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# Analysis of the evaporator heat flow rate in a water purification process integrated to an absorption heat transformer with energy recycling

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# ABSTRACT

An experimental system of an absorption heat transformer consists of four components: a helical double-pipe vertical evaporator, a helical double-pipe vertical condenser, an absorber and a generator. In a general case, the thermodynamic model estimates the coefficient of performance (COP) derived from the water purification process integrated with the absorption heat transformer, considering the heat flow rates of the absorber, the evaporator, the generator and the condenser. In this work, a theoretical evaporator model is incorporated into the thermodynamic model for the performance analysis in the absorption heat transformer with energy recycling. This model predicts the heat flow rate of the helical double-pipe vertical evaporator. It considers change of phase, equations of continuity, momentum and energy equations in the two water flows at counter-current and heat transfer by conduction in the internal tube wall and insulated external wall. This shows the effect upon the changes of the evaporator heat flow rate in the COP estimation. This analysis of COP versus evaporator heat flow rate was done in steady state that allows knowing the quantity of heat that the evaporator needs — both from energy recycling and a thermal source. In addition, these coupling models (theoretical and thermodynamic) give the opportunity to control the performance of the system on-line.

Keywords: Water purification; Absorption heat transformer; Helical double-pipe vertical evaporator

## 1. Introduction

An experimental system of an absorption heat transformer consists of four components: a helical double-pipe vertical evaporator, a helical double-pipe vertical condenser, an absorber and a generator [1]. As in previous works [1,2] the importance of the COP variable in order to determine the performance of the system was clearly established; then the COP is defined as the heat delivered in the absorber per unit of heat load supplied to the generator and the evaporator [Eq. (1)]. The absorption heat transformer is a system that consists of a thermodynamic device capable of producing useful heat at a thermal level, superior to the one provided by the source [1,3,4]. The advantage of this particular absorption heat transformer is that it may be incorporated in any other

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Fig. 1. Water purification process integrated to an absorption heat transformer with the incorporation of waste heat from the auxiliary condenser to the evaporator and generator of the absorption heat transformer. Continuous line (–) represents the absorption heat transformer; interrupted line (– . –) represents the water purification process; points (...) represent recycled energy.

system that may require a temperature higher than the one given by the source. In the past, other studies [1–3,5,6] reported the possibility of integrating the absorption heat transformer to a water purification process. Escobar et al. [2] presented a portable water purification process integrated with an absorption heat transformer. The authors mentioned that the coefficient of performance (COP) could be increased thanks to the incorporation of waste heat from the water purification condenser into both the absorption heat transformer evaporator and generator (see Fig. 1). This allowed a fraction of heat obtained by the absorption heat transformer to be recycled, thus obtaining a higher value of COP, while only the evaporator and generator temperatures were slightly increased.

Therefore, the relevance to analyze in steady state the effect upon the changes of the evaporator heat flow rate in the COP calculation comes into sight. The study will determine the quantity of heat that the evaporator needs from both energy recycling and a thermal source in the system.

Consequently, the aim of the present work is to know the effect of the evaporator heat flow rate over the system performance. A theoretical detailed helical evaporator model [7] is incorporated into the thermodynamic model to obtain a performance analysis in the absorption heat transformer with energy recycling. This theoretical model predicts the heat flow rate of the helical double-pipe vertical evaporator (see Fig. 2), considering change of phase, equations of continuity, momentum and energy in the two water flows at counter-current, heat transfer by conduction in the internal tube wall and insulated external wall as well. In addition, these coupling models (theoretical and thermodynamic) allow on-line control of the performance of the system.

# 2. Mathematical models

## 2.1. Thermodynamic model

The following assumptions have been made in the development of the thermodynamic model for an absorption heat transformer system (with reference to Fig. 1) [2]:

1. the entire system is in thermodynamic equilibrium,

2. the analysis is carried out under steady-state conditions,

3. a rectifier is not required since the absorbent does not evaporate under the operating temperature range of the system,

4. the solution that leaves the generator and the absorber is saturated; similarly, the working fluid leaving the condenser and the evaporator is also saturated,

5. heat losses and pressure drops in the tubing and the components are considered negligible,

6. the flow through the valves is isenthalpic,

7. outlet temperatures of the main components  $T_{5'}$ ,  $T_{8'}$ ,  $T_{10}$  and  $T_{3'}$  and heat load in the evaporator  $Q_{EV}$  are known,

8. a heat supply is delivered by industrial waste heat,

From these assumptions it is possible to define the enthalpy-based COP by the following equation.

$$COP = \frac{Q_{AB}}{Q_{GE} + Q_{EV}} \tag{1}$$



Fig. 2. Helical double-pipe experimental evaporator used in the absorption heat transformer.

where  $Q_{AB}$ ,  $Q_{GE}$  and  $Q_{EV}$  are the heat flow rate (kW) for each one of the energy cycle components. Eq. (1) is developed in Appendix A, as is reported by Escobar et al. [2].

#### 2.2. Theoretical helical evaporator model

A theoretical detailed helical evaporator model has been included in the thermodynamic model to understand this theoretical model. It is necessary to define the "control volume" concept. A control volume is a finite volume that delimits a physical space corresponding to a partial or global zone of a thermal unit [8], in this case, the helical evaporator. The terms *i* and *i*+1 in a control volume represent the inlet and outlet section respectively.

The governing equation has been developed following these assumptions [7]:

For a water fluid:

- one-dimensional flow: p(z, t), h(z, t), T(z, t), etc.;
- non-participant radiation, medium and negligible radiant heat exchanger between surfaces;
- axial heat conduction inside the fluid is neglected;
- internal and coiled diameter constant and uniform roughness surface.

The semi-integrated governing equations (mass, momentum, energy and entropy) over control volume were presented by García-Valladares et al. [9]. For every control volume, a set of algebraic equations is obtained by a discretization of the governing equations. The transient terms of the governing equations are discretized using the following approximation

$$\frac{\partial \phi}{\partial t} \cong \frac{\left(\phi - \phi^0\right)}{\Delta t}$$

where  $\phi$  represents a generic dependent variable ( $\phi = h, p, T$ , etc.) and the superscript 0 indicates the value of the previous instant. The average of the variables ( $\phi = h, p, T$ ,

etc.) has been estimated by the arithmetic mean between their values at the inlet and outlet sections, that is,

$$\widetilde{\phi}_i \cong \overline{\phi} \equiv \frac{\phi_i + \phi_{i+1}}{2}$$

Based on the numerical approaches indicated above, the governing equations can be discretized to obtain the value of the dependent variables (mass flow rate, pressure and enthalpy) at the outlet section of each control volume.

The mass flow rate is the result of the following discrete continuity equation:

$$\dot{m}_{i+1} = \dot{m}_i - \frac{A\Delta z}{\Delta t} \left( \overline{\rho}_{tp} - \overline{\rho}_{tp}^0 \right)$$
<sup>(2)</sup>

where the two-phase flow density is obtained from:

$$\rho_{tp} = \varepsilon_g \rho_g + (1 - \varepsilon_g) \rho_1$$

In terms of the mass flow rate, gas and liquid velocities are calculated as

$$V_{g} = \frac{\dot{m}x_{g}}{\rho_{\sigma}\varepsilon_{\sigma}A}$$
(3)

$$V_1 = \frac{\dot{m}(1 - x_g)}{\rho_1(1 - \varepsilon_1)A} \tag{4}$$

The discrete momentum equation is solved for the outlet pressure,

$$p_{i+1} = p_i - \frac{\Delta z}{A} \left\{ \phi_{l_0}^2 \frac{\overline{f_l}}{4} \frac{\overline{m}^2}{2\overline{\rho_l}A^2} P + \overline{\rho_{tp}} Agsin\phi + \frac{\left[ \frac{\dot{m}(x_g v_g + (1 - x_g)v_l)}{\Delta z} \right]_i^{i+1}}{\Delta z} + \frac{\overline{m} - \overline{m}^0}{\Delta t} \right\}$$
(5)

With the inlet and outlet pressure obtained from the momentum Eq. (5), the pressure drop in each control volume is also evaluated.

From the energy and continuity equation, the following equation was obtained for the outlet enthalpy:

$$h_{i+1} = \frac{2\dot{q}P\Delta z - \dot{m}_{i+1}a + \dot{m}_ib + \frac{A\Delta z}{\Delta t}c}{\dot{m}_{i+1} + \dot{m}_i + \frac{\left[\tilde{\varepsilon}_g^0\tilde{\rho}_g^0 + \left(1 - \tilde{\varepsilon}_g^0\right)\tilde{\rho}_1^0\right]A\Delta z}{\Delta t}}{\Delta t}$$
(6)

where:

$$a = \left[x_{g}V_{g} + (1 - x_{g})V_{1}\right]_{i+1}^{2} + g\sin\theta\Delta z - h_{i}$$
<sup>(7)</sup>

$$b = \left[x_g V_g + (1 - x_g) V_1\right]_i^2 + g \sin \theta \Delta z + h_i$$
(8)

$$c = (p_{i} + p_{i+1}) - (p_{i}^{0} + p_{i+1}^{0}) + \left[\tilde{\varepsilon}_{g}^{0} \tilde{\rho}_{g}^{0} + (1 - \tilde{\varepsilon}_{g}^{0}) \tilde{\rho}_{1}^{0}\right] \left[(h_{i}^{0} + h_{i+1}^{0}) - h_{i}\right] \\ - \left[\tilde{\rho} \left(\frac{V_{i} + V_{i+1}}{2}\right)^{2} - \tilde{\rho}^{0} \left(\frac{V_{i}^{0} + V_{i+1}^{0}}{2}\right)^{2}\right]$$
(9)

Temperature, mass fraction, density and all thermophysical properties are produced from the NIST/ASME Steam Properties Database [10]. The one-dimensional model requires the knowledge of the two-phase flow structure, which is evaluated by means of the void fraction  $\varepsilon_{g}$ . The evaluation of the shear stress is performed by means of a friction factor *f*, which is defined from the expression

$$\tau_w = \Phi_{1o}^2 \frac{\overline{f}_1 G^2}{4 \, 2\overline{\rho}_1}$$

.

where  $\Phi_{l_0}^2$  is the two-phase factor multiplier [11]. The inertial force, centrifugal force and gravity influences are embroiled in the calculation of friction factor for helical coil. Heat transfer through the helical tube wall and fluid temperature are related to the convective heat transfer coefficient  $\alpha$ , which is defined as:

$$\alpha = \frac{q_{\text{wall}}}{T_{\text{wall}} - T_{\text{fluid}}} \tag{10}$$

In the same way, the calculation of convective heat transfer involves previously mentioned helical effects. The above conservation equations are applicable to transient two-phase flow. Situation of steady flow and/or singlephase flow (liquid or gas) are particular cases for these formulations. In the internal pipe of the helical evaporator, different regions are present in the process. The differentiations between the three principal regions are given by the enthalpy, pressure and vapor quality, following these criteria:

- liquid region: when  $h(p) < h_l(p)$ , then  $x_g = 0$
- two-phase region: when  $h_l(p) \le h(p) \le h_g(p)$ , then  $0 < x_g < 1$
- vapor region: when  $h(p) > h_g$ , then  $x_g = 1$ .

where  $h_i(p)$  and  $h_g(p)$  represent the saturation liquid and gas enthalpy for a given pressure *p*.

# 2.2.1. Heat conduction in the internal and external tube wall

In the internal tube, the physical space that separates the flows is treated assuming the following hypotheses: one-dimensional transient temperature distribution and negligible heat exchanger by radiation. The energy balance used to describe heat transfer in the wall is obtained through:

$$\left(\overline{\dot{q}}_{ns}P_{ns}-\overline{\dot{q}}_{nn}P_{nn}\right)\Delta z + \left(\overline{\dot{q}}_{nw}-\overline{\dot{q}}_{ne}\right)A = m\frac{\partial h}{\partial t}$$
(11)

~

where  $(\overline{\dot{q}}_{ns})$  and  $(\overline{\dot{q}}_{nn})$  are estimated using the respective convective heat transfer coefficient in each zone, and the convective heat transfer fluxes are evaluated from the Fourier law, which is:

$$\overline{\dot{q}}_{ne} = -\lambda_{ne} \left( \frac{\partial T}{\partial z} \right)_{ne} \text{ and } \overline{\dot{q}}_{nw} = -\lambda_{nm} \left( \frac{\partial T}{\partial z} \right)_{nw}$$
(12)

Heat conduction in the external tube, insulation and natural convection with the environment are considered in the model according to García-Valladares [8], although this is estimated to be much smaller than heat transfer between both fluids. Therefore, it is necessary to know the amount of this heat because in the absorption heat transformer heat losses occur in each component (evaporator, absorber, generator and condenser). Consequently, the conduction equation for the insulation has been written assuming transient asymmetric temperature distribution and negligible heat exchanged (by radiation) with the external ambient. The north and south interfaces are evaluated from Fourier's law [Eq. (12)], except in the tube-insulation interface (where a harmonic mean thermal conductivity is used) and in the insulation-ambient interface (where the heat transfer by natural convection is introduced) [9].

#### 2.2.2. Empirical coefficients used

For the single-phase region — liquid and vapor in an internal pipe. The friction factor is evaluated from the following expression proposed by Schmidt [11] in the laminar region for Re <2300 in curved pipes:

$$\frac{f_c}{f_s} = \left[1 + 0.14 \left(\frac{d}{D}\right)^{0.97} \text{Re}\right]^{1/2}$$

where  $k = 1-0.644(d/D)^{0.312}$  and  $f_s = 64$ /Re. In the case of turbulent region the friction factor is evaluated with the equation proposed by Ito for Re >2300:

$$f_c = 1.216 \text{Re}^{-0.25} + 0.116 \left(\frac{d}{D}\right)^{0.5}$$

Guo et al. [12] recommend this equation as the standard formula for the turbulent region. The convective heat transfer coefficient was calculated using the equation proposed by Churchill [13] for the laminar region in helical coiled tubes (Re <2300):

$$Nu = \left[ \left( \frac{48}{11} + \frac{51/11}{1 + \frac{1342}{PrHe^2}} \right)^3 + 1.816 \left( \frac{He}{1 + \frac{1.15}{Pr}} \right)^{3/2} \right]^{1/3}$$

For the turbulent region, the equation developed by Sebas and McLaughlin [14] is used for Re >2300:

$$Nu=0.023Re^{0.8}Pr^{0.4}\left[Re(d/D)^2\right]^{0.05}$$

For two phase flow region in an internal pipe — The void fraction is evaluated according to semi-empirical relations proposed by Rouhani and Axelsson [15]:

$$\varepsilon_{g} = \frac{x_{g}}{\rho_{s}} \left\{ \left[ \left[ 1 + 0.12(1 - x_{g}) \right] \left( \frac{x_{g}}{\rho_{g}} + \frac{1 - x_{g}}{\rho_{l}} \right) \right] + \frac{1.18(1 - x_{g}) \left[ g\sigma_{i}(\rho_{l} - \rho_{g}) \right]^{0.25}}{G\rho_{l}^{0.5}} \right\}^{-1}$$
(13)

Using the correction factor (two-phase frictional multiplier) according to Friedel [16]:

$$\phi_{l0}^2 = \frac{\tau_w}{(\tau_w)_{l0}} = E + \frac{3.2FH}{Fr^{0.045}We^{0.035}}$$
(14)

where *E*, *F*, *H*, and Fr are reported by Colorado-Garrido et al. [7]. Finally, the convective heat transfer coefficient is calculated by Kozeki's correlation [14]. This correlation involving the Martinelli parameter is used for the laminar and turbulent region:

$$\frac{h_{tp}}{h_{l0}} = 2.5 \left(\frac{1}{Xtt}\right)^{0.75}$$

# 2.2.3. Annulus

Due to the fact that no information is available in the technical literature about a specific correlation used to get the friction factor and the heat transfer coefficient in the annulus of helical tubes, they are calculated using the expressions corresponding to single-phase flow evaluated with the hydraulic diameter.

# 3. Numerical solution

The main objective is the determination of the COP and evaporator heat flow rate  $(Q_{EV})$  values. The discretized equations have been coupled using a fully implicit step-by-step method in the flow direction. From the known values at the inlet section, the variables values at the outlet of each control volume are obtained iteratively. These solution outlet values become the inlet values for the next control volume. The procedure is carried out until the end of the helical evaporator is reached. The conditions of differentiation between regions mentioned in the mathematical formulation of two phase flow and the differentiation between regions, the control volume, where the transition occurs, are detected. The criterion of transition used in internal pipes is associated with the outlet of the control volume, and it is important to calculate the appropriated empirical coefficients in each zone. Inside each control volume the convergence working with the following criteria was verified as follows:

$$\frac{\left|\frac{\phi_{i+1}^{*}-\phi_{i}}{\Delta\phi}\right|}{\left|\Delta\phi\right|} < \delta \tag{15}$$

Here,  $\phi$  is the mass flow rate, pressure and enthalpy. The superscript \* denotes the values at the previous iteration. The referent values  $\Delta \phi$  are estimated in each control volume:  $\phi_{i+1}-\phi_i$ . When this value tends to be zero, it is substituted by  $\phi_i$ . Heat conduction in the tube wall and insulated tube wall were discretized in accordance with García-Valladares et al. [9]. The set of heat conduction discretized equations is solved applying the tri-diagonal matrix algorithm. For both models (thermodynamic and theoretical helical evaporator) were solved using the Matlab software.

#### 4. Results and discussion

The COP [Eq. (1)] is defined as the ration of heat delivered in the absorber divided by the heat load supplied to the generator plus evaporator. The thermodynamic model is used to estimate the COP for the absorption heat transformer integrated to the water purification process. This model follows assumptions for example: the entire system is in thermodynamic equilibrium, the analysis is only carried out under steady-sate, heat losses and pressure drop in the tubing and the components are considered negligible. These assumptions might be too difficult to fulfill in practice. Heat flux to AB, EV and GE are calculated under steady-state conditions. Consequently, the physical model to describe heat transfer  $(Q_{EV})$  and fluid dynamic behavior inside the evaporator must be incorporated into the thermodynamic model to estimate the COP of the system when  $Q_{EV}$  values are modified in a theoretical way with applications for optimization. It is clear that when  $Q_{EV}$  variations are carried out, the COP also changes because evaporator heat flux is summed to generator heat flux in the denominator. Therefore, the COP can be modified by the alterations of the  $Q_{EV}$  with applications to optimize the absorption heat transformer integrated to the water purification process.

Hernandez et al. [17] reported COP values from experimental data, and Escobar et al. [2] reported also COP values from the thermodynamic model for a water purification process integrated to an absorption heat transformer. In this work, we used the thermodynamic model to estimate the COP values considering the evaporator heat flow rate simulated (in steady state), which were compared with the experimental data. Under similar conditions, we present the result obtained from the estimation made from the coupled models.

According to developed algorithms reported by Escobar et al. [2], Figs. 3 and 4 were show temperature

and pressure experimental data of the absorption heat transformer integrated into a water puri-fication process. The obtained temperature values happened to be close to the experimental temperature values reported by Escobar [2].

With the measured data of temperature, pressure and the theoretical model, it is possible to calculate the  $Q_{EV}$ values at each time to calculate the COP values. In the coupled model this variable is estimated in each update acquired, which is shown in Fig. 5. Furthermore, with the theoretical model we can follow the variable of temperature at each pipe distance as it is shown in the evaporator in Fig. 6. Similar results of the evaporator heat flux were obtained by Colorado-Garrido et al [18] considering the physical model. The authors [18] compared temperature and pressure distribution in the flows obtained with the model and with experimental data in steady-state. This demonstrates that the physical dynamic model can be used to modify the  $Q_{EV}$  values when it is necessary to optimize the process. It is based on the applications of governing equations and use general empirical correlations to analyze the COP values of the system.

Fig. 7 shows the COP comparison between both models (thermodynamic model [2] and coupled model). The coupled model gives more information about the



Fig. 3. Experimental output temperatures measured in the four components of the absorption heat transformer integrated into a water purification process.



Fig. 4. Experimental pressures in the absorption heat transformer.



Fig. 5. Heat flow rate estimated with the coupled model for an absorption heat transformer.



Fig. 6. Theoretical helical evaporator model (water and vapor temperature behavior through the pipe).



Fig. 7. COP comparison between thermodynamic model and coupled thermodynamic–theoretical model for an absorption heat transformer.

system behavior. This developed model allows manipulating any evaporator variable. Furthermore, the theoretical model considers heat loss in the evaporator, a fact which diminishes the assumptions of the thermodynamic model.

# 5. Conclusions

This work compared the COP on-line results of the thermodynamic model and the coupled thermodynamictheoretical model, which allowed carrying out studies with variations of inlet parameters in the evaporator. The coupled model gives an estimation of the COP variations related to the dynamics of the helical evaporator equipment, but the complexity of the thermodynamic model was increased by the coupling. However, we have more information about system behavior. The model allows the estimation of the COP on-line considering changes in the evaporator, and in future works it can lead to control studies about the process. It is important to mention that the  $Q_{EV}$  and  $M_{WF}$  in the thermodynamic model are considered constant whereas the  $Q_{EV'}$   $M_{WF}$  in the coupling models are estimated for each update time (t). In addition, the theoretical model considers heat loss in the evaporator which diminishes the assumptions of the thermodynamic model.

# 6. Symbols

		- 2
Α		Cross area section, m <sup>2</sup>
AB	—	Absorber
b		Coil pitch
СО		Condenser
COP	—	Coefficient of performance
Ср	—	Specific heat at constant pressure, J/kg°C
ĊV	—	Control volume
D	—	Helical diameter, m
d	_	Internal diameter, m

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Dn	_	Dean number, $Dn = Re\left(\frac{D}{D}\right)$	
e		Specific energy, $h + \frac{V^2}{m} + gz \sin\theta$ , $L/kg$	
c		2	
EV	—	Evaporator	
f	—	Friction factor	
G	—	Mass velocity, kg/m <sup>2</sup> s	
8	—	Acceleration due to gravity, m/s <sup>2</sup>	
GE	—	Generator	
h	—	Enthalpy, J/kg	
He	_	Helical number He=Dn	
IIC .		$\left[1+b/2\pi(d/2)^2\right]^2$	
m	_	Mass flow rate, kg/s	
m	_	Mass. kg	
n	_	Number of control volumes	
· z		ad	
Nu	—	Nusselt number, $Nu = \frac{\alpha u}{2}$	
10		A Drosserno hor	
р D	_	Pressure, Dar	
Р		Perimeter, m	
Pr		Prandtl number, $\Pr = \frac{C_p \cdot \mu}{Pr}$	
		λ	
q	—	Heat flux per unit of area, W/m <sup>2</sup>	
Q	—	Heat flux, kW	
р.		$\mathbf{P}_{\mathbf{r}} = \mathbf{G}d$	
ĸe		$\mu$	
t	_	Time s	
T		Temperature, °C	
v	_	Velocity, m/s	
X	_	Concentration	
X		Martinelli parameter.	
11		$(1)^{0.9}$ $(1)^{0.5}$ $(1)^{0.1}$	
		$X_{tt} = \left(\frac{1 - x_g}{x_g}\right) \left(\frac{\rho_g}{\rho_l}\right) \left(\frac{\mu_g}{\mu_l}\right)$	
r	_	Vapor mass fraction	
7.	_	Axial coordinated	
~			

Greek

θ	—	Angle, rad
ρ	—	Density, kg/m <sup>3</sup>
δ	—	Convergence criterion
σ	—	Superficial tension, N/m
Φ	—	Two-phase frictional multiplier
α	—	Heat transfer coefficient, W/m <sup>2</sup> °C
τ	—	Shear stress, Pa
λ	—	Thermal conductivity, W/m °C
μ	—	Dynamic viscosity, Pas
$\mathbf{e}_{g}$	—	Void fraction
$\Delta t$	—	Temporal discretization step, s
$\Delta z$	—	Axial discretization step, m

Subscripts

Anu	—	Annulus
С	—	Coiled
exp		Experimental
g	—	Vapor
Int	—	Internal pipe
1		Liquid
s		Straight

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# Appendix A — Mathematical description of the COP

1. From assumption (1) of the section (thermodynamic model):

$$T_5 = T_6 \tag{A.1}$$

2. Since the working flow is considered saturated leaving the condenser and evaporator, then the pressure at state points may be obtained by the following equations:

$$P_8 = P(T_8)$$
 and  $P_{10} = P(T_{10})$  (A.2, A.3)

3. Since the pressure drops in the piping and the components are considered negligible,

$$P_7 = P_8 = P_4 = P_6$$
 and  $P_9 = P_{10} = P_3 = P_1$  (A.4, A.5)

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4. From assumption (4) the concentration of the LiBr concentrated solution leaving the generator can be estimated as follows:

$$X_6 = X(P_8, T_6)$$
(A.6)

5. In the same way, the concentration of the LiBr diluted solution leaving the absorber can be estimated as:

$$X_3 = X(P_{10}, T_3) \tag{A.7}$$

6. Because there is no mass transfer between generator and absorber,

$$X_6 = X_1$$
 and  $X_3 = X_4$  (A.8, A.9)

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7. From assumption (3), the concentration of the absorbent in the vapor leaving the generator is zero; then,

$$X_5 = X_8 = X_9 = X_{10} = 0 \tag{A.10}$$

8. The enthalpies at the exit of the absorber and generator can be estimated by

$$H_3 = H(T_3, X_3)$$
 and  $H_6 = H(T_6, X_6)$  (A11, A.12)

9. From assumption (4), the enthalpies at the output of the condenser and the evaporator can be estimated as follows:

$$H_8 = H(T_8)$$
 and  $H_{10} = H(T_{10})$  (A.13, A.14)

10. Considering that the vapor upon leaving the generator is superheated

$$H_5 = H(P_8, T_5)$$
(A.15)

11. From assumption (3) the absorbent does not evaporate in the temperature range under operating conditions; from mass balance in generator and absorber, the flow ratio (FR) for this system can be defined by:

$$FR = \frac{M_{AB}}{M_{WF}} = \frac{X_6}{X_6 - X_4}$$
(A.16)

12. Carrying out mass and energy balances in the absorber and using the *FR* definition, the mass flow rate of the working fluids can be calculated by the following equation:

$$M_F = \frac{Q_{AB}}{H_{10} - H_1 + FR(H_1 - H_3)}$$
(A.17)

13. From the *FR* definition:

$$M_{\rm AB} = M_{\rm WF} \, FR \tag{A.18}$$

14. From the mass balance in the absorber or generator:

$$M_{GE} = M_{AB} - M_{WF} \tag{A.19}$$

15. Considering that the process through the expansion valve is isenthalpic (6):

$$H_3 = H_4 \tag{A.20}$$

16. From mass and energy balances in the main components, the amount of heat supplied or delivered can be estimated from the following equations:

$$Q_{CO} = M_{WF} (H_5 - H_8) \tag{A.21}$$

$$Q_{EV} = M_{WF} (H_{10} - H_9)$$
(A.22)

$$Q_{GE} = M_{WF}H_5 + M_{GE}H_6 - M_{AB}H_4$$
 (A.23)

17. Then the coefficient of performance for a singlestage heat transformer is defined by the following equation:

$$COP = \frac{Q_{AB}}{Q_{GE} + Q_{EV}} \tag{A.24}$$

18. Heat,  $Q_{GE}$  and  $Q_{EV}$ , are provided from some industrial process, which is waste heat (assumption 8); in this case, waste heat delivered by the auxiliary condenser of the water purification processes reduces required heat supply.

19. The work of the pumps is negligible for the calculations.

20. Total delivered heat of the absorber is completely consumed for water purification at atmospheric pressure, which is considered as 1 atm; then this corresponds to 100°C for boiling temperature for impure water (with low salt concentration).