



## Numerical study of evaporation of liquid film by mixed convection in partially wetted vertical channel

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### ABSTRACT

This study focuses on a numerical investigation of steady conjugated heat and mass transfer by mixed convection in a finite vertical channel. The two channel walls are symmetrically heated with uniform heat flux. One wall is partially wetted by an extremely thin water film, while the other is dry and impermeable. The partially humid plate is divided into  $2.n$  equal regions which are alternately humid and dry zones. The effect of the number of wetted zones and their positions on the flow and on the heat and mass transfer is analysed. The results are reported in terms of axial distribution of wall temperature, relative heat fluxes and evaporation rate for different wetted zone positions. It is observed that the change in the wetted zone position has no significant effect on moist air flow. However, the heat and mass transfer are extremely influenced by the presence of the wetted zones and their positions. In fact, the evaporation rate is more intense when the wetted section is situated at the channel exit. Finally, it is observed that the evaporation is intensified by increasing the number of wetted zones. The importance of the studied system lies in the fact that it can describe several desalination cells associated in series.

*Keywords:* Evaporation; Mixed convection; Heat and mass transfer; Thin film; Vertical plates

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### 1. Introduction

The combined heat and mass transfer over vertical ducts has a wide range of applications in the field of science and technology, such as desalting, cooling technology, drying processes and air conditioning. Owing to these widespread applications, several researches have been investigated in these topics. Most of the previous theoretical and experimental studies are concerned with the boundary conditions of locally wall heat flux, uniform wall heat flux, uniform mass flux, uniform wall temperature and uniform wall concentration. Only those related to this work are briefly reviewed here.

A vast amount of work is about the evaporation in the case of local heating liquid films [1–9], local wetted

wall [10], of uniform wall temperature or uniform wall concentration [11–16] and in the case of uniform wall heat or mass fluxes [17–29]. Gatapova and Kabov [1] have presented a study of the flow of a liquid film sheared by gas flow in a channel with a heater placed at the bottom wall. They have shown that the influence of convective heat transfer mechanism is much more prominent for relatively high values of the liquid Reynolds number. The liquid–gas interface Biot number is shown to be a sectional-hyperbolic function of a longitudinal axis variable. They presented some qualitative and quantitative comparisons with experimental results. Frank [2] performed a direct three-dimensional non-linear simulation of the flows in a locally heated film using the particle method, and the numerical results were in agreement with the experimental data. A review of most of the results concerning the effect of

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non-uniform heating on film flow can be found in the introduction of [3]. However, non-uniform heating effects have been only partially understood for shear-driven liquid film flows [4,5], and the influence of the gas phase on the interfacial phenomena remains a challenging issue for modelling. Marchuk and Kabov [6] have studied the numerical modelling of thermo-capillary reverse flow in thin liquid films under local heating. Skotheim et al. [7] analysed the influence of localised heating on the instability of a falling film. Gatapova et al. [8] illustrated the thermo-capillary deformation of a locally heated liquid film moving under the action of a gas flow. Gatapova et al. [5] studied the heat transfer and two-dimensional deformations in a locally heated liquid film with co-current gas flow. Lee [9] presented a numerical analysis to investigate the effects of heat transfer in partially heated vertical parallel plates. In that study, both boundary conditions of uniform wall temperature/ uniform wall concentration and uniform heat flux/ uniform mass flux were considered. Their analysis showed that the presence of unheated entry and unheated exit severely affects the heat and mass transfer. They present theoretical correlations for average Nusselt number and Sherwood number. Mammou et al. [10] presented a numerical study of laminar heat and mass transfer from an inclined flat plate with a dry zone inserted between two wet zones. They concluded that the inclination angle has a small influence on the local Nusselt and Sherwood numbers. Baumann [11] studied the heat and mass transfer in two-component film evaporation in a vertical tube. Wei-Mon Yan [12] studied the convective heat and mass transfer along an inclined heated plate with film evaporation. Debissi et al. [23], Baumann and Thiele [24], Chow and Chung [25], Daif et al. [26], Yan [27], Lee et al. [28] and Shembharkar and Pai [29] presented numerical studies of finite liquid film evaporation on laminar convection heat and mass transfer in a vertical parallel plate channel with adiabatic wall. Salah El-Din [30] has examined the effect of mass buoyancy forces on the development of laminar mixed convection between vertical parallel plates with uniform wall heat and mass fluxes. The author studied the effect of the buoyancy ratio heat and mass transfer between plates.

To our knowledge, the heat and mass transfer along a partially wetted plate which is composed, respectively, by an alternation of humid and dry zones has not been studied yet. The main objective of this work is to study the evaporation of water into mixed convection flow of humid air. Particular attention will be paid to the effect of the wetted zones positions on the effectiveness of water evaporation. The importance of the

studied system lies in the fact that it can describe several desalination cells associated in series.

## 2. Analysis

This study presents a numerical analysis of heat and mass transfer during water evaporation by mixed convection in a finite vertical channel. The channel studied is made up of two parallel plates symmetrically heated with uniform heat fluxes. The left plate ( $y=0$ ) is made with a succession of  $2.n$  zones alternately wet and dry, while the right plate ( $y=d$ ) is dry.

At the channel entrances, the moist air flows upward with the ambient conditions of temperature  $T_0$ , pressure  $p_0$ , velocity  $u_0$  and water vapour concentration  $c_0$ . The geometry of the problem under consideration (for  $n=1$ ) is shown in Fig. 1a. In this case

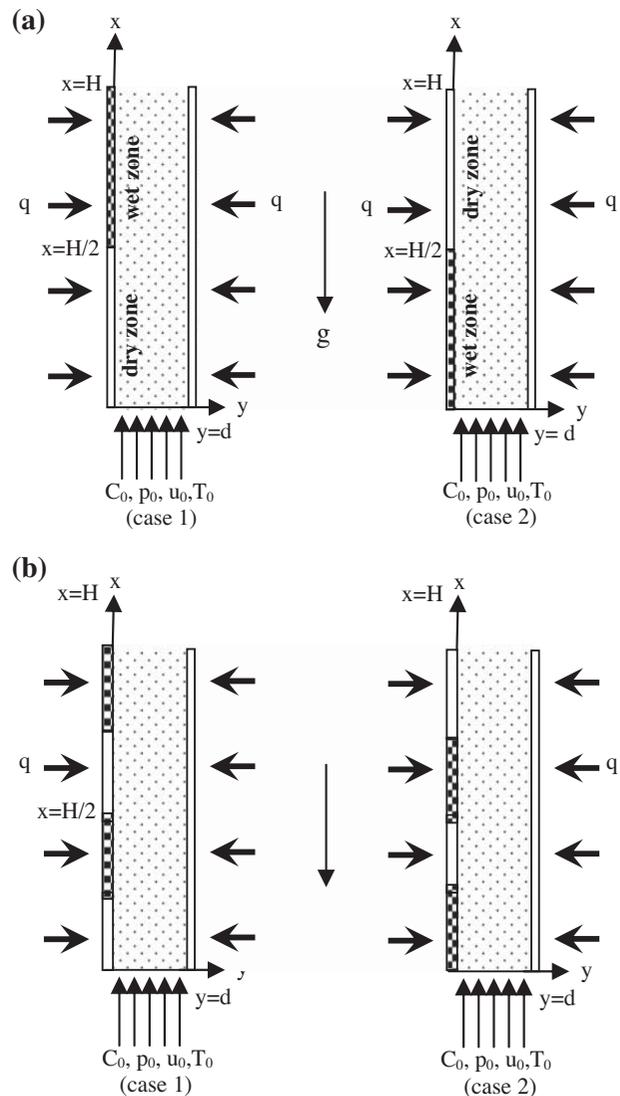


Fig. 1. Schematic diagram of the physical system.

( $n=1$ ), the left plate is divided into two regions with equal lengths ( $H/2$ ) which are alternatively wet and dry zones. Two configurations were considered in this study: in the first case (wet exit), the wetted zone is at the channel exit and the dry zone is at the first half of the plate ( $x=0$ ); in the second case (dry exit), the configuration is reversed.

In order to set the partial differential system of equation describing momentum, heat and mass transfer, few simplifying assumptions are taken into consideration [19–21,30,31]:

- The fluid flow is steady and laminar.
- For wet zones, the moist air is assumed to be at thermodynamic equilibrium so that the wall temperature and water concentration can be related through the saturated vapour pressure [1].
- The humid zone can be modelled by considering an extremely thin liquid film. Thus, transport in the liquid film can be replaced by approximate boundary conditions for gas flow [12,13].
- The boundary layer approximations are generally used to study the downward flow in the channel induced by mixed convection.
- The viscous dissipation and the pressure work are negligible.
- The Dufour and Soret effects are negligible.
- The thermal radiation is negligible.
- The vapour mixture behaves as an ideal gas.

From the above assumptions, the bi-dimensional flow of the gas mixture is described in the ( $x, y$ ) coordinate system by the continuity equation and the balances for momentum, heat and species concentration:

Continuity equation:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (1)$$

$x$ -momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \beta g(T - T_0) + \beta^* g(C - C_0) + (1/\rho) \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (2)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho C_p} \left[ \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \rho D (C_{pv} - C_{pa}) \frac{\partial T \partial C}{\partial y \partial y} \right] \quad (3)$$

Species diffusion equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \rho \frac{\partial C}{\partial y} \right) \quad (4)$$

( $\beta g(T - T_0) + \beta^* g(C - C_0)$ ) represents the momentum transfer caused by the combined buoyancy forces. Thermo-physical properties of gas mixture are considered as variable with temperature and concentration of water vapour.

In this study of steady mixed channel flow, the overall mass balance described by the following equation should be satisfied at every axial location:

$$\int_0^d \rho u(x, y) dy = d \rho_0 u_0 + \int_0^x \rho v(x, 0) dx \quad (5)$$

Boundary conditions:

- At  $x=0$  (inlet conditions):

$$-u = u_0, T = T_0, C = C_0 \text{ and } P = P_0 \quad (6)$$

- At  $y=0$  (wet and dry zones):

$$u = 0;$$

The transverse velocity of gas is deduced by assuming that the air-water interface is semi-permeable:

$$v(x, 0) = \varepsilon \left( \frac{-D}{1 - C(x, 0)} \frac{\partial C}{\partial y} \Big|_{y=0} \right) \quad (7a)$$

where the value of  $\varepsilon$  is zero for the case of dry zone and unity for the case of wetted zone.

The energy balance at the interface ( $y=0$ ) is evaluated by:

$$-\lambda \frac{\partial T}{\partial y} - \varepsilon \left( \frac{\rho L_v D}{1 - C(x, 0)} \frac{\partial C}{\partial y} \right) = q \quad (7b)$$

It is clear that the imposed heat flux  $q$  is the sum of a sensible ( $q_s$ ) and a latent ( $q_l$ ) component.

According to Dalton's law and by assuming the interface to be at thermodynamic equilibrium and the air-vapour mixture is an ideal gas mixture, concentration of the vapour can be evaluated by [20,21]:

$$C(x, 0) = \frac{M_v / M_a}{p / p_{vs} + M_v / M_a - 1} \quad (7c)$$

where  $p_{vs}$  is the equilibrium pressure of vapour given by the following equation [19–21]:

$$\log_{10} p_{vs} = 28,59,051 - 8.2 \log T + 2,4804.10^{-3}T - 3142.32/T$$

- At  $y=d$  (dry plate):

$$u = 0, v = 0, -\lambda \frac{\partial T}{\partial y} \Big|_{y=d} = q \tag{7d}$$

The impermeability of the dry plate ( $y=d$ ) to the water vapour can be described by:

$$\frac{\partial C}{\partial y} = 0 \tag{7e}$$

In order to validate the numerical scheme adopted in the present study, we have defined the following dimensionless coefficients used by Debissi et al. [21] and Shah and London [32].

The local Nusselt number is defined as:

$$Nu_x = \frac{h_x 2d}{\lambda} = -\frac{2d[(\partial T / \partial y)_{y=0}]_x}{T(x,0) - T_m} \tag{8}$$

where  $h_x$  is the local heat transfer coefficient.  $T_m$  is the fluid bulk temperature at a cross section:

$$T_m = \frac{\int_0^d \rho u \cdot T \cdot dy}{\int_0^d \rho u \cdot dy} \tag{9}$$

The mean Nusselt number is defined as:

$$Nu_m = \frac{1}{x} \int_0^x Nu_x dx \tag{10}$$

Similarly and for mass transfer, the local Sherwood number is defined as:

Table 1

Comparison of the total evaporative rate of water at the exit (case when the left plate ( $y=0$ ) is entirely wetted) for various grid arrangements ( $T_0=373.15$  K,  $T_w=373.15$  K,  $q=0$ ,  $p_0=1$ ,  $d/H=0.015$ )

$I \times J$ grid point	$\bar{m}(H):c_0=0.005$	$\bar{m}(H):c_0=0.01$
71 × 51	$5.152 \times 10^{-4}$	$5.021 \times 10^{-4}$
71 × 71	$5.175 \times 10^{-4}$	$5.043 \times 10^{-4}$
101 × 71	$5.167 \times 10^{-4}$	$5.035 \times 10^{-4}$
101 × 101	$5.058 \times 10^{-4}$	$5.032 \times 10^{-4}$

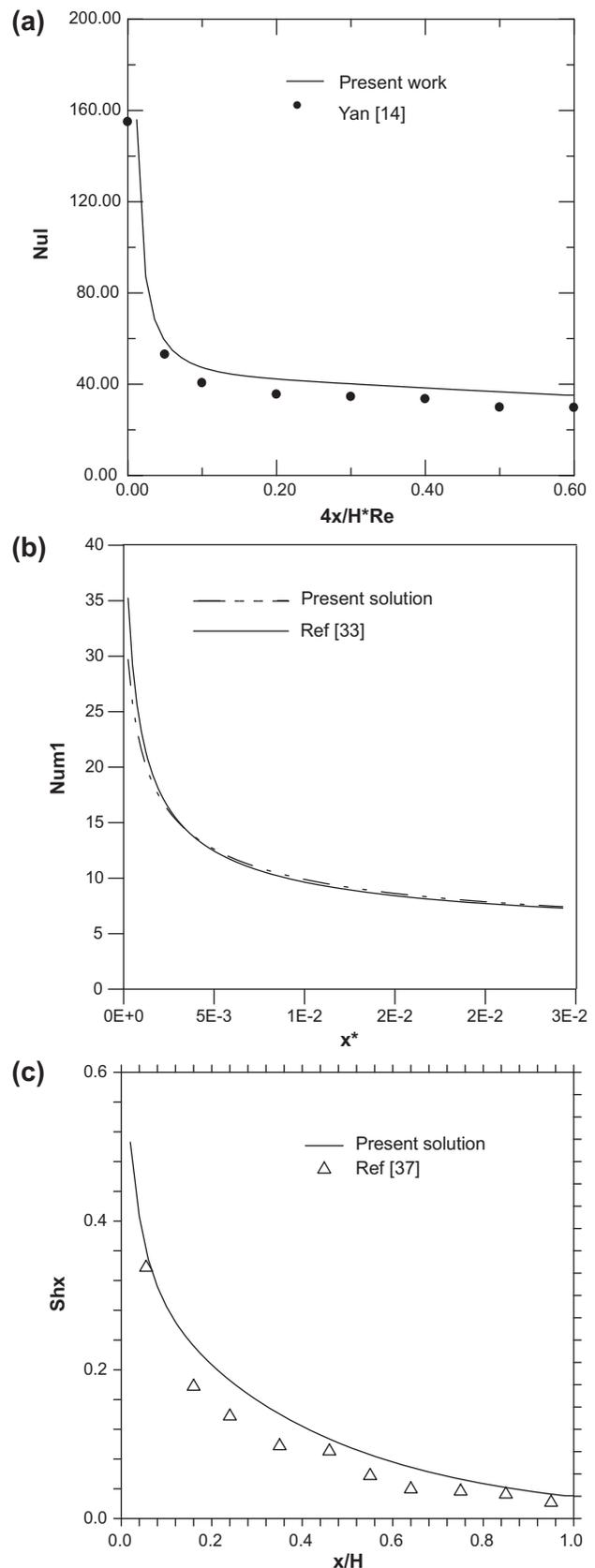


Fig. 2. Axial evolution of heat and mass coefficients.

$$Sh_x = -\frac{2d[(\partial C/\partial y)_{y=0}]_x}{C(x,0) - C_m} \quad (11)$$

$C_m$  is the fluid bulk concentration at a cross section:

$$C_m = \int_0^d \rho u \cdot C \cdot dy / \int_0^d \rho u \cdot dy$$

The total average evaporated mass flux is given by:

$$\bar{m} = \frac{1}{H} \int_0^H \rho v(x,0) dx \quad (12)$$

The water evaporating rate, commonly used in the previous studies [21,32–36], is expressed as:

$$Re_v = \frac{10^4 \bar{m}}{\sqrt{\rho_0 u_0 / H}} \quad (13)$$

For the dry zone: In this region, the governing equations for flow and heat transfer were obtained by

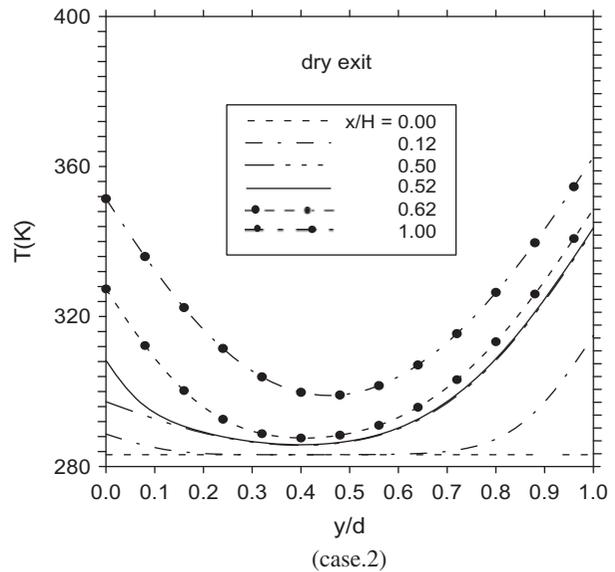
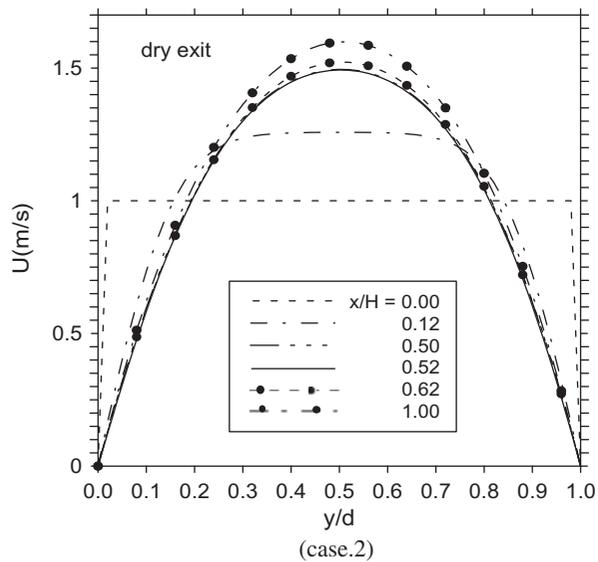
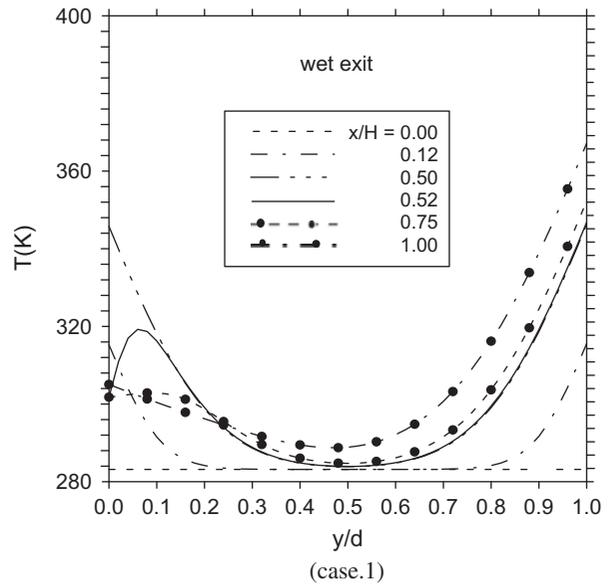
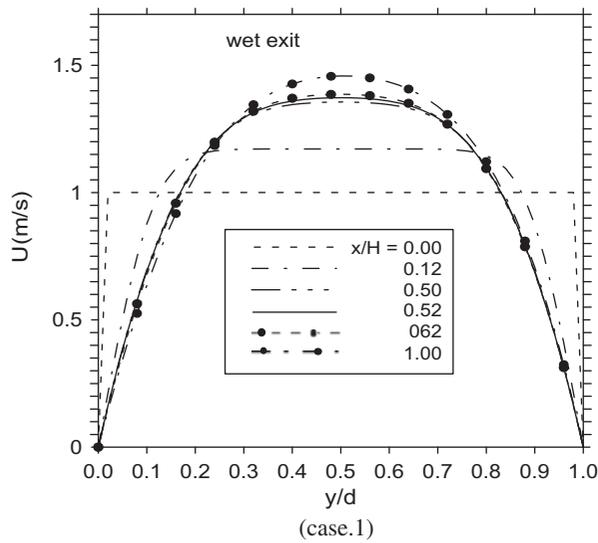


Fig. 3. Effect of the wetted zone position on the  $x$ -velocity component profiles for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

Fig. 4. Effect of the wetted zone position on the temperature profiles for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

adjusting the above equations (boundary conditions). In this case, the left plate was considered to be impermeable.

### 3. Solution method

The presented system of Eqs. (1)–(5) is solved numerically using finite difference method. The flow area is divided into a regular mesh placed in axial and transverse directions and a (71,71) grid is retained for computations. A fully implicit marching scheme, where the axial convection terms were approximated by the upstream difference and the

transverse convection, and diffusion terms by the central difference, is employed to transform the governing equation into finite difference equations. The resolution of the obtained algebraic equations was marched in the downstream direction as the flow under consideration is of a boundary-layer type. The discrete equations are resolved line by line from the inlet to the outlet of the channel. The solution procedure is briefly outlined as follows:

1. Give the flow, thermal and mass boundary conditions.
2. For the given axial location  $i$ , guess the wetted wall temperature  $T^*$  and solve the finite difference form of concentration equation of water vapour.
3. Solve the finite difference form of energy equation and compare the new value  $T$  of wetted temperature to  $T^*$  by testing if:

$$\left| \frac{T(i, 1) - T^*(i, 1)}{T(i, 1)} \right| < 10^{-6}$$

If this criterion is not satisfied, return to (2) and modify the wetted wall temperature by using the bisection method.

4. Guess a pressure  $P^*$  at the  $i$  axial location and solve the momentum and continuity finite difference equations, then verify the satisfaction of the overall conservation of mass expressed by the following criteria:

$$\left| \int_0^d \rho u(x, y) dy - (d \cdot \rho_0 u_0 + \int_0^x \rho v(x, 0) dx) \right| / (d \rho_0 u_0) < 10^{-6}$$

If this condition is not satisfied, return to Step 4 and modify the pressure value  $P^*$ .

5. For the dry plate, the concentration equation of water vapour and the evaporative rate in the overall conservation of mass were omitted.

To ensure that the results were grid independent, the solution was obtained for different grid sizes for typical case program test. Table 1 shows that the differences in the evaporative rate obtained using  $71 \times 51$  and  $101 \times 101$  grids are always less than 1%.

### 4. Results and discussion

To validate the numerical scheme adopted in the present study, different limiting cases for laminar mixed and free convection were considered. The results for the case of mixed convective heat and mass transfer inside a channel have been provided. The channel wall is maintained isothermal: the first plate

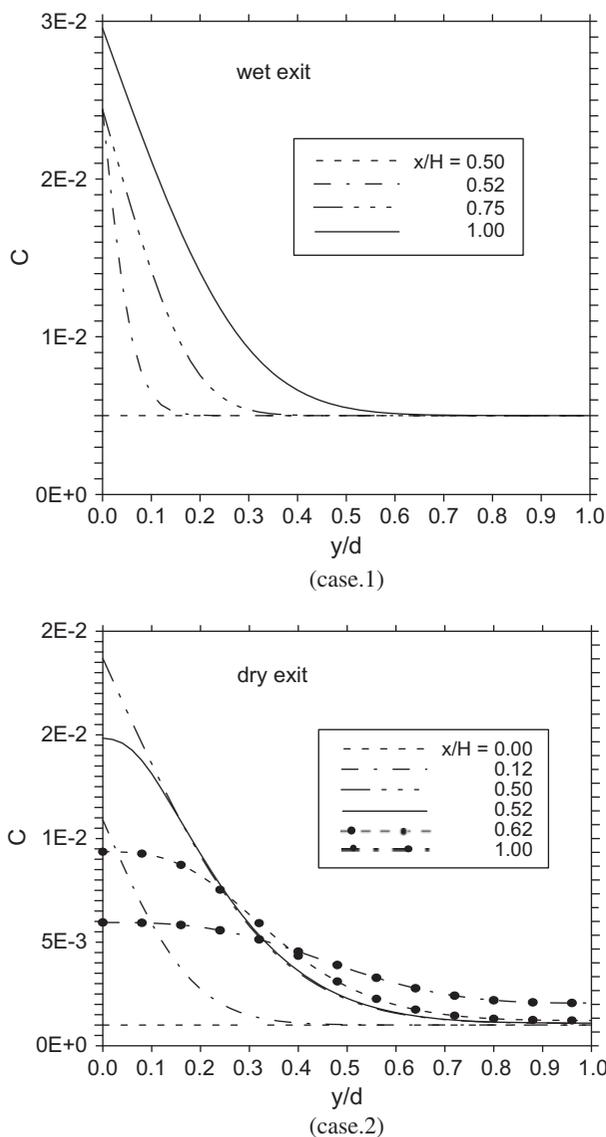


Fig. 5. Effect of the wetted zone position on the vapour concentration profiles for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

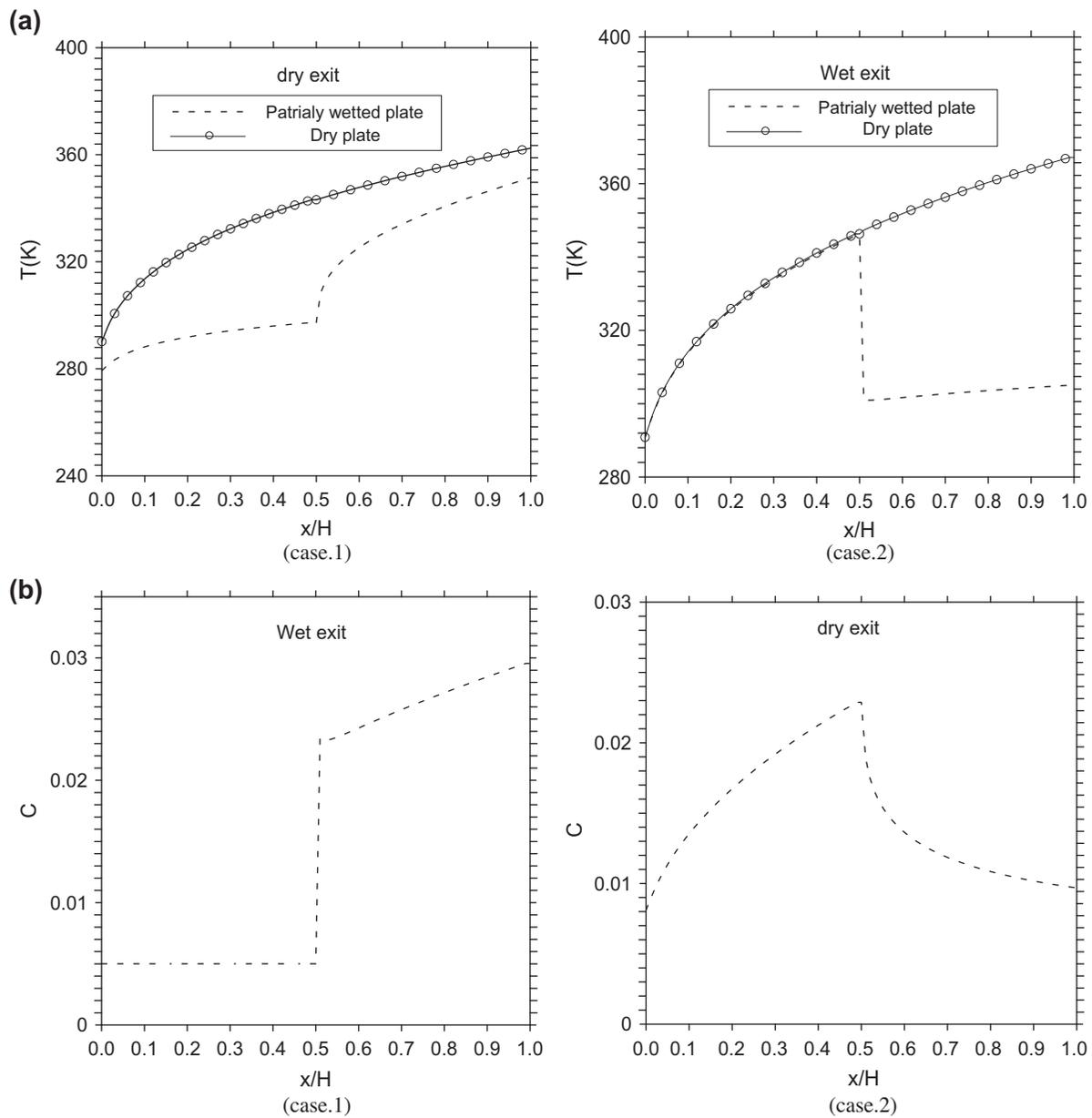


Fig. 6. Effect of the wetted zone position on the interfacial temperature and concentration evolution for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

is entirely wetted by an extremely thin water film and the second one is dry and kept at the ambient temperature  $T_0$ . The procedure has been tested by comparing the present results for mean and latent Nusselt numbers to those of Yan [13]. Fig. 2a shows a satisfactory conformity between our results and those obtained by Yan [13]. Furthermore, the numerical code has been tested successfully by comparing the present results for mean Nusselt number  $Num$  (Fig. 2b) at the isothermal wall to the analytical solution obtained by Mercier et al. found in Shah and London [32]. In the

case of forced convection, the procedure was tested by comparing the present results of local Sherwood number to the experimental data found in [36] for the problem of evaporation between two humid parallel plates. The experimental results of Sherwood number defined in [36] were obtained in the case of reduced Reynolds number  $Re=1.38$ . The results of this comparison, presented in Fig. 2c, show a good agreement.

All the results of this study have been carried out for a channel plunged into an upward flow of humid air with the ambient conditions:  $u_0=1$  m/s;  $C_0=0.005$ ;

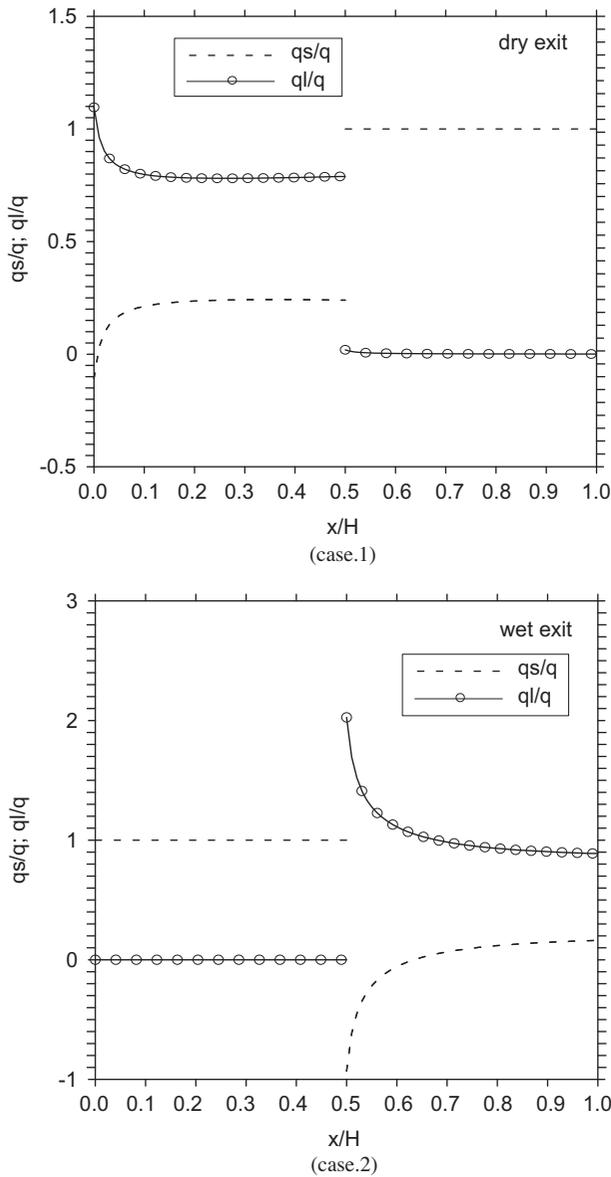


Fig. 7. Effect of the wetted zone position on the interfacial relative heat fluxes for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

$T_0=283.15$  K. The imposed wall heat flux is  $q=500$  W/m and the geometrical ratio is  $d/H=0.015$ . The left plate ( $y=0$ ) is divided along the channel into  $2.n$  equally wet and dry zones. The second plate is dry and heated with uniform heat flux  $q$ . In order to study the effect of the number of wet zones and their positions on the flow as well as on the effectiveness of water evaporation along the partially wetted plate, the cases of  $n=1$  and  $n=2$  are examined. The first part of this investigation deals with the case  $n=1$ , including two configurations. For the first one (case 1), the dry zone is located in the first half of the plate ( $x=0$ ),

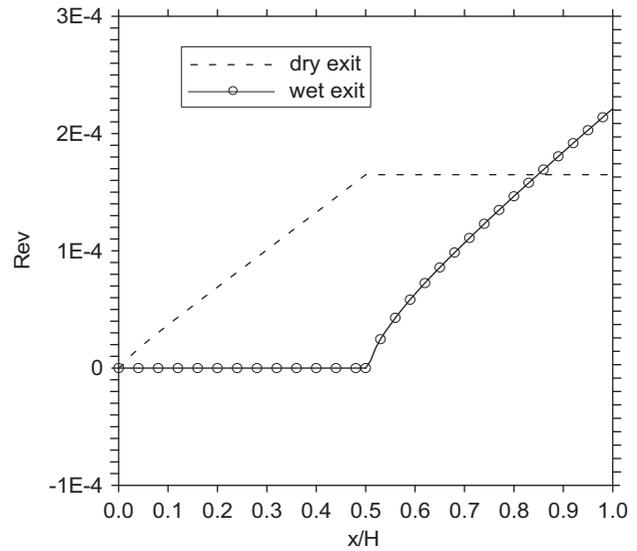


Fig. 8. Effect of the wetted zone position on the local evaporative rate for  $n=1$ :  $u_0=1$  m/s;  $C_0=0.005$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

whereas the wet zone is located at the channel exit. For the second one (case 2), the configuration is inverted.

For several channel sections, Figs. 3–5 present the  $x$ -velocity component, temperature and concentration profiles along the flow. From Fig. 3, we can see that the axial velocity profile gradually develops from uniform distribution at the inlet to parabolic ones as the flow goes upstream. By comparing the results of both cases, it is obvious that the axial velocity profile keeps decreasing at the centreline and is slightly affected by the separation zone between the dry and the wet regions. This can be justified by the difference in fluid viscosities.

On the other hand, the temperature and vapour concentration profiles are extremely influenced by the inversion of the wetted zone position. As can be seen from Fig. 4 and for the first case, the temperature profiles present a sudden decrease near the wetted zone at the channel mid-section. Fig. 5 shows that, in the first case, the water vapour concentration increases along the wetted zone. But in the second case, the concentration increases along the wetted zone and decreases along the dry zone. This result can be explained by the fact that in the dry zone, there is no evaporation process.

It is clear in Fig. 6a and for the two studied cases that near the left plate ( $y=0$ ), the parietal temperature keeps increasing along the channel, but this increase is much larger along the dry zone. The right plate ( $y=d$ ) is not affected by the presence of the wetted zone and the parietal temperature evolution is contin-

uous. In line with the above results and according to the relation (7c), Fig. 6b shows that for both cases and along the humid zones, the parietal vapour concentration increases since the parietal temperature in these zones keeps increasing. But for the first case and along the dry zone, the parietal water vapour concentration is constant because the water vapour concentration in the channel entrance is uniform. In the second case, the parietal vapour concentration along the dry zone decreases because the water vapour concentration at the entrance of this zone (dry zone) is not uniform.

To investigate the relative importance of the sensible and the latent heat exchange along the partially wetted plates, Fig. 7 gives the reduced heat transfer rates along the interface. This figure shows that for the wetted zones, the most part of the imposed heat flux serves for water evaporation. Thus, the heat transfer at the humid interfaces is dominated by the latent heat transport in association with the water vaporisation. On the other hand, and according to the imposed thermal boundary conditions, the figure shows that the sensible and latent heat fluxes are symmetrical along the humid zone. By comparing the two configurations, we can note that the latent heat at humid regions is more effective in the second case, especially at the entrance of the humid zone ( $x=0.5$ ). This can be attributed to the fact that humid air reaches this zone with a higher temperature allowing, accordingly, more evaporation.

Fig. 8 presents the effect of the wetted zone position on the local evaporative rate. This figure shows that in accordance with the above result for latent heat transfer, the evaporative rate at the channel exit is more effective when the humid zone is located at the channel exit (case 1). This result can be mainly justified by the fact that for this case, the air enters the humid region at higher temperature. Also, the axial velocity in the first case (wet exit) is more important than the one in the second case (dry exit); this tendency has the effect of activating the evaporation process.

In order to compare the efficiency of the water evaporation in the case of two wetted zones ( $n=2$ ) to those of one wetted zone ( $n=1$ ) for the same wetted length and to the case where the plate is entirely wetted, the evaporation rate is represented in Fig. 9. From the results given in Fig. 9a, we notice that, for the same wetted length, the evaporation rate in the case of dry exit is intensified by increasing the number of wetted zones and approaches the one in the case when the right plate ( $y=0$ ) is entirely wetted. In the case when  $n=2$  (two wetted zones), Fig. 9b shows that the evaporation rate is more pronounced for the case

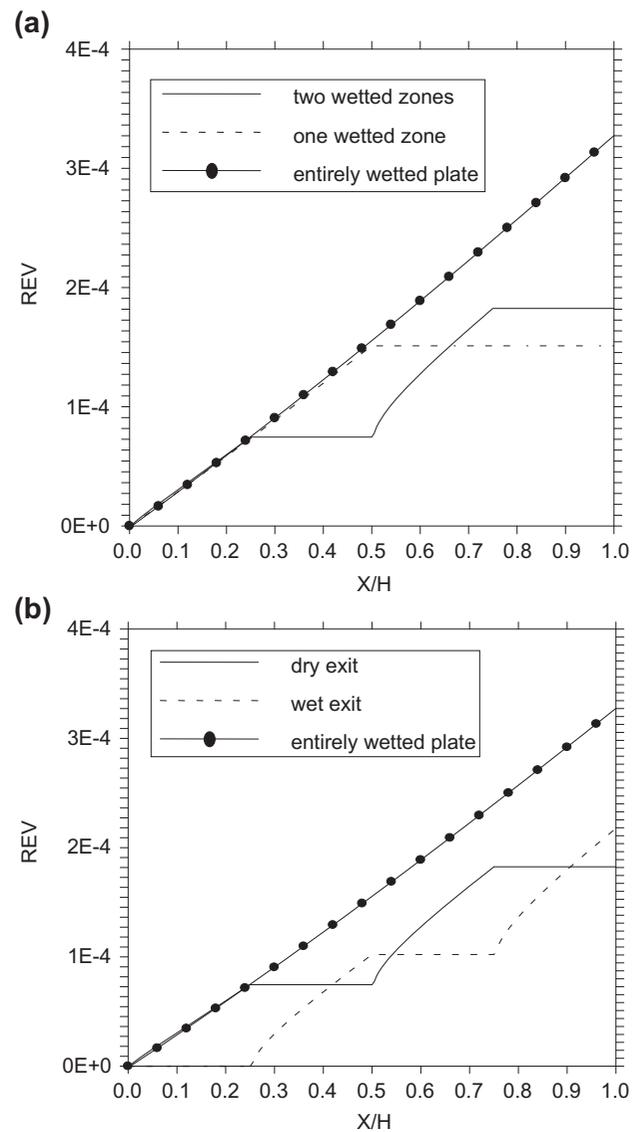


Fig. 9. Effect of the wetted zone position on the local evaporative rate for  $n=2$ :  $u_0=1$  m/s;  $C_0=0.001$ ;  $T_0=283.15$  K;  $q=500$  W/m;  $d/H=0.015$ .

where the humid zone is located at the channel exit. In this case, the whole quantity of energy provided by the heating contributes to liquid evaporation. It is shown that when the number of wetted zones increases, the evaporation rate approaches the one in the case when the right plate ( $y=0$ ) is entirely wetted.

## 5. Conclusion

This work presents an analysis of the transfer coupled of heat and mass during the evaporation by mixed convection in a vertical channel. The two plates of the channel are heated symmetrically with uniform heat fluxes. One of the plates is divided into  $2.n$  equal

regions which are alternately humid and dry zones and the other one is dry. The effect of the position of the humid zone on the outflow and the transfers combined of heat and mass have been analysed. It is observed that the position of the humid zone does not have a meaningful effect on the streamlined profile. On the other hand, it is shown that the heat and mass transfer is influenced strongly by the presence of the wetted zone. This investigation also shows that the evaporation is more efficient in the case where the humid zone is situated at the channel exit. The study also shows that, following an increase in the number of wetted zones, the evaporation rate increases and approaches the one in the case where the entire plate is wetted.

### Notations

$c$	— mass fraction vapor
$C_p$	— specific heat at constant pressure, $\text{kJ kg}^{-1} \text{K}^{-1}$
$C_{pa}$	— specific heat for air, $\text{kJ kg}^{-1} \text{K}^{-1}$
$C_{pv}$	— specific heat for water vapour, $\text{kJ kg}^{-1} \text{K}^{-1}$
$D$	— mass diffusivity, $\text{m}^2 \text{s}^{-1}$
$d$	— channel width, m
$g$	— gravitational acceleration, $\text{m s}^{-2}$
$H$	— channel length, m
$L_v$	— latent heat per mass unit, $\text{kJ kg}^{-1}$
$M_v$	— molar mass of vapour, $\text{kg mol}^{-1}$
$M_a$	— molecular weight of air, $\text{kg mol}^{-1}$
$n$	— number of wetted section
$p$	— pressure, Pa
$m(x)$	— local evaporation rate, $\text{kg s}^{-1} \text{m}^{-2}$
$R_{ev}$	— average evaporating rate, $[\text{kg m}^{-2} \text{s}^{-1}]^{0.5}$
$Re$	— gas Reynolds number, $2d \cdot u_0 / \nu_0$
$Nu_x$	— peripheral local Nusselt number
$Nu_l$	— local Nusselt number for latent heat transfer
$T$	— temperature, K
$T_w$	— dry wall temperature, K
$u$	— axial velocity, $\text{m s}^{-1}$
$v$	— transversal velocity, $\text{m s}^{-1}$
$x$	— axial coordinate, m
$y$	— transversal coordinate, m

### Greek symbols

$\mu$	— dynamic viscosity, $\text{kg m}^{-1} \text{s}^{-1}$
$\rho$	— density, $\text{kg m}^{-3}$
$\lambda$	— thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
$\beta$	— thermal expansion coefficient $-1/\rho(\partial\rho/\partial T)_{p,c}$ , $\text{K}^{-1}$
$\beta^*$	— mass expansion coefficient $-1/\rho(\partial\rho/\partial c)_{p,T}$

### Subscripts

0	— inlet condition
w	— wall

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