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The IAPWS industrial formulation for the thermodynamic properties of seawater

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ABSTRACT

In 2008, the International Association for the Properties of Water and Steam (IAPWS) adopted a standard formulation for the thermodynamic properties of seawater as a sum of contributions to the Gibbs free energy from pure water and from dissolved sea salt. For pure water, the IAPWS formulation for general and scientific use (IAPWS-95) was used. However, for industrial uses such as desalination and seawater power-plant cooling, it is likely to be more convenient to use the computationally simpler IAPWS formulation for industrial use (IAPWS-IF97), which is standard in the steam power industry. This paper documents this approach and gives formulas for calculating thermodynamic properties of seawater and steam (volume, enthalpy, isobaric heat capacity, etc.). The calculation of colligative properties (such as boiling and freezing points and osmotic pressure) is also described, as is the calculation of properties of two-phase states such as brine-vapor and brine-ice (sea ice). The computing speeds for these calculations are faster than those using IAPWS-95 by factors on the order of 100–200. The use of IAPWS-IF97 instead of IAPWS-95 for industrial seawater calculations is endorsed in IAPWS Advisory Note No. 5: Industrial Calculation of the Thermodynamic Properties of Seawater. This use is valid for IAPSO

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Standard Seawater with sea salt of the Reference Composition in specified regions inside the pressure, temperature, and salinity ranges of 0.3 kPa ≤ p ≤ 100 MPa, 261 K ≤ T ≤ 353 K, and 0 ≤ S ≤ 0.12 kg kg^{-1}.

Keywords: Seawater; Thermodynamics; Industrial use; Gibbs function; Brine ice; Brine vapor

1. Introduction

The calculation of thermodynamic properties of seawater is important not only for ocean science, but also for industrial applications such as desalination and seawater cooling of power plants. In 2008, the International Association for the Properties of Water and Steam (IAPWS) adopted the "Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater" (for short, IAPWS-08) [1–4]. This formulation was also adopted as an international standard by the Intergovernmental Oceanographic Commission (IOC) in Paris in 2009 and by the International Union of Geodesy and Geophysics (IUGG) in Melbourne in 2011. In this formulation, the thermodynamic properties of seawater are calculated from an equation of state consisting of a water contribution and a saline contribution. The water contribution of IAPWS-08 is computed from the Helmholtz free energy equation of the "Revised Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use" (IAPWS-95) [5,6]. The saline contribution is formulated as a Gibbs free energy equation.

However, the iterative calculation of required properties from the IAPWS-95 Helmholtz free energy equation is computationally intensive, making its use less desirable in applications where processing speed is important. For modeling desalination and cooling processes, it is more convenient to use the Gibbs free energy equation of region 1 of the "Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [7,8] for calculating the water contribution of the seawater formulation. An additional reason for using IAPWS-IF97 in the seawater formulation is that the IAPWS-IF97 formulation is commonly used by industry for calculating properties of pure steam and water for power plants and their components.

The purpose of this paper is to document the IAPWS "Advisory Note No. 5: Industrial Calculation of the Thermodynamic Properties of Seawater" [9], which describes the use of IAPWS-IF97 instead of IAPWS-95 for calculating thermodynamic properties of the water contribution of the seawater formulation in order to enable more consistent and efficient calculations in the cases described in the preceding paragraph. This industrial calculation of the thermodynamic properties of seawater will herein be called the industrial seawater formulation. All requisite equations and parameters are provided in this paper.

It should be noted that this usage differs from the use of the simplified water Gibbs free energy formulation adopted as the "IAPWS Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use" [2,3,10]. Ref. [10] documents a formulation for liquid-water thermodynamics to be used specifically for computational efficiency in oceanographic calculations, while the present paper describes the application of the existing IAPWS "industrial" formulation to calculate the water contribution to seawater properties in industrial settings where the use of IAPWS-IF97 is desired for other reasons.

Sharqawy et al. [11] published a review of correlations and experimental data available for the calculation of thermophysical properties of seawater. In addition, those authors developed regression equations for the calculation of different properties of seawater as functions of temperature and salinity only. These relatively simple equations have a limited range of application due to their pressure independence. Also, their equations were correlated separately, so they are not mutually thermodynamically consistent, which may conceivably cause problems in process calculations. As a consequence, it may be preferable to use the more accurate algorithms of IAPWS-08 in combination with IAPWS-IF97 to calculate the thermodynamic properties of seawater for industrial use.

In this paper, the terms "seawater" and "brine" refer to a solution of sea salt in liquid water; the term "sea salt" refers to a mixed solute with the chemical Reference Composition described in [12]; "brine-vapor mixture" is a two-phase composite of brine with water vapor, and "brine-ice mixture" is a two-phase composite of brine with ice Ih. For water vapor, the IAPWS "Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [7,8] is used, and for ice Ih, the IAPWS "Revised Release on the Equation of State 2006 for H_2O Ice Ih" (for short, IAPWS-06) [13,14] is used. Note that salinity S here refers to the Absolute Salinity of seawater which differs from the

Practical Salinity that is obtained from common salinometers or oceanographic devices [1,4,12,15].

2. Thermodynamic properties of seawater

2.1. The fundamental equation of state for seawater

The equation of state of IAPWS for the liquid mixture “seawater” [1–4] is in the form of the specific Gibbs free energy as a function of pressure p, temperature T, and salinity S, and reads

$$g(p,T,S) = g^{\mathrm{W}}(p,T) + g^{\mathrm{S}}(p,T,S) \tag{1}$$

As described in the following Sections 2.2 and 2.3, the water contribution $g^{\mathrm{W}}(p,T)$ is computed from the Industrial Formulation IAPWS-IF97, and the saline contribution $g^{\mathrm{S}}(p,T,S)$ from the IAPWS-08 seawater formulation. The temperature T is based on the International Temperature Scale ITS-90 [16,17]. Salinity S is the mass fraction of sea salt in seawater

$$S = \frac{m_{\mathrm{S}}}{m} \tag{2}$$

where m_{S} is the mass of sea salt and m is the mass of the mixture seawater. The composition of sea salt is assumed to be the Reference Composition for IAPSO Standard Seawater [12]. For pure water with $S = 0$, the saline contribution vanishes, $g^{\mathrm{S}}(p,T,0) = 0$.

Using salinity S, the molality of seawater, defined as the moles of the solute sea salt per mass of the solvent water, is computed as

$$b = \frac{S}{M_{\mathrm{S}}(1-S)} \tag{3}$$

where $M_{\mathrm{S}} = 31.403\ 821\ 8\,\mathrm{kg\ kmol^{-1}}$ is the molar mass, or the “mole weighted average atomic weight of the elements of sea salt” as given by Millero et al. [12].

All thermodynamic properties and derivatives can be calculated from Eq. (1) by using the appropriate combinations of the Gibbs free energy equation and its derivatives with respect to p, T, and S. Relations between relevant thermodynamic properties and $g(p,T,S)$, Eq. (1), and its derivatives are summarized in Table 1. To assist the user in computer-program verification, test values are given in Table A1, which contains values for the specific Gibbs free energy, $g(p,T,S)$, together with the corresponding derivatives and some thermodynamic properties. For an easy check of the Gibbs free energy functions of the water contribution $g^{\mathrm{W}}(p,T)$ and of the saline contribution $g^{\mathrm{S}}(p,T,S)$, the results of both contributions are reported separately in Table A1.

In addition, any thermodynamic derivative of the variables p, T, v, s, u, h, f, and g, for example, $(\partial h/\partial p)_v$, can be determined from algebraic combinations of v, s, c_p, α_v, and κ_T as described in detail in the IAPWS Advisory Note No. 3 [19] and comprehensively in [20].

Backward functions, which use other input variables, can also be calculated by iteration from the explicit equations listed in Table 1. For example, for given pressure p, specific enthalpy h, and salinity S, the temperature T is calculated from the equation for $h(p,T,S)$ by iteration.

2.2. Water contribution

The water contribution of Eq. (1) is calculated from the specific Gibbs free energy equation of IAPWS-IF97 region 1 (liquid water) [7,8]

$$\frac{g^{\mathrm{W}}(p,T)}{R_{\mathrm{W}}T} = \gamma(\pi,\tau) = \sum_{i=1}^{34} n_i (7.1-\pi)^{I_i} (\tau - 1.222)^{J_i} \tag{4}$$

where the reduced Gibbs free energy is $\gamma = g^{\mathrm{W}}/(R_{\mathrm{W}}T)$, the reduced pressure is $\pi = p/p^*$ and the inverse reduced temperature is $\tau = T^*/T$ with $p^* = 16.53\,\mathrm{MPa}$, and $T^* = 1{,}386\ \mathrm{K}$. $R_{\mathrm{W}} = 0.461\,526\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$ is the specific gas constant of water given in [7]. The coefficients n_i and the exponents I_i and J_i are given in Table 2. The equations of the Industrial Formulation IAPWS-IF97 are also described in various steam-table books [20–24].

Table 3 contains the relations for determining $g^{\mathrm{W}}(p,T)$ and its pressure and temperature derivatives from the reduced Gibbs free energy equation $\gamma(\pi,\tau)$ and its derivatives. The equations for the derivatives of $\gamma(\pi,\tau)$ are taken from Table 4. With the equations of Table 3, the water contributions in the equations of Table 1 can be determined.

Note that for temperatures less than the melting (freezing) temperature of pure water [25,26], or greater than the saturation (boiling) temperature of pure water [7,8], the IAPWS-IF97 Gibbs free energy equation of liquid water region 1, Eq. (4), is evaluated at conditions where the liquid phase of pure water is metastable. Investigations have shown that Eq. (4) can be used at temperatures in this metastable region and even below 273.15 K, the minimum temperature of IAPWS-IF97. The accuracy of the thermodynamic properties of seawater calculated from IAPWS-IF97 is sufficient for industrial calculations. Section 6 contains a description of the deviations of IAPWS-IF97 from IAPWS-95.

Table 1
Relations between the relevant thermodynamic properties of seawater and $g(p,T,S)$, Eq. (1), and its derivatives[a,b]

Property	Relation
Specific volume v	$v(p,T,S) = g_p$
Density $\rho = v^{-1}$	$\rho(p,T,S) = \frac{1}{g_p}$
Specific internal energy u	$u(p,T,S) = g - p\,g_p - T\,g_T$
Specific enthalpy $h = u + pv$	$h(p,T,S) = g - T\,g_T$
Specific entropy s	$s(p,T,S) = -g_T$
Specific isobaric heat capacity $c_p = c_p = (\partial h/\partial T)_p$	$c_p(p,T,S) = -T\,g_{TT}$
Specific isochoric heat capacity $c_v = (\partial u/\partial T)_v$	$c_v(p,T,S) = T\left(\frac{g_{pT}^2}{g_{pp}} - g_{TT}\right)$
Cubic isobaric expansion coefficient $\alpha_v = v^{-1}(\partial v/\partial T)_p$	$\alpha_v(p,T,S) = \frac{g_{pT}}{g_p}$
Isothermal compressibility $\kappa_T = -v^{-1}(\partial v/\partial p)_T$	$\kappa_T(p,T,S) = -\frac{g_{pp}}{g_p}$
Speed of sound $w = v\sqrt{-(\partial p/\partial v)_s}$	$w(p,T,S) = g_p\sqrt{\frac{g_{TT}}{(g_{pT}^2 - g_{pp}g_{TT})}}$
Isentropic exponent $\kappa = -vp^{-1}(\partial p/\partial v)_s$	$\kappa(p,T,S) = \frac{1}{p}\frac{g_p g_{TT}}{(g_{pT}^2 - g_{pp}g_{TT})}$
Relative chemical potential $\mu = (\partial g/\partial S)_{p,T}$	$\mu(p,T,S) = g_S$
Chemical potential of water $\mu_W = g - S\,\mu$	$\mu_W(p,T,S) = g - S\,g_S$
Chemical potential of sea salt $\mu_S = \mu + \mu_W$	$\mu_S(p,T,S) = g + (1-S)g_S$
Osmotic coefficient $\phi = (g^W - \mu_W)b^{-1}R_m^{-1}T^{-1}$	$\phi(p,T,S) = -\frac{(g^S - S\,g_S)}{b\,R_m\,T}$
Haline contraction coefficient $\beta = \rho^{-1}(\partial\rho/\partial S)_{p,T}$	$\beta(p,T,S) = -\frac{g_{pS}}{g_p}$

[a] $g_p = \left(\frac{\partial g}{\partial p}\right)_{T,S} = \left(\frac{\partial g^W}{\partial p}\right)_T + \left(\frac{\partial g^S}{\partial p}\right)_{T,S}$, $g_{pp} = \left(\frac{\partial^2 g}{\partial p^2}\right)_{T,S} = \left(\frac{\partial^2 g^W}{\partial p^2}\right)_T + \left(\frac{\partial^2 g^S}{\partial p^2}\right)_{T,S}$, $g_T = \left(\frac{\partial g}{\partial T}\right)_{p,S} = \left(\frac{\partial g^W}{\partial T}\right)_p + \left(\frac{\partial g^S}{\partial T}\right)_{p,S}$, $g_{TT} = \left(\frac{\partial^2 g}{\partial T^2}\right)_{p,S} = \left(\frac{\partial^2 g^W}{\partial T^2}\right)_p + \left(\frac{\partial^2 g^S}{\partial T^2}\right)_{p,S}$, $g_{pT} = \left(\frac{\partial^2 g}{\partial p\partial T}\right)_S = \left(\frac{\partial^2 g^W}{\partial p\partial T}\right) + \left(\frac{\partial^2 g^S}{\partial p\partial T}\right)_S$, $g_S = \left(\frac{\partial g}{\partial S}\right)_{p,T} = \left(\frac{\partial g^S}{\partial S}\right)_{p,T}$, $g_{pS} = \left(\frac{\partial^2 g}{\partial p\partial S}\right)_T = \left(\frac{\partial^2 g^S}{\partial p\partial S}\right)_T$.

[b] The value of the molar gas constant $R_m = 8.314\,472$ kJ kmol^{-1} K^{-1} was taken from CODATA 2006 [18] and is used in the 2008 IAPWS formulation for seawater.

Numerical check values for the water contribution computed from $g^W(p,T)$, Eq. (4), and its derivatives, for the seawater properties computed from the Gibbs function $g(p,T,S)$, Eq. (1), and its derivatives as well as for selected seawater properties of Table 1 at given points (p,T,S) are given in Table A1.

2.3. Saline contribution

The saline contribution of Eq. (1) is calculated from the Gibbs free energy equation of the IAPWS formulation for seawater properties [1].

$$\frac{g^S(p,T,S)}{g^*} = \sum_{k=0}^{5}\sum_{j=0}^{6}\left(n_{1jk}\xi^2\ln\xi + \sum_{i=2}^{7} n_{ijk}\xi^i\right)(\theta-\theta_0)^j(\pi-\pi_0)^k \tag{5}$$

where the reduced pressure is $\pi = p/p^*$, the reduced atmospheric pressure is $\pi_0 = p_0/p^*$, the reduced temperature is $\theta = T/T^*$, the reduced Celsius zero-point temperature is $\theta_0 = T_0/T^*$, and the square root of the reduced salinity is $\xi = \sqrt{S/S^*}$. The values of the

Table 2
Numerical values of the coefficients and exponents of the dimensionless Gibbs free energy for region 1, Eq. (4)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	–2	0.146 329 712 131 67	18	2	3	–0.441 418 453 308 46 × 10^{-5}
2	0	–1	–0.845 481 871 691 14	19	2	17	–0.726 949 962 975 94 × 10^{-15}
3	0	0	–0.375 636 036 720 40 × 10^{1}	20	3	–4	–0.316 796 448 450 54 × 10^{-4}
4	0	1	0.338 551 691 683 85 × 10^{1}	21	3	0	–0.282 707 979 853 12 × 10^{-5}
5	0	2	–0.957 919 633 878 72	22	3	6	–0.852 051 281 201 03 × 10^{-9}
6	0	3	0.157 720 385 132 28	23	4	–5	–0.224 252 819 080 00 × 10^{-5}
7	0	4	–0.166 164 171 995 01 × 10^{-1}	24	4	–2	–0.651 712 228 956 01 × 10^{-6}
8	0	5	0.812 146 299 835 68 × 10^{-3}	25	4	10	–0.143 417 299 379 24 × 10^{-12}
9	1	–9	0.283 190 801 238 04 × 10^{-3}	26	5	–8	–0.405 169 968 601 17 × 10^{-6}
10	1	–7	–0.607 063 015 658 74 × 10^{-3}	27	8	–11	–0.127 343 017 416 41 × 10^{-8}
11	1	–1	–0.189 900 682 184 19 × 10^{-1}	28	8	–6	–0.174 248 712 306 34 × 10^{-9}
12	1	0	–0.325 297 487 705 05 × 10^{-1}	29	21	–29	–0.687 621 312 955 31 × 10^{-18}
13	1	1	–0.218 417 171 754 14 × 10^{-1}	30	23	–31	0.144 783 078 285 21 × 10^{-19}
14	1	3	–0.528 383 579 699 30 × 10^{-4}	31	29	–38	0.263 357 816 627 95 × 10^{-22}
15	2	–3	–0.471 843 210 732 67 × 10^{-3}	32	30	–39	–0.119 476 226 400 71 × 10^{-22}
16	2	0	–0.300 017 807 930 26 × 10^{-3}	33	31	–40	0.182 280 945 814 04 × 10^{-23}
17	2	1	0.476 613 939 069 87 × 10^{-4}	34	32	–41	–0.935 370 872 924 58 × 10^{-25}

Table 3
Relations for determining $g^{\mathrm{W}}(p,T)$ and pressure and temperature derivatives from the reduced Gibbs free energy equation $\gamma(\pi,\tau)$ and its derivatives of IAPWS-IF97 region 1[a]

$$g^{\mathrm{W}} = R_{\mathrm{W}} T\gamma, \quad \left(\frac{\partial g^{\mathrm{W}}}{\partial p}\right)_T = \frac{R_{\mathrm{W}} T}{p}\pi\gamma_\pi, \quad \left(\frac{\partial^2 g^{\mathrm{W}}}{\partial p^2}\right)_T = \frac{R_{\mathrm{W}}\,T}{p^2}\pi^2\gamma_{\pi\pi},$$

$$\left(\frac{\partial g^{\mathrm{W}}}{\partial T}\right)_p = R_{\mathrm{W}}(\gamma - \tau\gamma_\tau), \quad \left(\frac{\partial^2 g^{\mathrm{W}}}{\partial T^2}\right)_p = \frac{R_{\mathrm{W}}}{T}\tau^2\gamma_{\tau\tau}, \quad \left(\frac{\partial^2 g^{\mathrm{W}}}{\partial p \partial T}\right) = \frac{R_{\mathrm{W}}\pi}{p}(\gamma_\pi - \tau\gamma_{\pi\tau})$$

[a] $\gamma_\pi = \left(\frac{\partial \gamma}{\partial \pi}\right)_\tau, \quad \gamma_{\pi\pi} = \left(\frac{\partial^2 \gamma}{\partial \pi^2}\right)_\tau, \quad \gamma_\tau = \left(\frac{\partial \gamma}{\partial \tau}\right)_\pi, \quad \gamma_{\tau\tau} = \left(\frac{\partial^2 \gamma}{\partial \tau^2}\right)_\pi, \quad \gamma_{\pi\tau} = \left(\frac{\partial^2 \gamma}{\partial \pi \partial \tau}\right).$

Table 4
The dimensionless Gibbs free energy γ and its derivatives[a] according to Eq. (4)

$$\gamma = \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} (\tau - 1.222)^{J_i}$$

$$\gamma_\pi = -\sum_{i=1}^{34} n_i I_i (7.1 - \pi)^{I_i - 1} (\tau - 1.222)^{J_i} \quad \gamma_{\pi\pi} = \sum_{i=1}^{34} n_i I_i (I_i - 1)(7.1 - \pi)^{I_i - 2} (\tau - 1.222)^{J_i}$$

$$\gamma_\tau = \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} J_i (\tau - 1.222)^{J_i - 1} \quad \gamma_{\tau\tau} = \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} J_i (J_i - 1)(\tau - 1.222)^{J_i - 2}$$

$$\gamma_{\pi\tau} = -\sum_{i=1}^{34} n_i I_i (7.1 - \pi)^{I_i - 1} J_i (\tau - 1.222)^{J_i - 1}$$

[a] $\gamma_\pi = \left(\frac{\partial \gamma}{\partial \pi}\right)_\tau, \quad \gamma_{\pi\pi} = \left(\frac{\partial^2 \gamma}{\partial \pi^2}\right)_\tau, \quad \gamma_\tau = \left(\frac{\partial \gamma}{\partial \tau}\right)_\pi, \quad \gamma_{\tau\tau} = \left(\frac{\partial^2 \gamma}{\partial \tau^2}\right)_\pi, \quad \gamma_{\pi\tau} = \left(\frac{\partial^2 \gamma}{\partial \pi\, \partial \tau}\right).$

constants $g^* = 10^{-3}$ kJ kg^{-1}, $p^* = 100$ MPa, $T^* = 40$ K, and $S^* = 40/35 \times S_{\mathrm{n}}$ were taken from Feistel [4] while the normal salinity of seawater $S_{\mathrm{n}} = 0.035\,165\,04$ kg kg^{-1} was taken from Millero et al. [12]. The constants $p_0 = 0.101\,325$ MPa [27] and $T_0 = 273.15$ K [16], respectively, are the standard atmospheric pressure and the Celsius zero point. The coefficients n_{ijk} are given in Table 5.

Table 6 contains the relations for determining the following derivatives:

$$g_p^{\mathrm{S}} = \left(\frac{\partial g^{\mathrm{S}}}{\partial p}\right)_{T,S}, \quad g_{pp}^{\mathrm{S}} = \left(\frac{\partial^2 g^{\mathrm{S}}}{\partial p^2}\right)_{T,S}, \quad g_T^{\mathrm{S}} = \left(\frac{\partial g^{\mathrm{S}}}{\partial T}\right)_{p,S},$$

$$g_{TT}^{\mathrm{S}} = \left(\frac{\partial^2 g^{\mathrm{S}}}{\partial T^2}\right)_{p,S}, \quad g_{pT}^{\mathrm{S}} = \left(\frac{\partial^2 g^{\mathrm{S}}}{\partial p \partial T}\right)_S, \quad g_S^{\mathrm{S}} = \left(\frac{\partial g^{\mathrm{S}}}{\partial S}\right)_{p,T},$$

$$g_{pS}^{\mathrm{S}} = \left(\frac{\partial^2 g^{\mathrm{S}}}{\partial p \partial S}\right)_T$$

from Eq. (5) which are used in Table 1.

Table 5
Coefficients of the saline contribution of the Gibbs function, as given by Eq. (5). Coefficients not contained in this table have the value $n_{ijk} = 0$

i	j	k	n_{ijk}	i	j	k	n_{ijk}
1	0	0	$0.581\,281\,456\,626\,732 \times 10^{4}$	2	2	1	$-0.860\,764\,303\,783\,977 \times 10^{3}$
2	0	0	$0.141\,627\,648\,484\,197 \times 10^{4}$	3	2	1	$0.383\,058\,066\,002\,476 \times 10^{3}$
3	0	0	$-0.243\,214\,662\,381\,794 \times 10^{4}$	2	3	1	$0.694\,244\,814\,133\,268 \times 10^{3}$
4	0	0	$0.202\,580\,115\,603\,697 \times 10^{4}$	3	3	1	$-0.460\,319\,931\,801\,257 \times 10^{3}$
5	0	0	$-0.109\,166\,841\,042\,967 \times 10^{4}$	2	4	1	$-0.297\,728\,741\,987\,187 \times 10^{3}$
6	0	0	$0.374\,601\,237\,877\,840 \times 10^{3}$	3	4	1	$0.234\,565\,187\,611\,355 \times 10^{3}$
7	0	0	$-0.485\,891\,069\,025\,409 \times 10^{2}$	2	0	2	$0.384\,794\,152\,978\,599 \times 10^{3}$
1	1	0	$0.851\,226\,734\,946\,706 \times 10^{3}$	3	0	2	$-0.522\,940\,909\,281\,335 \times 10^{2}$
2	1	0	$0.168\,072\,408\,311\,545 \times 10^{3}$	4	0	2	$-0.408\,193\,978\,912\,261 \times 10^{1}$
3	1	0	$-0.493\,407\,510\,141\,682 \times 10^{3}$	2	1	2	$-0.343\,956\,902\,961\,561 \times 10^{3}$
4	1	0	$0.543\,835\,333\,000\,098 \times 10^{3}$	3	1	2	$0.831\,923\,927\,801\,819 \times 10^{2}$
5	1	0	$-0.196\,028\,306\,689\,776 \times 10^{3}$	2	2	2	$0.337\,409\,530\,269\,367 \times 10^{3}$
6	1	0	$0.367\,571\,622\,995\,805 \times 10^{2}$	3	2	2	$-0.541\,917\,262\,517\,112 \times 10^{2}$
2	2	0	$0.880\,031\,352\,997\,204 \times 10^{3}$	2	3	2	$-0.204\,889\,641\,964\,903 \times 10^{3}$
3	2	0	$-0.430\,664\,675\,978\,042 \times 10^{2}$	2	4	2	$0.747\,261\,411\,387\,560 \times 10^{2}$
4	2	0	$-0.685\,572\,509\,204\,491 \times 10^{2}$	2	0	3	$-0.965\,324\,320\,107\,458 \times 10^{2}$
2	3	0	$-0.225\,267\,649\,263\,401 \times 10^{3}$	3	0	3	$0.680\,444\,942\,726\,459 \times 10^{2}$
3	3	0	$-0.100\,227\,370\,861\,875 \times 10^{2}$	4	0	3	$-0.301\,755\,111\,971\,161 \times 10^{2}$
4	3	0	$0.493\,667\,694\,856\,254 \times 10^{2}$	2	1	3	$0.124\,687\,671\,116\,248 \times 10^{3}$
2	4	0	$0.914\,260\,447\,751\,259 \times 10^{2}$	3	1	3	$-0.294\,830\,643\,494\,290 \times 10^{2}$
3	4	0	$0.875\,600\,661\,808\,945$	2	2	3	$-0.178\,314\,556\,207\,638 \times 10^{3}$
4	4	0	$-0.171\,397\,577\,419\,788 \times 10^{2}$	3	2	3	$0.256\,398\,487\,389\,914 \times 10^{2}$
2	5	0	$-0.216\,603\,240\,875\,311 \times 10^{2}$	2	3	3	$0.113\,561\,697\,840\,594 \times 10^{3}$
4	5	0	$0.249\,697\,009\,569\,508 \times 10^{1}$	2	4	3	$-0.364\,872\,919\,001\,588 \times 10^{2}$
2	6	0	$0.213\,016\,970\,847\,183 \times 10^{1}$	2	0	4	$0.158\,408\,172\,766\,824 \times 10^{2}$
2	0	1	$-0.331\,049\,154\,044\,839 \times 10^{4}$	3	0	4	$-0.341\,251\,932\,441\,282 \times 10^{1}$
3	0	1	$0.199\,459\,603\,073\,901 \times 10^{3}$	2	1	4	$-0.316\,569\,643\,860\,730 \times 10^{2}$
4	0	1	$-0.547\,919\,133\,532\,887 \times 10^{2}$	2	2	4	$0.442\,040\,358\,308\,000 \times 10^{2}$
5	0	1	$0.360\,284\,195\,611\,086 \times 10^{2}$	2	3	4	$-0.111\,282\,734\,326\,413 \times 10^{2}$
2	1	1	$0.729\,116\,529\,735\,046 \times 10^{3}$	2	0	5	$-0.262\,480\,156\,590\,992 \times 10^{1}$
3	1	1	$-0.175\,292\,041\,186\,547 \times 10^{3}$	2	1	5	$0.704\,658\,803\,315\,449 \times 10^{1}$
4	1	1	$-0.226\,683\,558\,512\,829 \times 10^{2}$	2	2	5	$-0.792\,001\,547\,211\,682 \times 10^{1}$

Table 6
Equations for the saline contribution of the specific Gibbs free energy, Eq. (5), and its derivatives

$g^{\mathrm{S}}(p,T,S) = g^* \sum_{k=0}^{5} \sum_{j=0}^{6} (n_{1jk}\xi^2 \ln \xi + \sum_{i=2}^{7} n_{ijk}\xi^i)(\theta-\theta_0)^j(\pi-\pi_0)^k$
$g^{\mathrm{S}}_{T}(p,T,S) = \frac{g^*}{T^*} \sum_{k=0}^{5} \sum_{j=1}^{6} j(n_{1jk}\xi^2 \ln \xi + \sum_{i=2}^{7} n_{ijk}\xi^i)(\theta-\theta_0)^{j-1}(\pi-\pi_0)^k$
$g^{\mathrm{S}}_{p}(p,T,S) = \frac{g^*}{p^*} \sum_{k=1}^{5} \sum_{j=0}^{6} \sum_{i=2}^{7} k n_{ijk}\xi^i(\theta-\theta_0)^j(\pi-\pi_0)^{k-1}$
$g^{\mathrm{S}}_{TT}(p,T,S) = \frac{g^*}{(T^*)^2} \sum_{k=0}^{5} \sum_{j=2}^{6} \sum_{i=2}^{7} j(j-1) n_{ijk}\xi^i(\theta-\theta_0)^{j-2}(\pi-\pi_0)^k$
$g^{\mathrm{S}}_{pT}(p,T,S) = \frac{g^*}{T^* p^*} \sum_{k=1}^{5} \sum_{j=1}^{6} \sum_{i=2}^{7} jk n_{ijk}\xi^i(\theta-\theta_0)^{j-1}(\pi-\pi_0)^{k-1}$
$g^{\mathrm{S}}_{pp}(p,T,S) = \frac{g^*}{(p^*)^2} \sum_{k=2}^{5} \sum_{j=0}^{6} \sum_{i=2}^{7} k(k-1) n_{ijk}\xi^i(\theta-\theta_0)^j(\pi-\pi_0)^{k-2}$
$g^{\mathrm{S}}_{S}(p,T,S) = \frac{g^*}{2S^*} \sum_{k=0}^{5} \sum_{j=0}^{6} [n_{1jk}(2 \ln \xi + 1) + \sum_{i=2}^{7} i n_{ijk}\xi^{i-2}](\theta-\theta_0)^j(\pi-\pi_0)^k$
$g^{\mathrm{S}}_{pS}(p,T,S) = \frac{g^*}{2S^* p^*} \sum_{k=1}^{5} \sum_{j=0}^{6} \sum_{i=2}^{7} ik n_{ijk}\xi^{i-2}(\theta-\theta_0)^j(\pi-\pi_0)^{k-1}$

Numerical check values for the saline contribution computed from $g^S(p,T,S)$, Eq. (5), and its derivatives are given in Table A1.

3. Colligative properties

3.1. Phase equilibrium between seawater and water vapor

For computation of the equilibrium between seawater and water vapor, the required thermodynamic condition is the equality of the chemical potential of H_2O in seawater, μ_W, with the specific Gibbs free energy of water vapor, g^{vap}. In thermodynamic equilibrium, the amount of sea salt in the vapor phase is assumed to be negligible. Here, chemical potentials are expressed on a mass basis, which is the usual (molar) chemical potential divided by molar mass.

For the phase equilibrium between seawater and water vapor, the following condition must be fulfilled:

$$\mu_W(p,T,S) = g^{vap}(p,T) \tag{6}$$

which can be written equivalently in terms of the osmotic coefficient ϕ as

$$bR_m T\phi(p,T,S) = g^W(p,T) - g^{vap}(p,T) \tag{7}$$

where $\mu_W(p,T,S)$ and $\phi(p,T,S)$ are from Table 1, and $g^W(p,T)$ is from Eq. (4). The molality b is determined from Eq. (3) and the value of the molar gas constant $R_m = 8.314\,472\ \mathrm{kJ\,kmol^{-1}\,K^{-1}}$ is taken from CODATA 2006 [18]. Note that the slightly different recently updated value of R_m [28] is not used here for consistency with IAPWS-08. The specific Gibbs free energy of water vapor $g^{vap}(p,T)$ is calculated from the Gibbs free energy equation of the IAPWS-IF97 region 2 (water vapor),

$$\frac{g^{vap}(p,T)}{R_W T} = \gamma(\pi,\tau) = \gamma^o(\pi,\tau) + \gamma^r(\pi,\tau) \tag{8}$$

with the ideal-gas contribution,

$$\gamma^o = \ln\pi + \sum_{i=1}^{9} n_i^o \tau^{J_i^o} \tag{9}$$

and the residual contribution,

$$\gamma^r = \sum_{i=1}^{43} n_i\,\pi^{I_i}(\tau - 0.5)^{J_i} \tag{10}$$

In Eqs. (8)–(10), the reduced Gibbs free energy is $\gamma = g^{vap}/(R_W T)$, the reduced pressure is $\pi = p/p^*$, and the inverse reduced temperature is $\tau = T^*/T$ with $p^* = 1$ MPa and $T^* = 540$ K. The specific gas constant of water $R_W = 0.461\,526\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$ is given in [7]. The coefficients n_i^o and n_i, and the exponents I_i, J_i^o, and J_i are given in Tables 7 and 8.

All thermodynamic properties of pure water vapor can be derived from Eq. (8) by using the appropriate combinations of the ideal-gas contribution γ^o, Eq. (9), and the residual contribution γ^r, Eq. (10), of the dimensionless Gibbs free energy and their derivatives. All required derivatives of the ideal-gas contribution and of the residual contribution of the dimensionless Gibbs free energy are given in Tables 9 and 10, respectively. Relations between the relevant thermodynamic properties and γ^o and γ^r and their derivatives are summarized in Table 11.

Using Eqs. (6) or (7), the boiling temperature of seawater $T = T_b$ can be calculated by iteration from pressure p and salinity S, or the boiling pressure of seawater $p = p_b$ from temperature T and salinity S, or the brine salinity of seawater $S = S_b$ from p and T. At a given equilibrium state between seawater and water vapor, the properties of the seawater (brine) phase are calculated from Table 1 and the properties of the water vapor from Table 11.

At brine-vapor equilibrium, vapor is sub-saturated; i.e. at a given pressure, the temperature is higher than the saturation (boiling) temperature of pure water, and the pressure at a given temperature is below the saturation (boiling) pressure of pure water.

Note that the specific Gibbs free energy of liquid water $g^W(p,T)$ in Eq. (7) is evaluated from IAPWS-IF97 at conditions where the liquid phase of pure water is metastable. Due to salinity, the boiling-point elevation can be up to 2 K (see Feistel [4] for experimental results from the literature). Investigations have shown that Eq. (7) can reasonably be extrapolated to these metastable states.

Table A2 shows selected boiling temperatures for given pressures and salinities.

3.2. Phase equilibrium between seawater and ice

For the phase equilibrium between seawater and ice Ih, the following condition must be fulfilled:

$$\mu_W(p,T,S) = g^{Ih}(p,T) \tag{11}$$

or equivalently:

Table 7
Numerical values of the coefficients and exponents of the ideal-gas contribution γ^{o} of the dimensionless Gibbs free energy for region 2, Eq. (9)

i	J_i^{o}	n_i^{o}	i	J_i^{o}	n_i^{o}
1	0	$-0.969\,276\,865\,002\,17 \times 10^{1}$	6	-2	$0.142\,408\,191\,714\,44 \times 10^{1}$
2	1	$0.100\,866\,559\,680\,18 \times 10^{2}$	7	-1	$-0.438\,395\,113\,194\,50 \times 10^{1}$
3	-5	$-0.560\,879\,112\,830\,20 \times 10^{-2}$	8	2	$-0.284\,086\,324\,607\,72$
4	-4	$0.714\,527\,380\,814\,55 \times 10^{-1}$	9	3	$0.212\,684\,637\,533\,07 \times 10^{-1}$
5	-3	$-0.407\,104\,982\,239\,28$			

Table 8
Numerical values of the coefficients and exponents of the residual contribution γ^{r} of the dimensionless Gibbs free energy for region 2, Eq. (10)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	0	$-0.177\,317\,424\,732\,13 \times 10^{-2}$	23	7	0	$-0.590\,595\,643\,242\,70 \times 10^{-17}$
2	1	1	$-0.178\,348\,622\,923\,58 \times 10^{-1}$	24	7	11	$-0.126\,218\,088\,991\,01 \times 10^{-5}$
3	1	2	$-0.459\,960\,136\,963\,65 \times 10^{-1}$	25	7	25	$-0.389\,468\,424\,357\,39 \times 10^{-1}$
4	1	3	$-0.575\,812\,590\,834\,32 \times 10^{-1}$	26	8	8	$0.112\,562\,113\,604\,59 \times 10^{-10}$
5	1	6	$-0.503\,252\,787\,279\,30 \times 10^{-1}$	27	8	36	$-0.823\,113\,408\,979\,98 \times 10^{1}$
6	2	1	$-0.330\,326\,416\,702\,03 \times 10^{-4}$	28	9	13	$0.198\,097\,128\,020\,88 \times 10^{-7}$
7	2	2	$-0.189\,489\,875\,163\,15 \times 10^{-3}$	29	10	4	$0.104\,069\,652\,101\,74 \times 10^{-18}$
8	2	4	$-0.393\,927\,772\,433\,55 \times 10^{-2}$	30	10	10	$-0.102\,347\,470\,959\,29 \times 10^{-12}$
9	2	7	$-0.437\,972\,956\,505\,73 \times 10^{-1}$	31	10	14	$-0.100\,181\,793\,795\,11 \times 10^{-8}$
10	2	36	$-0.266\,745\,479\,140\,87 \times 10^{-4}$	32	16	29	$-0.808\,829\,086\,469\,85 \times 10^{-10}$
11	3	0	$0.204\,817\,376\,923\,09 \times 10^{-7}$	33	16	50	$0.106\,930\,318\,794\,09$
12	3	1	$0.438\,706\,672\,844\,35 \times 10^{-6}$	34	18	57	$-0.336\,622\,505\,741\,71$
13	3	3	$-0.322\,776\,772\,385\,70 \times 10^{-4}$	35	20	20	$0.891\,858\,453\,554\,21 \times 10^{-24}$
14	3	6	$-0.150\,339\,245\,421\,48 \times 10^{-2}$	36	20	35	$0.306\,293\,168\,762\,32 \times 10^{-12}$
15	3	35	$-0.406\,682\,535\,626\,49 \times 10^{-1}$	37	20	48	$-0.420\,024\,676\,982\,08 \times 10^{-5}$
16	4	1	$-0.788\,473\,095\,593\,67 \times 10^{-9}$	38	21	21	$-0.590\,560\,296\,856\,39 \times 10^{-25}$
17	4	2	$0.127\,907\,178\,522\,85 \times 10^{-7}$	39	22	53	$0.378\,269\,476\,134\,57 \times 10^{-5}$
18	4	3	$0.482\,253\,727\,185\,07 \times 10^{-6}$	40	23	39	$-0.127\,686\,089\,346\,81 \times 10^{-14}$
19	5	7	$0.229\,220\,763\,376\,61 \times 10^{-5}$	41	24	26	$0.730\,876\,105\,950\,61 \times 10^{-28}$
20	6	3	$-0.167\,147\,664\,510\,61 \times 10^{-10}$	42	24	40	$0.554\,147\,153\,507\,78 \times 10^{-16}$
21	6	16	$-0.211\,714\,723\,213\,55 \times 10^{-2}$	43	24	58	$-0.943\,697\,072\,412\,10 \times 10^{-6}$
22	6	35	$-0.238\,957\,419\,341\,04 \times 10^{2}$				

Table 9
The ideal-gas contribution γ^{o} of the dimensionless Gibbs free energy and its derivatives[a] according to Eq. (9)

$\gamma^{\mathrm{o}} = \ln\pi + \sum_{i=1}^{9} n_i^{\mathrm{o}} \tau^{J_i^{\mathrm{o}}}$	$\gamma_{\tau}^{\mathrm{o}} = 0 + \sum_{i=1}^{9} n_i^{\mathrm{o}} J_i^{\mathrm{o}} \tau^{J_i^{\mathrm{o}}-1}$
$\gamma_{\pi}^{\mathrm{o}} = 1/\pi + 0$	$\gamma_{\tau\tau}^{\mathrm{o}} = 0 + \sum_{i=1}^{9} n_i^{\mathrm{o}} J_i^{\mathrm{o}} (J_i^{\mathrm{o}} - 1) \tau^{J_i^{\mathrm{o}}-2}$
$\gamma_{\pi\pi}^{\mathrm{o}} = -1/\pi^2 + 0$	$\gamma_{\pi\tau}^{\mathrm{o}} = 0 + 0$

[a] $\gamma_{\pi}^{\mathrm{o}} = \left(\frac{\partial\gamma^{\mathrm{o}}}{\partial\pi}\right)_{\tau}, \quad \gamma_{\pi\pi}^{\mathrm{o}} = \left(\frac{\partial^2\gamma^{\mathrm{o}}}{\partial\pi^2}\right)_{\tau}, \quad \gamma_{\tau}^{\mathrm{o}} = \left(\frac{\partial\gamma^{\mathrm{o}}}{\partial\tau}\right)_{\pi}, \quad \gamma_{\tau\tau}^{\mathrm{o}} = \left(\frac{\partial^2\gamma^{\mathrm{o}}}{\partial\tau^2}\right)_{\pi}, \quad \gamma_{\pi\tau}^{\mathrm{o}} = \left(\frac{\partial^2\gamma^{\mathrm{o}}}{\partial\pi\partial\tau}\right).$

$$bR_{\mathrm{m}}T\phi(p,T,S) = g^{\mathrm{W}}(p,T) - g^{\mathrm{Ih}}(p,T) \tag{12}$$

with $\mu_{\mathrm{W}}(p,T,S)$ and $\phi(p,T,S)$ from Table 1 and $g^{\mathrm{W}}(p,T)$ from Table 3. The molality b is determined from Eq. (3) and the value of the molar gas constant $R_{\mathrm{m}} = 8.314\,472\ \mathrm{kJ\,kmol^{-1}\,K^{-1}}$ is taken from [18]. In thermodynamic equilibrium, the amount of sea salt in the ice phase is assumed to be negligible. The specific Gibbs free energy of ice $g^{\mathrm{Ih}}(p,T)$ is calculated from the Gibbs free energy equation of the IAPWS-06 Release [13]:

Table 10
The residual contribution γ^{r} of the dimensionless Gibbs free energy and its derivatives[a] according to Eq. (10)

$\gamma^{\mathrm{r}} = \sum_{i=1}^{43} n_i\, \pi^{I_i} (\tau - 0.5)^{J_i}$	$\gamma^{\mathrm{r}}_{\tau} = \sum_{i=1}^{43} n_i\, \pi^{I_i} J_i (\tau - 0.5)^{J_i - 1}$
$\gamma^{\mathrm{r}}_{\pi} = \sum_{i=1}^{43} n_i\, I_i\, \pi^{I_i - 1} (\tau - 0.5)^{J_i}$	$\gamma^{\mathrm{r}}_{\tau\tau} = \sum_{i=1}^{43} n_i\, \pi^{I_i} J_i (J_i - 1)(\tau - 0.5)^{J_i - 2}$
$\gamma^{\mathrm{r}}_{\pi\pi} = \sum_{i=1}^{43} n_i\, I_i (I_i - 1) \pi^{I_i - 2} (\tau - 0.5)^{J_i}$	$\gamma^{\mathrm{r}}_{\pi\tau} = \sum_{i=1}^{43} n_i\, I_i \pi^{I_i - 1} J_i (\tau - 0.5)^{J_i - 1}$

[a] $\gamma^{\mathrm{r}}_{\pi} = \left(\frac{\partial \gamma^{\mathrm{r}}}{\partial \pi}\right)_{\tau}$, $\gamma^{\mathrm{r}}_{\pi\pi} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \pi^2}\right)_{\tau}$, $\gamma^{\mathrm{r}}_{\tau} = \left(\frac{\partial \gamma^{\mathrm{r}}}{\partial \tau}\right)_{\pi}$, $\gamma^{\mathrm{r}}_{\tau\tau} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \tau^2}\right)_{\pi}$, $\gamma^{\mathrm{r}}_{\pi\tau} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \pi\, \partial \tau}\right)$.

Table 11
Relations of thermodynamic properties to the ideal-gas contribution γ^{o} and the residual contribution γ^{r} of the dimensionless Gibbs free energy and their derivatives[a] when using Eq. (8)

Property	Relation
Specific volume $v = (\partial g/\partial p)_T$	$v(\pi,\tau) = \frac{RT}{p}\pi\left(\gamma^{\mathrm{o}}_{\pi} + \gamma^{\mathrm{r}}_{\pi}\right)$
Density $\rho = (\partial g/\partial p)_T^{-1}$	$\rho(\pi,\tau) = \frac{p}{RT}\frac{1}{\pi\left(\gamma^{\mathrm{o}}_{\pi} + \gamma^{\mathrm{r}}_{\pi}\right)}$
Specific internal energy $u = g - T(\partial g/\partial T)_p - p(\partial g/\partial p)_T$	$u(\pi,\tau) = RT\left[\tau\left(\gamma^{\mathrm{o}}_{\tau} + \gamma^{\mathrm{r}}_{\tau}\right) - \pi\left(\gamma^{\mathrm{o}}_{\pi} + \gamma^{\mathrm{r}}_{\pi}\right)\right]$
Specific entropy $s = -(\partial g/\partial T)_p$	$s\ (\pi,\tau) = R\left[\tau\left(\gamma^{\mathrm{o}}_{\tau} + \gamma^{\mathrm{r}}_{\tau}\right) - \left(\gamma^{\mathrm{o}} + \gamma^{\mathrm{r}}\right)\right]$
Specific enthalpy $h = g - T(\partial g/\partial T)_p$	$h(\pi,\tau) = RT\tau\left(\gamma^{\mathrm{o}}_{\tau} + \gamma^{\mathrm{r}}_{\tau}\right)$
Specific isobaric heat capacity $c_p = (\partial h/\partial T)_p$	$c_p(\pi,\tau) = -R\tau^2\left(\gamma^{\mathrm{o}}_{\tau\tau} + \gamma^{\mathrm{r}}_{\tau\tau}\right)$
Specific isochoric heat capacity $c_v = (\partial u/\partial T)_v$	$c_v(\pi,\tau) = R\left[-\tau^2\left(\gamma^{\mathrm{o}}_{\tau\tau} + \gamma^{\mathrm{r}}_{\tau\tau}\right) - \frac{\left(1+\pi\gamma^{\mathrm{r}}_{\pi} - \tau\pi\gamma^{\mathrm{r}}_{\pi\tau}\right)^2}{1-\pi^2\gamma^{\mathrm{r}}_{\pi\pi}}\right]$
Speed of sound $w = v\sqrt{-(\partial p/\partial v)_s}$	$w(\pi,\tau) = \sqrt{RT\frac{1+2\pi\gamma^{\mathrm{r}}_{\pi}+\pi^2{\gamma^{\mathrm{r}}_{\pi}}^2}{\left(1-\pi^2\gamma^{\mathrm{r}}_{\pi\pi}\right)+\frac{\left(1+\pi\gamma^{\mathrm{r}}_{\pi}-\tau\pi\gamma^{\mathrm{r}}_{\pi\tau}\right)^2}{\tau^2\left(\gamma^{\mathrm{o}}_{\tau\tau}+\gamma^{\mathrm{r}}_{\tau\tau}\right)}}}$
Isentropic exponent $\kappa = -vp^{-1}(\partial p/\partial v)_s$	$\kappa(\pi,\tau) = \frac{1}{\pi\left(\gamma^{\mathrm{o}}_{\pi}+\gamma^{\mathrm{r}}_{\pi}\right)}\frac{1+2\pi\gamma^{\mathrm{r}}_{\pi}+\pi^2{\gamma^{\mathrm{r}}_{\pi}}^2}{\left(1-\pi^2\gamma^{\mathrm{r}}_{\pi\pi}\right)+\frac{\left(1+\pi\gamma^{\mathrm{r}}_{\pi}-\tau\pi\gamma^{\mathrm{r}}_{\pi\tau}\right)^2}{\tau^2\left(\gamma^{\mathrm{o}}_{\tau\tau}+\gamma^{\mathrm{r}}_{\tau\tau}\right)}}$
Isobaric cubic expansion coefficient $\alpha_v = v^{-1}(\partial v/\partial T)_p$	$\alpha_v(\pi,\tau) = \frac{1}{T}\frac{1+\pi\gamma^{\mathrm{r}}_{\pi}-\tau\pi\gamma^{\mathrm{r}}_{\pi\tau}}{1+\pi\gamma^{\mathrm{r}}_{\pi}}$
Isothermal compressibility $\kappa_T = -v^{-1}(\partial v/\partial p)_T$	$\kappa_T(\pi,\tau) = \frac{1}{p}\frac{1-\pi^2\gamma^{\mathrm{r}}_{\pi\pi}}{1+\pi\gamma^{\mathrm{r}}_{\pi}}$

[a] $\gamma^{\mathrm{r}}_{\pi} = \left(\frac{\partial \gamma^{\mathrm{r}}}{\partial \pi}\right)_{\tau}$, $\gamma^{\mathrm{r}}_{\pi\pi} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \pi^2}\right)_{\tau}$, $\gamma^{\mathrm{r}}_{\tau} = \left(\frac{\partial \gamma^{\mathrm{r}}}{\partial \tau}\right)_{\pi}$, $\gamma^{\mathrm{r}}_{\tau\tau} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \tau^2}\right)_{\pi}$, $\gamma^{\mathrm{r}}_{\pi\tau} = \left(\frac{\partial^2 \gamma^{\mathrm{r}}}{\partial \pi\, \partial \tau}\right)$, $\gamma^{\mathrm{o}}_{\tau} = \left(\frac{\partial \gamma^{\mathrm{o}}}{\partial \tau}\right)_{\pi}$, $\gamma^{\mathrm{o}}_{\tau\tau} = \left(\frac{\partial^2 \gamma^{\mathrm{o}}}{\partial \tau^2}\right)_{\pi}$.

$$\frac{g^{\mathrm{Ih}}(p,T)}{g^*} = \gamma^{\mathrm{Ih}}(\pi,\theta) = \gamma_0(\pi) - \sigma_0\theta + \theta_{\mathrm{t}}^{\mathrm{W}}\mathrm{Re}\left\{\sum_{k=1}^{2}\zeta_k\left[(\theta_k-\theta)\ln(\theta_k-\theta) + (\theta_k+\theta)\ln(\theta_k+\theta) - 2\theta_k\ln(\theta_k) - \frac{\theta^2}{\theta_k}\right]\right\} \tag{13}$$

with

$$\gamma_0(\pi) = \sum_{k=0}^{4}\gamma_{0k}(\pi-\pi_0)^k$$

$$\zeta_2(\pi) = \sum_{k=0}^{2}\zeta_{2k}(\pi-\pi_0)^k \text{ and}$$

$$\sigma_0 = s_0 T_{\mathrm{t}}^{\mathrm{W}}/g^*$$

where the reduced temperature is $\theta = T/T_{\mathrm{t}}^{\mathrm{W}}$, the reduced pressure is $\pi = p/p_{\mathrm{t}}^{\mathrm{W}}$, the reduced atmospheric pressure is $\pi_0 = p_0/p_{\mathrm{t}}^{\mathrm{W}}$, and the reduced triple-point temperature is $\theta_{\mathrm{t}}^{\mathrm{W}} = T_{\mathrm{t}}^{\mathrm{W}}/T^*$. The triple-point temperature of water is $T_{\mathrm{t}}^{\mathrm{W}} = 273.16$ K [16], the triple-point pressure of water is $p_{\mathrm{t}}^{\mathrm{W}} = 611.657 \times 10^{-6}$ MPa [5], the standard atmospheric pressure is $p_0 = 0.101\,325$ MPa [27], the reducing specific Gibbs free energy is $g^* = 1$ kJ kg^{-1}, and the reducing temperature is $T^* = 1$ K. The real constants γ_{00}–γ_{04} and s_0 as well as the complex constants θ_1, θ_2, ζ_1, and ζ_{20}–ζ_{22} are listed in Table 12. The complex logarithm in Eq. (13) should be taken as its principal value.

Thermodynamic properties can be derived from Eq. (13) using the appropriate combinations of the specific Gibbs free energy and its derivatives. All required derivatives of the specific Gibbs free energy

Table 12
Coefficients of the specific Gibbs free energy of ice, Eq. (13)

Coefficient	Real part	Imaginary part
γ_{00}	$-0.632\,020\,233\,335\,886 \times 10^{3}$	
γ_{01}	$0.655\,022\,213\,658\,955 \times 10^{-3}$	
γ_{02}	$-0.189\,369\,929\,326\,131 \times 10^{-10}$	
γ_{03}	$0.339\,746\,123\,271\,053 \times 10^{-17}$	
γ_{04}	$-0.556\,464\,869\,058\,991 \times 10^{-24}$	
s_0	-3.32733756492168 kJ kg^{-1} K^{-1}	
θ_1	$0.368\,017\,112\,855\,051 \times 10^{-1}$	$0.510\,878\,114\,959\,572 \times 10^{-1}$
θ_2	$0.337\,315\,741\,065\,416$	$0.335\,449\,415\,919\,309$
ζ_1	$0.447\,050\,716\,285\,388 \times 10^{-1}$	$0.656\,876\,847\,463\,481 \times 10^{-1}$
ζ_{20}	$-0.725\,974\,574\,329\,220 \times 10^{-1}$	$-0.781\,008\,427\,112\,870 \times 10^{-1}$
ζ_{21}	$-0.557\,107\,698\,030\,123 \times 10^{-7}$	$0.464\,578\,634\,580\,806 \times 10^{-7}$
ζ_{22}	$0.234\,801\,409\,215\,913 \times 10^{-13}$	$-0.285\,651\,142\,904\,972 \times 10^{-13}$

Table 13
Equations for the derivatives[a] of the specific Gibbs free energy of ice, Eq. (13)

$$\gamma_\theta = -\frac{\sigma_0}{\theta_t^W} + \mathrm{Re}\left\{\sum_{k=1}^{2} \zeta_k \left[-\ln(\theta_k - \theta) + \ln(\theta_k + \theta) - 2\frac{\theta}{\theta_k}\right]\right\}$$

$$\gamma_\pi = \gamma_{0,\pi} + \theta_t^W \mathrm{Re}\left\{\zeta_{2,\pi}\left[(\theta_2 - \theta)\ln(\theta_2 - \theta) + (\theta_2 + \theta)\ln(\theta_2 + \theta) - 2\theta_2\ln(\theta_2) - \frac{\theta^2}{\theta_2}\right]\right\}$$

$$\gamma_{\theta\theta} = \frac{1}{\theta_t^W}\mathrm{Re}\left[\sum_{k=1}^{2} \zeta_k \left(\frac{1}{\theta_k - \theta} + \frac{1}{\theta_k + \theta} - \frac{2}{\theta_k}\right)\right]$$

$$\gamma_{\pi\pi} = \gamma_{0,\pi\pi} + \theta_t^W \mathrm{Re}\left\{\zeta_{2,\pi\pi}\left[(\theta_2 - \theta)\ln(\theta_2 - \theta) + (\theta_2 + \theta)\ln(\theta_2 + \theta) - 2\theta_2\ln(\theta_2) - \frac{\theta^2}{\theta_2}\right]\right\}$$

$$\gamma_{\pi\theta} = \mathrm{Re}\left\{\zeta_{2,\pi}\left[-\ln(\theta_2 - \theta) + \ln(\theta_2 + \theta) - 2\frac{\theta}{\theta_2}\right]\right\}$$

$\gamma_0(\pi)$ equation and its derivatives[b]	$\zeta_2(\pi)$ equation and its derivatives[b]
$\gamma_0(\pi) = \sum_{k=0}^{4} \gamma_{0k}(\pi - \pi_0)^k$	$\zeta_2(\pi) = \sum_{k=0}^{2} \zeta_{2k}(\pi - \pi_0)^k$
$\gamma_{0,\pi} = \sum_{k=1}^{4} \gamma_{0k}\frac{k}{p_t^W}(\pi - \pi_0)^{k-1}$	$\zeta_{2,\pi} = \sum_{k=1}^{2} \zeta_{2k}\frac{k}{p_t^W}(\pi - \pi_0)^{k-1}$
$\gamma_{0,\pi\pi} = \sum_{k=2}^{4} \gamma_{0k}\frac{k(k-1)}{(p_t^W)^2}(\pi - \pi_0)^{k-2}$	$\zeta_{2,\pi\pi} = \zeta_{22}\frac{2}{(p_t^W)^2}$

[a] $g = g^*\gamma,\quad \left(\frac{\partial g}{\partial T}\right)_p = \frac{g^*}{T^*}\gamma_\theta,\quad \left(\frac{\partial g}{\partial p}\right)_T = \frac{g^*}{p^*}\gamma_\pi,\quad \left(\frac{\partial^2 g}{\partial T^2}\right)_p = \frac{g^*}{T^{*2}}\gamma_{\theta\theta},\quad \left(\frac{\partial^2 g}{\partial p^2}\right)_T = \frac{g^*}{p^{*2}}\gamma_{\pi\pi},\quad \left(\frac{\partial^2 g}{\partial p\partial T}\right) = \frac{g^*}{p^*T^*}\gamma_{\pi\theta}.$

[b] $\gamma_0,\pi = \left(\frac{\partial\gamma_0}{\partial\pi}\right)_\theta,\quad \gamma_{0,\pi\pi} = \left(\frac{\partial^2\gamma_0}{\partial\pi^2}\right)_\theta,\quad \zeta_{2,\pi} = \left(\frac{\partial\zeta_2}{\partial\pi}\right)_\theta,\quad \zeta_{2,\pi\pi} = \left(\frac{\partial^2\zeta_2}{\partial\pi^2}\right)_\theta.$

are explicitly given in Table 13. Relations between thermodynamic properties and $g(p,T)$ and its derivatives are summarized in Table 14.

Using Eqs. (11) or (12), the freezing temperature $T = T_f$ can be calculated by iteration from pressure p and salinity S, or the freezing pressure $p = p_f$ from T and S, or brine salinity $S = S_f$, from p and T. At a given equilibrium state between seawater and ice, the properties of the seawater (brine) phase are calculated from Table 1 and the properties of the ice phase from Table 14.

At brine-ice equilibrium, the temperature is lower than the melting (freezing) temperature at a given pressure of pure ice, and the pressure at a given temperature is below the melting (freezing) pressure of pure ice.

Note that the specific Gibbs free energy of liquid water $g^W(p,T)$ in Eq. (12) is evaluated from IAPWS-IF97 at conditions where the liquid phase of pure water is metastable. Due to salinity, the freezing-point depression can be up to 8 K (see Feistel [4] for experimental results from the literature). Investigations have shown that Eq. (12) can reasonably be extrapolated to these metastable states.

Table A3 shows selected freezing temperatures for given pressures and salinities.

3.3. Triple-point temperatures and pressures

The triple-point temperatures $T = T_t(S)$ and related triple-point pressures $p = p_t(S)$ of seawater are calculated for given salinity S by iteration from both Eqs.

Table 14
Relations of the thermodynamic properties to the equation for the specific Gibbs free energy of ice, Eq. (13), and its derivatives[a]

Property	Relation
Specific volume $v(p,T) = (\partial g/\partial p)_T$	$v(\pi,\theta) = \gamma_\pi$
Density $\rho(p,T) = (\partial g/\partial p)_T^{-1}$	$\rho(\pi,\theta) = \gamma_\pi^{-1}$
Specific internal energy $u(p,T) = g + T\,s - p\,v$	$u(\pi,\theta) = \gamma - T\,\gamma_\theta - p\,\gamma_\pi$
Specific entropy $s(p,T) = -(\partial g/\partial T)_p$	$s(\pi,\theta) = -\gamma_\theta$
Specific enthalpy $h(p,T) = g + T\,s$	$h(\pi,\theta) = \gamma - T\,\gamma_\theta$
Specific isobaric heat capacity $c_p(p,T) = T\,(\partial s/\partial T)_p$	$c_p(\pi,\theta) = -\theta\gamma_{\theta\theta}$
Specific isochoric heat capacity $c_v(p,T) = (\partial u/\partial T)_v$	$c_v(\pi,\theta) = \theta\left(\frac{\gamma_{\pi\theta}^2}{\gamma_{\pi\pi}} - \gamma_{\theta\theta}\right)$
Speed of sound $w(p,T) = v\sqrt{-(\partial p/\partial v)_s}$	$w(\pi,\theta) = \gamma_\pi\sqrt{\frac{\gamma_{\theta\theta}}{\gamma_{\pi\theta}^2 - \gamma_{\theta\theta}\,\gamma_{\pi\pi}}}$
Isentropic exponent $\kappa(p,T) = -vp^{-1}(\partial p/\partial v)_s$	$\kappa(\pi,\theta) = \frac{1}{p}\frac{\gamma_\pi\,\gamma_{\theta\theta}}{\gamma_{\pi\theta}^2 - \gamma_{\theta\theta}\,\gamma_{\pi\pi}}$
Isobaric cubic expansion coefficient $\alpha_v(p,T) = v^{-1}(\partial v/\partial T)_p$	$\alpha_v(\pi,\theta) = \frac{\gamma_{\pi\theta}}{\gamma_\pi}$
Isothermal compressibility $\kappa_T(p,T) = -v^{-1}(\partial v/\partial p)_T$	$\kappa_T(\pi,\theta) = -\frac{\gamma_{\pi\pi}}{\gamma_\pi}$

[a] $\gamma_\theta = \left(\frac{\partial\gamma}{\partial\theta}\right)_\pi, \quad \gamma_\pi = \left(\frac{\partial\gamma}{\partial\pi}\right)_\theta, \quad \gamma_{\theta\theta} = \left(\frac{\partial^2\gamma}{\partial\theta^2}\right)_\pi, \quad \gamma_{\pi\pi} = \left(\frac{\partial^2\gamma}{\partial\pi^2}\right)_\theta, \quad \gamma_{\pi\theta} = \left(\frac{\partial^2\gamma}{\partial\pi\partial\theta}\right).$

(6) and (11) or from both Eqs. (7) and (12). Table A4 contains selected triple-point temperatures and triple-point pressures for given salinities.

3.4. Osmotic pressure

On the two sides of a membrane permeable to water but not to sea salt, equilibrium between liquid water and seawater causes an excess pressure of seawater, the osmotic pressure p_{osm}, computed from the condition

$$\mu_{\mathrm{W}}[(p + p_{\mathrm{osm}}), T, S] = g^{\mathrm{W}}(p,T) \tag{14}$$

or equivalently from

$$bR_{\mathrm{m}}T\;\phi[(p + p_{\mathrm{osm}}), T, S] = g^{\mathrm{W}}[(p + p_{\mathrm{osm}}), T] - g^{\mathrm{W}}(T,p) \tag{15}$$

with μ_{W} and ϕ from Table 1 and g^{W} from Eq. (4). The molality b is determined from Eq. (3) and the value of the molar gas constant $R_{\mathrm{m}} = 8.314\ 472$ kJ kmol^{-1} K^{-1} is taken from [18].

Using Eqs. (14) or (15), the osmotic pressure p_{osm} is calculated by iteration for pressure p, temperature T, and salinity S.

3.5. Properties of brine-vapor mixture

The properties for the two-phase composite of brine and water vapor, commonly termed brine-vapor mixture, can be computed from the combined Gibbs free energy equation

$$g^{\mathrm{BV}}(p,T,S) = (1-x)g(p,T,S_{\mathrm{b}}) + xg^{\mathrm{vap}}(p,T) \tag{16}$$

where p is the pressure, T is the temperature, and S is the salinity of the mixture including water vapor. The boiling-brine salinity $S_{\mathrm{b}} = S_{\mathrm{b}}(p,T)$ is calculated for pressure p and temperature T using Eq. (6) by iteration. The mass fraction of water vapor in the mixture (vapor fraction) x can be determined from the equation

$$x = 1 - \frac{S}{S_{\mathrm{b}}(p,T)} \tag{17}$$

The specific Gibbs free energy of the brine $g(p,T,S_{\mathrm{b}})$ is calculated from Eq. (1) and the specific Gibbs free energy of water vapor $g^{\mathrm{vap}}(p,T)$ from Eq. (8).

Brine-vapor properties can be computed from the equation $g^{\mathrm{BV}}(p,T,S)$, Eq. (16), and its derivatives in analogy to Table 1. The derivatives of the included $g^{\mathrm{vap}}(p, T)$, Eq. (8), for water vapor can be taken from [7]. As an example, the specific enthalpy of brine-vapor mixture is calculated from the equation

$$h^{\mathrm{BV}}(p,T,S) = (1-x)h(p,T,S_{\mathrm{b}}) + xh^{\mathrm{vap}}(p,T) \tag{18}$$

where the determination of $h(p,T,S_{\mathrm{b}})$ is given in Table 1 and of $h^{\mathrm{vap}}(p,T)$ in [7].

Further properties such as specific volume $v^{\mathrm{BV}}(p,T,S)$, specific internal energy $u^{\mathrm{BV}}(p,T,S)$, and specific entropy $s^{\mathrm{BV}}(p,T,S)$ are computed analogously.

Table A5 shows values for the brine contribution, the vapor contribution, and the brine-vapor mixture of selected properties at given pressure, temperature, and salinity.

3.6. Properties of brine-ice mixture

The properties for the two-phase composite of brine and ice Ih, termed brine-ice mixture or sea ice, can be computed from the combined Gibbs free energy equation

$$g^{\mathrm{BI}}(p,T,S) = (1-y)g(p,T,S_{\mathrm{f}}) + yg^{\mathrm{Ih}}(p,T) \tag{19}$$

where p is the pressure, T is the temperature, and S is the salinity of the mixture including solid ice Ih. The freezing-brine salinity $S_{\mathrm{f}} = S_{\mathrm{f}}(p,T)$ is calculated for pressure p and temperature T using Eq. (11) by iteration. The mass fraction of ice Ih in the mixture (ice fraction) y can be determined from the equation

$$y = 1 - \frac{S}{S_{\mathrm{f}}(p,T)} \tag{20}$$

The specific Gibbs free energy of the brine $g(p,T,S_{\mathrm{f}})$ is calculated from Eq. (1) and the specific Gibbs free energy of ice Ih $g^{\mathrm{Ih}}(p,T)$ from Eq. (13).

Brine-ice mixture properties can be computed from the equation $g^{\mathrm{BI}}(p,T,S)$, Eq. (19), and its derivatives in analogy to Table 1. The derivatives of the included $g^{\mathrm{Ih}}(p,T)$, Eq. (13), for ice Ih can be taken from [13]. As an example, the specific enthalpy of brine-ice mixture is calculated from the equation

$$h^{\mathrm{BI}}(p,T,S) = (1-y)h(p,T,S_{\mathrm{f}}) + yh^{\mathrm{Ih}}(p,T) \tag{21}$$

where the determination of $h(p,T,S_{\mathrm{f}})$ is given in Table 1 and of $h^{\mathrm{Ih}}(p,T)$ in [13].

Further properties such as specific volume $v^{\mathrm{BI}}(p,T,S)$, specific internal energy $u^{\mathrm{BI}}(p,T,S)$, and specific entropy $s^{\mathrm{BI}}(p,T,S)$ are computed analogously.

Table A6 shows values for the brine contribution, the ice contribution, and the brine-ice mixture of selected properties at given pressure, temperature, and salinity.

4. Reference states

According to the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater [1], the specific enthalpy and the specific entropy of seawater calculated from Eq. (1) have been set to zero,

$$h = 0 \text{ and } s = 0$$

at $p = p_0 = 0.101\,325$ MPa, $T = T_0 = 273.15$ K, and $S = S_{\mathrm{n}} = 0.035\,165\,04\ \mathrm{kg\,kg^{-1}}$, where S_{n} is the normal salinity of seawater [12].

In addition, the reference-state conditions of Eq. (1) for pure liquid water, water vapor, and ice require that the specific internal energy and the specific entropy of pure liquid water at the IAPWS-95 numerical triple point ($p_{\mathrm{t}}^{\mathrm{W}} = 611.654\,771\,007\,9 \times 10^{-6}$ MPa, $T_{\mathrm{t}}^{\mathrm{W}} = 273.16$ K, and $S = 0$) are:

$$u = 0 \text{ and } s = 0$$

The four independent conditions specify four numerical parameters, namely those related to the absolute energies and the absolute entropies of water substance and of dissolved sea salt which are not available from thermodynamic measurements. A detailed discussion of these conditions is given in Feistel et al. [29].

From $u = 0$ it follows that the specific enthalpy of liquid pure water is

$$h = 0.000\,611\,783\ \mathrm{kJ\,kg^{-1}}$$

at this triple point. These values also apply to the reference state of IAPWS-IF97 for pure water.

5. Range of validity

Fig. 1 shows a p-T-S graphical representation of the range of validity of the equation of state, Eq. (1), which is bordered by bold lines. In addition, the four two-dimensional plots of Figs. 2–5 display the range of validity of seawater in one T-S diagram at standard atmospheric pressure and in three p-T diagrams for salinities of $S = 0$, $S = 0.035\ \mathrm{kg\ kg^{-1}}$, and $S = 0.12\ \mathrm{kg\ kg^{-1}}$. The freezing surface (and the freezing line) is defined by Eqs. (11) and (12), the boiling

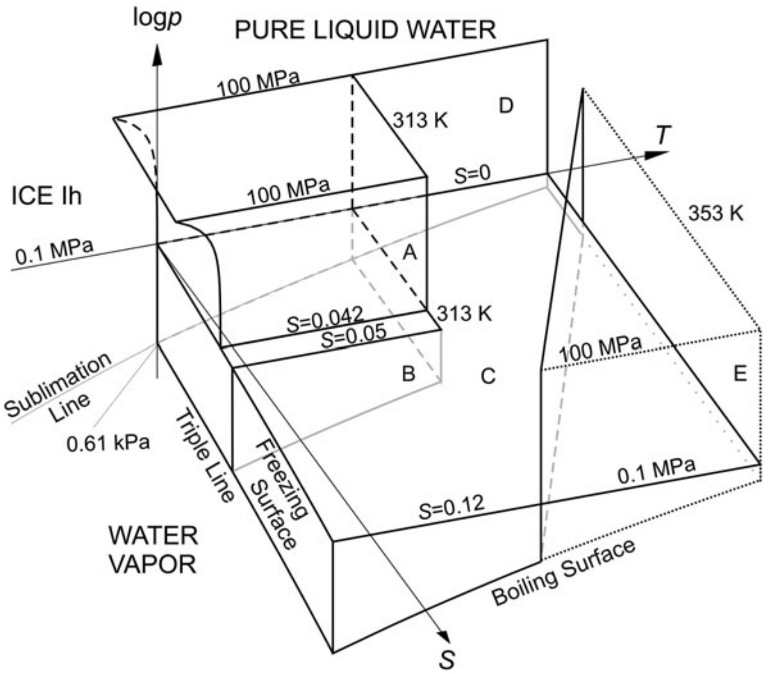

Fig. 1. Phase diagram of seawater with the regions A to E of p-T-S space (values of salinity S in kg kg^{-1}). Regions A, B, and E are three-dimensional volumes. Region C is planar at $p = 0.1$ MPa, region D is planar at $S = 0$. The intersection of the boiling surface with the plane $S = 0$ is the saturation curve of pure water (not indicated in the graph).

surface (and the boiling line) by Eqs. (6) and (7); the triple line (and the triple point) obeys the freezing and the boiling equations. The sublimation line is the phase equilibrium between water vapor and ice of pure water. The equation of state, Eq. (1), is valid for IAPSO Standard Seawater with sea salt of the Reference Composition [12] in specified regions inside the following pressure, temperature, and salinity ranges 0.3 kPa ≤ p ≤ 100 MPa, 261 K ≤ T ≤ 353 K, and 0 ≤ S ≤ 0.12 kg kg^{-1}.

All properties of Table 1 can be calculated with an accuracy, as specified in Section 6, in the temperature range $T_f(p,S) \le T \le 313$ K for 0.101 325 MPa ≤ p ≤ 100 MPa and 0 ≤ S ≤ 0.042 kg kg^{-1} (Region A in

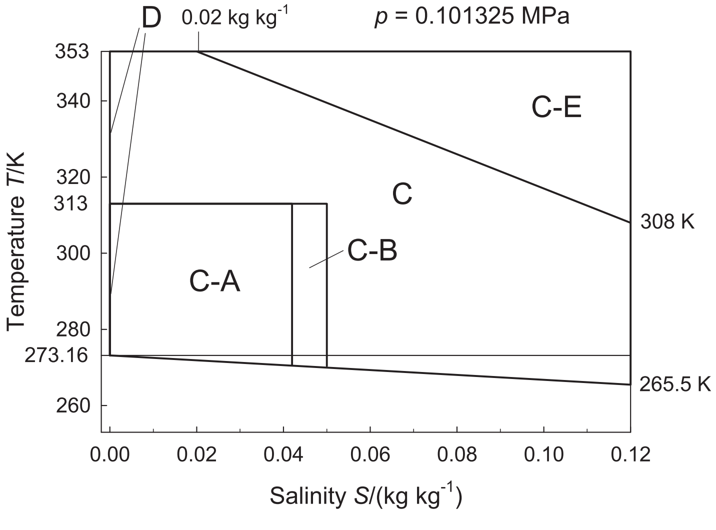

Fig. 2. T-S diagram of seawater at standard atmospheric pressure $p_0 = 0.101$ 325 MPa with the regions C and D of the T-S plane and the intersections of the regions A, B, and E with region C (C-A, C-B, C-E).

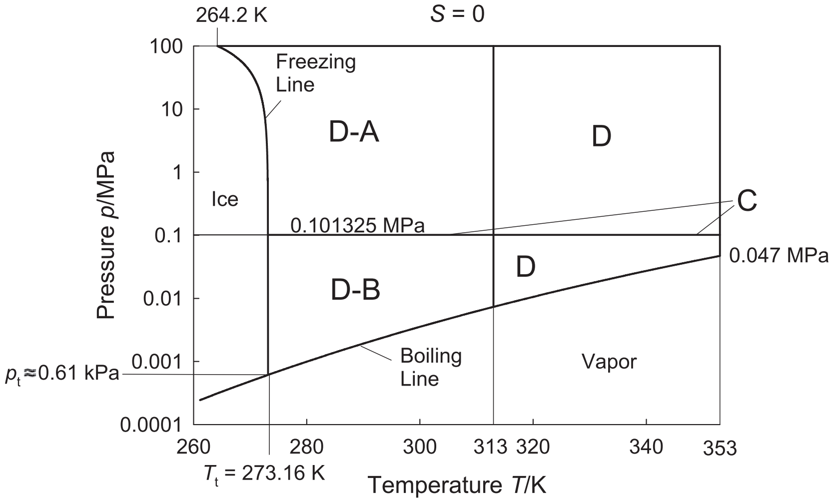

Fig. 3. p-T diagram of seawater at salinity $S = 0$ (pure water) with the regions C and D of the p-T plane and the intersections of the regions A and B with region D (D-A, D-B).

Figs. 1–4) or for $p_t(S) \leq p \leq 0.101\ 325$ MPa and $0 \leq S \leq 0.05\,\text{kg kg}^{-1}$ (Region B in Figs. 1–4), where $p_t(S)$ is the pressure at the triple line (Section 3.3), which varies between $p_t(0) = 0.61$ kPa for pure water and $p_t(0.05\ \text{kg kg}^{-1}) = 0.485$ kPa, and can be approximated by $p_t(S) = (0.61 - 2.5\,S)$ kPa, see Table A4.

Outside the regions A and B, properties of Table 1 except those depending on pressure derivatives can be calculated with an accuracy, as specified in Section 6, at standard atmospheric pressure $p_0 = 0.101\ 325$ MPa for $T_f(p_0,S) \leq T \leq 353$ K and $0 \leq S \leq 0.12$ kg kg^{-1} (Region C in Figs. 1–5).

At this T-S plane, outside its intersection with the regions A, B, or D (described below), the pressure derivatives of the Gibbs function, g_p, g_{pS}, g_{pT}, and g_{pp}, of Table 1 give values extrapolated beyond their range of validity for the density and its derivatives. Due to the lack of experimental data, no statements can be made about the accuracy at high pressures for temperatures greater than 313 K and salinities greater than 0.042 kg kg^{-1}.

As summarized in Table 15, the saline contribution of the specific Gibbs free energy at standard atmospheric pressure p_0, $g^S(p_0,T,S)$, represents the experimental thermal and colligative data for salinity from pure water to 0.12 kg kg^{-1} and for temperatures from the freezing point to 353 K in region C of Figs. 1 and 2. The pressure-dependent contribution of the Gibbs function, $g^S(p,T,S) - g^S(p_0,T,S)$, represents highly accurate experimental data of density and sound speed in the regions A and B of Fig. 1.

The validity region D is the zero-salinity limit of dilute seawater in the range $S = 0$, $T_f(p,S = 0) \leq T \leq 353$ K, and $p^{\text{vap}} \leq p \leq 100$ MPa (Region D in Figs. 1 and 3), which is inside the validity range of the IAPWS-IF97 formulation for temperatures $T \geq 273.15$ K. Fig. 6 displays in a p-T diagram of pure water (salinity $S = 0$) different Gibbs equations of IAPWS-IF97 and IAPWS-06. Extensive comparisons for the algorithms of IAPWS-IF97 to IAPWS-95 were carried out including temperatures below 273.15 K and showed (see Section 6), that the differences are very small and therefore the algorithms of IAPWS-IF97 can also be used in this specified range of validity for temperatures below 273.15 K (gray-shaded area in Fig. 6). In the two-dimensional p-T region D, outside its intersection with the regions A, B, or C, the salinity derivative of the Gibbs function, g_S, of Table 1 cannot be evaluated at zero salinity. For physical reasons, the ideal-solution contributions to the relative chemical potential and the chemical potential of sea salt, both from Table 1, possess logarithmic singularities for $S \to 0$. This singularity mathematically reflects the fact that perfect desalination of seawater requires infinite effort.

In the prism-shaped region E, the extrapolation of the density derivatives g_{pS}, g_{pT}, and g_{pp} (which appear in Table 1) outside their validity range may result in values that are obviously unreasonable or even invalid. For the pressure range $p_b(T,S) \leq p \leq 100$ MPa (Region E in Figs. 1, 2, 4, and 5), this region is found inside a triangle given by the conditions $T/\text{K} + 450\ S/(\text{kg kg}^{-1}) \geq 362$, $S \leq 0.12$ kg kg^{-1}, and $T \leq 353$ K.

In particular, the use of these derivatives is restricted to $T < 308$ K at the highest salinity, $S = 0.12$ kg kg^{-1}, and to $S < 0.02$ kg kg^{-1} at the highest temperature, $T = 353$ K.

The logarithmic term of Eq. (5) follows Planck's theory of ideal solutions. The series expansion of Eq.

Table 15
Uncertainties of selected quantities in specified regions of p-T-S space estimated for Eq. (1)

Quantity	S interval in kg kg^{-1}	T interval in K	p interval in MPa	u_{08}	Δ_{RMS}	u
$\lvert\frac{\Delta\rho}{\rho}\rvert$	0–0.04	273–313	0.1	4×10^{-6}	2.9×10^{-6}	5×10^{-6}
	0.04–0.05	288–303	0.1	1×10^{-5}	1.3×10^{-6}	1×10^{-5}
	0.005–0.04	273–313	0.1–10	1×10^{-5}	2.6×10^{-6}	1×10^{-5}
	0.005–0.04	273–313	10–100	2×10^{-5}	5.3×10^{-6}	2×10^{-5}
	0.04–0.12	293–313	0.1	3×10^{-4}	4.2×10^{-6}	3×10^{-4}
	0–0.04	313–333	0.1	4×10^{-4}	1.3×10^{-5}	4×10^{-4}
	0.04–0.08	313–333	0.1	9×10^{-4}	1.3×10^{-5}	9×10^{-4}
	0.08–0.12	313–333	0.1	3×10^{-3}	1.3×10^{-5}	3×10^{-3}
	0–0.04	333–353	0.1	1.4×10^{-3}	1.5×10^{-5}	1.4×10^{-3}
	0.04–0.08	333–353	0.1	4×10^{-3}	1.5×10^{-5}	4×10^{-3}
	0.08–0.12	333–353	0.1	1.3×10^{-2}	1.6×10^{-5}	1.3×10^{-2}
$\lvert\Delta\alpha_v\rvert$	0.01–0.03	267–274	0.7–33	$6\times10^{-7}\,\text{K}^{-1}$	$1\times10^{-6}\,\text{K}^{-1}$	$1\times10^{-6}\,\text{K}^{-1}$
$\lvert\frac{\Delta w}{w}\rvert$	0.029–0.043	273–308	0.1–2	3×10^{-5}	8.2×10^{-4}	8.2×10^{-4}
	0.029–0.043	273–303	0.1–5	3×10^{-5}	6.4×10^{-4}	6.4×10^{-4}
	0.033–0.037	273–278	0.1–100	5×10^{-5}	1.8×10^{-4}	1.8×10^{-4}
$\lvert\frac{\Delta p^{\text{vap}}}{p^{\text{vap}}}\rvert$	0.02–0.12	293–353	0.002–0.05	1×10^{-3}	3.9×10^{-5}	1×10^{-3}
	0.018–0.04	298	0.003	2×10^{-4}	1.5×10^{-5}	2×10^{-4}
$\lvert\Delta T_{\text{f}}\rvert$	0.004–0.04	271–273	0.1	2 mK	0.014 mK	2 mK
$\lvert\Delta T_{\text{b}}\rvert$	0.006–0.07	333–353	0.02–0.05	2 mK	1.2 mK	2.3 mK
$\lvert\frac{\Delta\phi}{\phi}\rvert$	0.004–0.04	273	0.1	2×10^{-3}	–[a]	2×10^{-3}
	0.0017–0.038	298	0.1	2×10^{-3}	–[a]	2×10^{-3}
	0.01–0.12	273–278	0.1	3×10^{-3}	–[a]	3×10^{-3}
$\lvert\Delta c_p^{\text{S}}\rvert$	0–0.04	273–313	0.1	$0.5\,\text{J kg}^{-1}\,\text{K}^{-1}$	–[a]	$0.5\,\text{J kg}^{-1}\,\text{K}^{-1}$
$\lvert\Delta c_p\rvert$	0–0.12	273–353	0.1	$4\,\text{J kg}^{-1}\,\text{K}^{-1}$	$1.3\,\text{J kg}^{-1}\,\text{K}^{-1}$	$4.2\,\text{J kg}^{-1}\,\text{K}^{-1}$

[a] The quantity is only dependent on the saline contribution of Eq. (1).

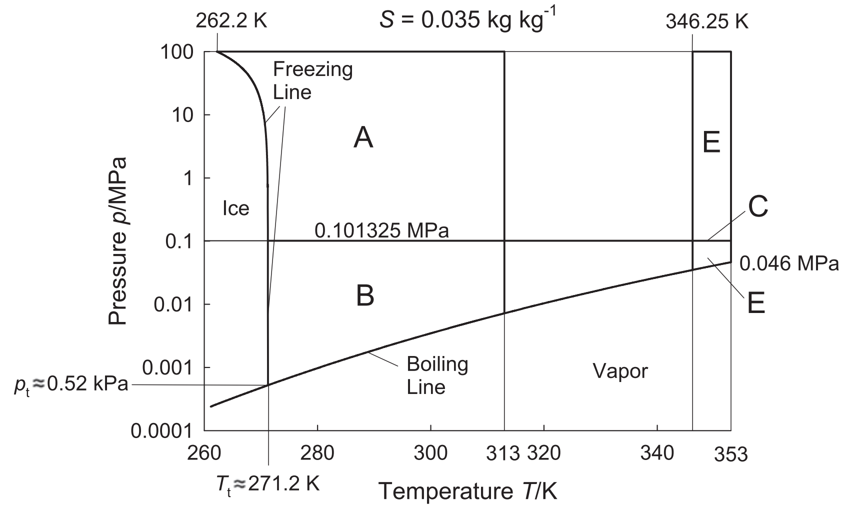

Fig. 4. p-T diagram of seawater at salinity $S = 0.035\,\text{kg kg}^{-1}$ with the regions A, B, C, and E of the p-T plane.

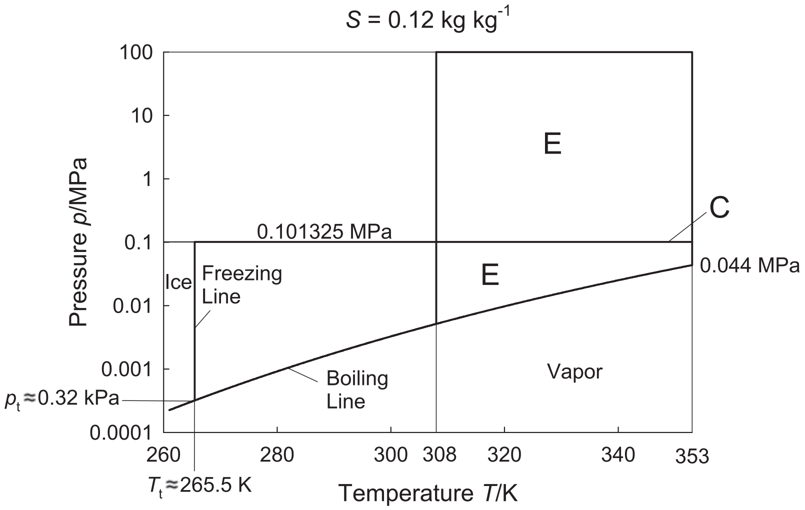

Fig. 5. p-T diagram of seawater at salinity $S = 0.12\,\mathrm{kg\,kg^{-1}}$ with the regions C and E of the p-T plane.

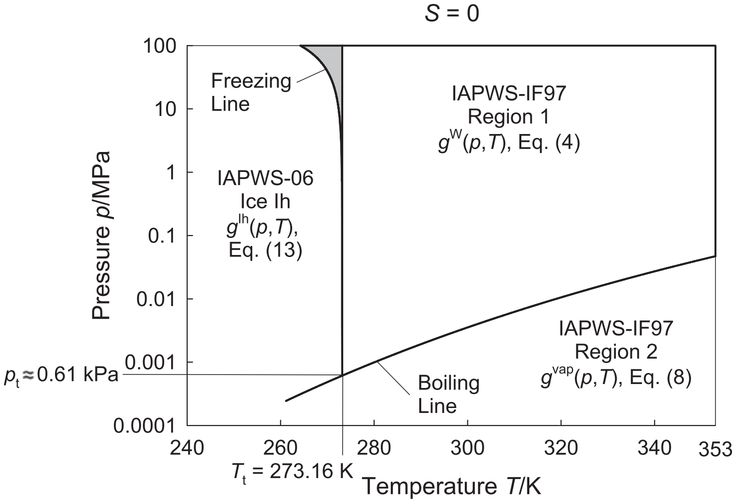

Fig. 6. p-T diagram of pure water (salinity $S = 0$) showing the Gibbs equations of Region 1 (liquid) and Region 2 (vapor) of IAPWS-IF97 as well as the Gibbs equation of the ice Ih region of IAPWS-06. In the gray-shaded area, the Gibbs equation of IAPWS-IF97 Region 1 is extrapolated to temperatures below its range of validity in the metastable region.

(5) in terms of the salinity root is consistent with the Debye-Hückel limiting law of dilute electrolytes. In this formulation, both theoretical expressions adopted from statistical thermodynamics support a smooth crossover of calculated seawater properties in the limiting case of vanishing salinity where measurements are unavailable.

In several regions of the world's oceans, in particular in coastal waters or estuaries, the composition of the local seawater differs from that of IAPSO Standard Seawater [12]. The related deviation between the directly measured density and the density computed from Eq. (1), based on salinity measurements carried out on the currently valid, conductivity-based international Practical Salinity Scale of 1978 [12], is up to 0.005% in the open ocean and up to 0.013% in the Baltic Sea.

The range of validity given here does not consider the solubility of sea salt components such as gypsum ($CaSO_4$) or lime ($CaCO_3$) which may become supersaturated or precipitate at a salinity even less than $0.12\,\mathrm{kg\,kg^{-1}}$. Currently, saturation concentrations are not sufficiently well known and are discussed in Marion et al. [30,31].

6. Uncertainty

A summary of estimated combined standard uncertainties u (coverage factor $k = 1$) of selected quantities for the industrial seawater formulation in specified regions of p-T-S space is given in Table 15. The total uncertainty of the industrial seawater formulation has been estimated by the equation

$$u = \sqrt{u_{08}^2 + \Delta_{\mathrm{RMS}}^2} \tag{22}$$

where u_{08} represents the uncertainty of the IAPWS-08 seawater formulation [1], which uses IAPWS-95 for the pure-water contribution, and Δ_{RMS} represents the root-mean-square deviation caused by the difference between IAPWS-IF97 and IAPWS-95 and is computed from $\Delta_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\Delta x_n)^2}$, where Δx_n can be either absolute or percentage difference between the corresponding quantities x; N is the number of Δx_n values.

Estimated values of u_{08}, Δ_{RMS}, and u are given in Table 15. 100,000 points uniformly distributed over the respective intervals given in Table 15 were used to calculate the values for Δ_{RMS}.

Table 15 shows that, for all salinities considered, the deviations in calculated properties caused by the use of IAPWS-IF97 instead of IAPWS-95 are small compared to the uncertainties in IAPWS-08. Therefore, the total uncertainties of the industrial seawater formulation are generally equivalent to those of the IAPWS-08 formulation.

$$\mathrm{CTR} = \frac{\text{computing time for a quantity calculated from IAPWS - 08}}{\text{computing time for this quantity calculated from the industrial formulation}} \tag{23}$$

7. Computing-time comparisons

One important reason to use IAPWS-IF97 instead of IAPWS-95 for calculating the water contribution of the IAPWS seawater functions is the computing-speed difference, which is particularly important in computer-intensive process modeling calculations. A comparison of the computing speed of the seawater functions of the 2008 IAPWS seawater formulation [1] (which uses IAPWS-95) with that of the industrial formulation for seawater documented in this work (using IAPWS-IF97) was reported in [32]. The relevant functions of the 2008 IAPWS seawater formulation and the industrial seawater formulation were programmed with regard to short computing times. The tests were performed using the same code as for the evaluation of IAPWS-IF97, so-called NIFBENCH [33], on a PC using an Intel Core i7 920 with 2.64 GHz as CPU in combination with a 32-bit Windows 7 operating system.[1]

The measured computing times were used to calculate computing-time ratios (CTR) between the 2008 IAPWS seawater formulation and the industrial formulation of this work:

Table 16
Computing-time ratios of important properties calculated from Eq. (23)

Property	CTR
Specific volume v	243
Specific enthalpy h	236
Specific entropy s	220
Specific isobaric heat capacity c_p	430
Chemical potential of water in seawater μ_{W}	134
Boiling temperature of seawater T_{b}	206
Freezing temperature of seawater T_{f}	32

The results of these calculations are shown in Table 16.

In summary, the calculation of seawater properties using the industrial seawater formulation is approximately 200 times faster than the calculation using the IAPWS 2008 formulation. It should also be noted that the observed improvement depends on the efficiency with which IAPWS-95 is programmed (and also on the machine and compiler used). Nevertheless, the use of IAPWS-IF97 clearly produces a significant reduction in computing time.

8. Summary and conclusions

In this paper, the industrial calculation of seawater properties was carried out using the algorithms of IAPWS-IF97 for pure water. This was a requirement

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[1] Certain commercial products are identified in this paper, but only to adequately specify the procedure. Such identification neither constitutes nor implies recommendation or endorsement by either the US government or the National Institute of Standards and Technology.

by industry which routinely uses the industrial formulation IAPWS-IF97 for process calculations that are often demanding on computer time. An advantage of using the algorithms of IAPWS-IF97 instead of IAPWS-95 is to avoid iterative calculations from the IAPWS-95 Helmholtz free energy equation, which is computationally intensive and therefore undesirable in applications where processing speed is important.

Because the algorithms to calculate different properties for the liquid region 1 of IAPWS-IF97 are expressed as a Gibbs free energy equation, their implementation into the seawater equation of state, which is also formulated in the Gibbs free energy, can be made with no additional conversions or adaptations. Furthermore, equations to calculate colligative properties are also given here, along with algorithms for the mixtures of brine with water vapor and with ice Ih.

To verify the accuracy of the industrial formulation of seawater, extensive comparisons were carried out for the difference in calculated properties between the algorithms of the industrial formulation for seawater and the scientific formulation IAPWS-08. The results showed that the differences between the two seawater formulations are very small. Consequently, the accuracy of this industrial formulation is nearly the same as that of IAPWS-08. The range of validity of the industrial formulation of seawater is the same as given for IAPWS-08. The computing-time comparison presented here leads to the conclusion that the calculation of seawater properties using the industrial seawater formulation is approximately 200 times faster than calculation using the original IAPWS-08 Formulation. In conclusion, the industrial formulation presented here is recommended for use in industrial applications, such as desalination or cooling processes.

For convenient use of this industrial formulation in the daily work of an engineer, the property library LibSeaWa [34] was developed. For the scientific formulation IAPWS-08, the Sea-Ice-Air (SIA) library [35,36] and the Gibbs-SeaWater Oceanographic Toolbox [37] can be used specifically for ocean and climate modeling. In addition, the properties of seawater using the industrial formulation can be calculated online [38].

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List of symbols

Symbol	Unit	physical quantity
b	$\mathrm{kmol\,kg^{-1}}$	molality of salt in seawater
c_p	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$	specific isobaric heat capacity of seawater
c_v	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$	specific isochoric heat capacity of seawater
CTR		computing-time ratio, determined by Eq. (23)
f	$\mathrm{kJ\,kg^{-1}}$	specific Helmholtz free energy of seawater
g	$\mathrm{kJ\,kg^{-1}}$	specific Gibbs free energy of seawater
g^{BI}	$\mathrm{kJ\,kg^{-1}}$	specific Gibbs free energy of brine-ice mixture (sea ice)
g^{BV}	$\mathrm{kJ\,kg^{-1}}$	specific Gibbs free energy of brine-vapor mixture
g^{Ih}	$\mathrm{kJ\,kg^{-1}}$	specific Gibbs free energy of ice Ih
g^{S}	$\mathrm{kJ\,kg^{-1}}$	saline contribution of the specific Gibbs free energy of seawater
g^{vap}	$\mathrm{kJ\,kg^{-1}}$	specific Gibbs free energy of water vapor
g^{W}	$\mathrm{kJ\,kg^{-1}}$	water contribution of the specific Gibbs free energy of seawater
h	$\mathrm{kJ\,kg^{-1}}$	specific enthalpy of seawater
h^{W}	$\mathrm{kJ\,kg^{-1}}$	specific enthalpy of water
M_{S}	$\mathrm{kg\,kmol^{-1}}$	molar mass of sea salt
m	kg	mass of seawater including water vapor or ice
m_{S}	kg	mass of sea salt in seawater
m_{W}	kg	mass of water in seawater
p	MPa	pressure
p^{vap}	MPa	pressure at the phase equilibrium between seawater and water vapor
p_{b}	MPa	boiling pressure of seawater
p_{f}	MPa	freezing pressure of seawater
p_{osm}	MPa	osmotic pressure of seawater
p_{t}	MPa	triple-point pressure of seawater
R_{m}	$\mathrm{kJ\,kmol^{-1}\,K^{-1}}$	molar gas constant
R_{W}	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$	specific gas constant of water
Re		real part of a complex number
RMS		root-mean-square value of a quantity
S	$\mathrm{kg\,kg^{-1}}$	salinity, mass fraction of sea salt in seawater including water vapor or ice
S_{b}	$\mathrm{kg\,kg^{-1}}$	salinity of boiling brine
S_{f}	$\mathrm{kg\,kg^{-1}}$	salinity of freezing brine
S_{n}	$\mathrm{kg\,kg^{-1}}$	normal salinity of seawater
s	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$	specific entropy of seawater
s^{W}	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$	specific entropy of water
T	K	absolute temperature (ITS-90)

T_b	K	boiling temperature of seawater
T_f	K	freezing temperature of seawater
T_t	K	triple-point temperature of seawater
u		uncertainty of a property given in Table 15
u	kJ kg^{-1}	specific internal energy of seawater
u^W	kJ kg^{-1}	specific internal energy of water
v	m^3 kg^{-1}	specific volume of seawater
w	m s^{-1}	speed of sound of seawater
x	kg kg^{-1}	mass fraction of the vapor phase in the brine-vapor mixture of seawater (vapor fraction)
y	kg kg^{-1}	mass fraction of the ice phase in the brine-ice mixture (ice fraction)
α_v	K^{-1}	isobaric cubic expansion coefficient (thermal expansion coefficient) of seawater
β	kg kg^{-1}	haline contraction coefficient of seawater
γ		reduced Gibbs free energy
θ		reduced temperature
κ		isentropic exponent of seawater
κ_T	Pa^{-1}	isothermal compressibility of seawater
μ	kJ kg^{-1}	relative chemical potential of seawater
μ_S	kJ kg^{-1}	chemical potential of sea salt in seawater
μ_W	kJ kg^{-1}	chemical potential of water in seawater
ξ		reduced salinity root
π		reduced pressure
ρ	kg m^{-3}	mass density of seawater
τ		inverse reduced temperature
ϕ		osmotic coefficient of seawater

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Appendix

Table A1.
Numerical check values for the water contribution computed from $g^{W}(p,T)$, Eq. (4), and its derivatives, for the saline contribution computed from $g^{S}(p,T,S)$, Eq. (5), and its derivatives, for the seawater properties computed from the Gibbs function $g(p,T,S)$, Eq. (1), and its derivatives and for selected seawater properties of Table 1 at given points (p,T,S)

Quantity	Water contribution	Saline contribution	Property of seawater	Unit
Properties at $p = 0.101\,325$ MPa, $T = 273.15$ K, $S = S_n = 0.035\,165\,04$ kg kg^{-1}				
g	$0.101\,359\,446$	$-0.101\,342\,742$	$0.167\,05\times10^{-4}$	kJ kg^{-1}
$(\partial g/\partial p)_{T,S}$	$0.100\,015\,572\times10^{-2}$	$-0.274\,957\,224\times10^{-4}$	$0.972\,659\,995\times10^{-3}$	m^3 kg^{-1}
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.508\,885\,499\times10^{-12}$	$0.581\,535\,172\times10^{-13}$	$-0.450\,731\,982\times10^{-12}$	m^3 kg^{-1} Pa^{-1}
$(\partial g/\partial T)_{p,S}$	$0.147\,711\,823\times10^{-3}$	$-0.147\,643\,376\times10^{-3}$	$0.684\,47\times10^{-7}$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.154\,473\,013\times10^{-1}$	$0.852\,861\,151\times10^{-3}$	$-0.145\,944\,401\times10^{-1}$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial p\partial T)_{S}$	$-0.676\,992\,620\times10^{-7}$	$0.119\,286\,787\times10^{-6}$	$0.515\,875\,254\times10^{-7}$	m^3 kg^{-1} K^{-1}
$(\partial g/\partial S)_{p,T}$	0	$0.639\,974\,067\times10^{2}$	$0.639\,974\,067\times10^{2}$	kJ kg^{-1}
$(\partial^2 g/\partial p\partial S)_{T}$	0	$-0.759\,615\,412\times10^{-3}$	$-0.759\,615\,412\times10^{-3}$	m^3 kg^{-1}
v	$0.100\,015\,572\times10^{-2}$	$-0.274\,957\,224\times10^{-4}$	$0.972\,659\,995\times10^{-3}$	m^3 kg^{-1}
u	$-0.403\,288\,161\times10^{-1}$	$-0.582\,279\,494\times10^{-1}$	$-0.985\,567\,655\times10^{-1}$	kJ kg^{-1}
h	$0.610\,119\,617\times10^{-1}$	$-0.610\,139\,535\times10^{-1}$	$-0.199\,16\times10^{-5}$	kJ kg^{-1}
s	$-0.147\,711\,823\times10^{-3}$	$0.147\,643\,376\times10^{-3}$	$-0.684\,47\times10^{-7}$	kJ kg^{-1} K^{-1}
c_p	$0.421\,943\,034\times10^{1}$	$-0.232\,959\,023$	$0.398\,647\,132\times10^{1}$	kJ kg^{-1} K^{-1}
w	$0.140\,243\,979\times10^{4}$	–[a]	$0.144\,907\,123\times10^{4}$	m s^{-1}
μ_W	$0.101\,359\,446$	$-0.235\,181\,411\times10^{1}$	$-0.225\,045\,466\times10^{1}$	kJ kg^{-1}
Properties at $p = 0.101\,325$ MPa, $T = 353$ K, $S = 0.1$ kg kg^{-1}				
g	$-0.446\,091\,363\times10^{2}$	$0.150\,871\,740\times10^{2}$	$-0.295\,219\,623\times10^{2}$	kJ kg^{-1}
$(\partial g/\partial p)_{T,S}$	$0.102\,891\,627\times10^{-2}$	$-0.579\,227\,286\times10^{-4}$	$0.970\,993\,546\times10^{-3}$	m^3 kg^{-1}
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.473\,220\,602\times10^{-12}$	$0.213\,086\,154\times10^{-12}$	$-0.260\,134\,448\times10^{-12}$	m^3 kg^{-1} Pa^{-1}
$(\partial g/\partial T)_{p,S}$	$-0.107\,357\,342\times10^{1}$	$0.156\,230\,907$	$-0.917\,342\,513$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.118\,849\,543\times10^{-1}$	$0.127\,922\,649\times10^{-2}$	$-0.106\,057\,278\times10^{-1}$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial p\partial T)_{S}$	$0.659\,344\,070\times10^{-6}$	$0.803\,061\,596\times10^{-6}$	$0.146\,240\,567\times10^{-5}$	m^3 kg^{-1} K^{-1}
$(\partial g/\partial S)_{p,T}$	0	$0.251\,957\,276\times10^{3}$	$0.251\,957\,276\times10^{3}$	kJ kg^{-1}
$(\partial^2 g/\partial p\partial S)_{T}$	0	$-0.305\,957\,802\times10^{-3}$	$-0.305\,957\,802\times10^{-3}$	m^3 kg^{-1}
v	$0.102\,891\,627\times10^{-2}$	$-0.579\,227\,286\times10^{-4}$	$0.970\,993\,546\times10^{-3}$	m^3 kg^{-1}
u	$0.334\,258\,026\times10^{3}$	$-0.400\,564\,673\times10^{2}$	$0.294\,201\,559\times10^{3}$	kJ kg^{-1}
h	$0.334\,362\,281\times10^{3}$	$-0.400\,623\,363\times10^{2}$	$0.294\,299\,945\times10^{3}$	kJ kg^{-1}
s	$0.107\,357\,342\times10^{1}$	$-0.156\,230\,907$	$0.917\,342\,513$	kJ kg^{-1} K^{-1}
c_p	$0.419\,538\,887\times10^{1}$	$-0.451\,566\,952$	$0.374\,382\,192\times10^{1}$	kJ kg^{-1} K^{-1}
w	$0.155\,710\,273\times10^{4}$	–[a]	$0.401\,505\,044\times10^{4}$	m s^{-1}
μ_W	$-0.446\,091\,363\times10^{2}$	$-0.101\,085\,536\times10^{2}$	$-0.547\,176\,899\times10^{2}$	kJ kg^{-1}
Properties at $p = 100$ MPa, $T = 273.15$ K, $S = S_n = 0.035\,165\,04$ kg kg^{-1}				
g	$0.977\,302\,204\times10^{2}$	$-0.260\,093\,051\times10^{1}$	$0.951\,292\,899\times10^{2}$	kJ kg^{-1}
$(\partial g/\partial p)_{T,S}$	$0.956\,686\,939\times10^{-3}$	$-0.229\,123\,842\times10^{-4}$	$0.933\,774\,555\times10^{-3}$	m^3 kg^{-1}
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.371\,594\,622\times10^{-12}$	$0.357\,345\,736\times10^{-13}$	$-0.335\,860\,049\times10^{-12}$	m^3 kg^{-1} Pa^{-1}
$(\partial g/\partial T)_{p,S}$	$0.858\,228\,709\times10^{-2}$	$0.754\,045\,685\times10^{-2}$	$0.161\,227\,439\times10^{-1}$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.142\,987\,096\times10^{-1}$	$0.488\,076\,974\times10^{-3}$	$-0.138\,106\,326\times10^{-1}$	kJ kg^{-1} K^{-1}
$(\partial^2 g/\partial p\partial T)_{S}$	$0.202\,974\,451\times10^{-6}$	$0.466\,284\,412\times10^{-7}$	$0.249\,602\,892\times10^{-6}$	m^3 kg^{-1} K^{-1}
$(\partial g/\partial S)_{p,T}$	0	$-0.545\,861\,581\times10^{1}$	$-0.545\,861\,581\times10^{1}$	kJ kg^{-1}
$(\partial^2 g/\partial p\partial S)_{T}$	0	$-0.640\,757\,619\times10^{-3}$	$-0.640\,757\,619\times10^{-3}$	m^3 kg^{-1}
v	$0.956\,686\,939\times10^{-3}$	$-0.229\,123\,842\times10^{-4}$	$0.933\,774\,555\times10^{-3}$	m^3 kg^{-1}
u	$-0.282\,725\,255$	$-0.236\,936\,788\times10^{1}$	$-0.265\,209\,313\times10^{1}$	kJ kg^{-1}
h	$0.953\,859\,687\times10^{2}$	$-0.466\,060\,630\times10^{1}$	$0.907\,253\,624\times10^{2}$	kJ kg^{-1}
s	$-0.858\,228\,709\times10^{-2}$	$-0.754\,045\,685\times10^{-2}$	$-0.161\,227\,439\times10^{-1}$	kJ kg^{-1} K^{-1}
c_p	$0.390\,569\,252\times10^{1}$	$-0.133\,318\,225$	$0.377\,237\,430\times10^{1}$	kJ kg^{-1} K^{-1}
w	$0.157\,552\,544\times10^{4}$	–[a]	$0.162\,218\,081\times10^{4}$	m s^{-1}
μ_W	$0.977\,302\,204\times10^{2}$	$-0.240\,897\,806\times10^{1}$	$0.953\,212\,423\times10^{2}$	kJ kg^{-1}

[a] This contribution cannot be computed from g^{S} alone because it is a nonlinear expression in g.

Table A2
Boiling temperatures T_b in K of seawater for selected points (p,S) calculated by iteration from Eq. (6) or Eq. (7)

	Salinity S in kg kg^{-1}						
Pressure p in MPa	0	0.02	0.04	0.06	0.08	0.10	0.12
0.001	280.12	280.27	280.43	280.61	280.80	281.01	281.25
0.005	306.03	306.21	306.41	306.63	306.87	307.14	307.44
0.010	318.96	319.16	319.38	319.62	319.89	320.19	320.52
0.015	327.12	327.34	327.57	327.83	328.11	328.42	328.78
0.020	333.21	333.44	333.68	333.95	334.24	334.57	334.94
0.025	338.11	338.35	338.60	338.88	339.18	339.52	339.91
0.030	342.25	342.49	342.75	343.03	343.34	343.69	344.09
0.035	345.83	346.08	346.34	346.63	346.95	347.31	347.72
0.040	349.01	349.26	349.53	349.83	350.15	350.52	350.94

Table A3
Freezing temperatures T_f in K of seawater for selected points (p,S) calculated by iteration from Eq. (11) or Eq. (12)

Pressure p in MPa	Salinity S in kg kg^{-1}						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0.001	273.1600	272.0823	270.9611	269.7618	268.4609	267.0397	265.4900
0.005	273.1597	272.0820	270.9608	269.7615	268.4606	267.0394	265.4897
0.01	273.1593	272.0816	270.9604	269.7612	268.4602	267.0390	265.4893
0.05	273.156	272.079	270.957	269.758	268.457	267.036	265.486
0.1	273.153	272.075	270.954	269.754	268.453	267.032	265.482
1	273.09	272.01	270.89	–[a]	–[a]	–[a]	–[a]
10	272.40	271.32	270.20	–[a]	–[a]	–[a]	–[a]
20	271.61	270.53	269.40	–[a]	–[a]	–[a]	–[a]
50	269.06	267.97	266.82	–[a]	–[a]	–[a]	–[a]
100	264.21	263.09	261.92	–[a]	–[a]	–[a]	–[a]

[a] This state point is out of the range of validity of Eq. (1).

Table A4
Triple-point temperatures T_t and related triple-point pressures p_t of seawater for selected salinities S calculated by iteration from both Eqs. (6) and (11) or from Eqs. (7) and (12)

Salinity S in kg kg^{-1}	0	0.02	0.04	0.06	0.08	0.10	0.12
Triple-point temperature T_t in K	273.16	272.08	270.96	269.76	268.46	267.04	265.49
Triple-point pressure p_t in kPa	0.61168	0.55953	0.50961	0.46073	0.41257	0.36524	0.31932

Table A5
Numerical check values for the brine contribution computed from $g(p, T, S_b)$, Eq. (1), and its derivatives, for the vapor contribution computed from $g^W(p, T)$, Eq. (4), and its derivatives, and for the brine-vapor properties computed from $g^{BV}(p, T, S)$, Eq. (16), for specific volume, specific internal energy, specific enthalpy, and specific entropy at $p = 0.04$ MPa, $T = 350.15$ K, and $S = 0.06$ kg kg^{-1} ($S_b = 0.079\ 781\ 386\ 1$ kg kg^{-1} and $x = 0.247\ 944\ 879$)

Quantity	Brine contribution	Vapor contribution	Property of brine-vapor mixture	Unit
v	$0.975\ 269\ 274 \times 10^{-3}$	$0.400\ 679\ 744 \times 10^{1}$	$0.994\ 198\ 363$	m^3 kg^{-1}
u	$0.291\ 183\ 170 \times 10^{3}$	$0.247\ 806\ 401 \times 10^{4}$	$0.833\ 409\ 076 \times 10^{3}$	kJ kg^{-1}
h	$0.291\ 222\ 180 \times 10^{3}$	$0.263\ 833\ 591 \times 10^{4}$	$0.873\ 177\ 011 \times 10^{3}$	kJ kg^{-1}
s	$0.922\ 324\ 233$	$0.767\ 550\ 581 \times 10^{1}$	$0.259\ 674\ 102 \times 10^{1}$	kJ kg^{-1} K^{-1}

Table A6
Numerical check values for the brine contribution computed from $g(p, T, S_f)$, Eq. (1), and its derivatives, for the ice contribution computed from $g^{Ih}(p, T)$, Eq. (13), and its derivatives, and for the brine-ice properties computed from $g^{BI}(p, T, S)$, Eq. (19), for specific volume, specific internal energy, specific enthalpy, and specific entropy at $p = 0.04$ MPa, $T = 270.15$ K, and $S = 0.04$ kg kg^{-1} ($S_f = 0.053\ 643\ 523\ 5$ kg kg^{-1} and $y = 0.254\ 336\ 826$)

Quantity	Brine contribution	Ice contribution	Property of brine-ice mixture	Unit
v	$0.958\ 594\ 509 \times 10^{-3}$	$0.109\ 033\ 070 \times 10^{-2}$	$0.992\ 099\ 875 \times 10^{-3}$	m^3 kg^{-1}
u	$-0.122\ 140\ 337 \times 10^{2}$	$-0.339\ 724\ 684 \times 10^{3}$	$-0.955\ 120\ 530 \times 10^{2}$	kJ kg^{-1}
h	$-0.121\ 756\ 899 \times 10^{2}$	$-0.339\ 681\ 071 \times 10^{3}$	$-0.954\ 723\ 690 \times 10^{2}$	kJ kg^{-1}
s	$-0.500\ 174\ 577 \times 10^{-1}$	$-0.124\ 381\ 107 \times 10^{1}$	$-0.353\ 643\ 136$	kJ kg^{-1} K^{-1}