysis has been done on how the dispersion of copper oxide nanoparticles affects the effectiveness of heat exchangers and the evaporation process.

The main findings of the present work are the following:

- The addition of nanoparticles to the base fluid causes further enhancement of film evaporation. Consequently, the water film evaporation is enhanced by 23% when the volume fraction $\phi_0$ of copper oxide is elevated by 5%.
- Increasing the volume fraction $\phi_0$ of copper oxide by 5% has been led to a 7% improvement in latent heat transfer.

- It is shown that when the volume fraction $\phi_0$ of copper oxide is increased by 5%, sensible heat transfer improves and is close to 11%.
- Increasing inlet gas temperature improves the water film evaporation.
- Increasing inlet liquid temperature enhances the water film evaporation.
- Increasing supplied heat flux improves the water film evaporation.

Symbols

- $c$ — Mass fraction for water vapor
- $c_{pv}$ — Heat capacity of water vapor, J/kg·K
- $c_{pa}$ — Heat capacity of water dry air, J/kg·K
- $D$ — Mass diffusivity, m$^2$/s
- $D_B$ — Brownian coefficient
- $D_T$ — Thermal diffusion coefficient
- $L_v$ — Latent heat of water evaporation, J/kg
- $P$ — Pressure, N/m$^2$
- $T$ — Temperature, K
- $g$ — Gravitational acceleration, m/s$^2$
- $u$ — Axial velocity, m/s
- $v$ — Transverse velocity, m/s
- $x$ — Coordinate in the axial direction, m
- $y$ — Coordinate in the transverse direction, m
- $k_b$ — Boltzmann’s constant

Greeks

- $d_p$ — Nanoparticles diameter
- $\psi$ — Volume fraction of nanoparticles
- $\lambda$ — Thermal conductivity, W/m·K
- $\mu$ — Dynamic viscosity, kg/m·s
- $\delta$ — Thickness of liquid film, m
- $\rho$ — Density, kg/m$^3$
- $(c_p)_T$ — Heat capacity, J/K
- $\eta$ — Dimensionless coordinate in the transverse direction
- $\eta_L$ — Dimensionless transverse coordinate in the liquid ($\eta_L = y/\delta$)
- $\zeta$ — Dimensionless coordinate in the flow direction
- $\beta$ — Thermal expansion coefficient, $-1/\rho(\partial \rho / \partial T)_{p,c}$ K$^{-1}$
- $\beta^*$ — mass expansion coefficient, $-1p(\partial \rho / \partial c)_p,T$

Subscripts

- $n$ — Nanoparticle
- $nf$ — Nanofluid
- $0$ — Inlet condition
- $L$ — Liquid

References


