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Stochastic optimization of water pipes for optimal replacement strategy

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ABSTRACT

In a pipeline system, aging of the pipeline due to a variety of internal and external factors reduces its functionality as a water supply system and increases the risk of pipe failure. Failure of aging pipelines leads to greater social and economic damage, thus through proper repair and replacement, the pipeline systems must be managed to ensure safe water quality and structural performance. In this study, the authors propose a methodology for estimating the replacement time to minimize the life-cycle cost of the pipeline systems. It is assumed that failures in the pipeline system are classified into break and destruction. The occurrence probabilities of break and destruction in a pipeline were estimated using the competing deterioration hazard model. The time to break and destruction are explained by using the exponential hazard model and Weibull hazard model, respectively. The optimal replacement strategies are estimated using life-cycle cost approach. In order to evaluate the applicability of the proposed methodology in this study, an empirical analysis was carried out with the actual data of the pipeline system of S city, Korea.

Keywords: Competing deterioration hazard model; Pipe break; Pipe destruction; Life-cycle cost approach; Optimal replacement strategy; Pipeline systems

1. Introduction

Pipelines deteriorate with the lapse of time after installation; along with deterioration, leakage occurs due to cracking or break and eventually, the pipeline can also be completely destroyed. Because pipe failures, break, and destruction cause enormous social and economic damage, system managers repair the damaged pipe and replace the aging pipeline before it is completely destroyed. However, because frequent rehabilitation of aging pipeline increases maintenance cost, an optimal rehabilitation strategy is required to minimize life-cycle cost (LCC), which is a summation of the total social cost and rehabilitation cost. In this study, we predict the probability of occurrence of break and destruction in a pipeline, and develop an optimal replacement strategy model considering need-based repairs of breaks and replacements of destroyed pipes during the life time of the pipeline.

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Predicting the deterioration of a pipeline system and the optimal maintenance strategy based on LCC analysis are necessary for pipeline system management. Until now, several studies on the optimal maintenance strategy in pipeline systems have been reported. Shamir and Howard [1] estimated the optimal replacement time, which minimizes the sum of the repair cost and replacement cost. The repair cost was calculated based on pipe break rate. Following this study, many studies have been carried out based on similar approaches (e.g., Walski and Pelliccia [2], Kleiner et al. [3,4], Kleiner and Rajani [5]). Gustafson and Clancy [6] estimated the break order for optimal replacement time which minimizes the economic loss with a Monte Carlo simulation. Kleiner [7] forecasted the pipe deterioration using a semi-Markov model and estimated the optimal schedule of inspection and renewal of a large infrastructure asset that minimizes the sum of cost of intervention, inspection, and failure. Luong and Fujiwara [8] proposed an optimal repair strategy which determines the priority of repair that maximize net benefit between repair cost and water saving due to repair in the limited budget. Mailhot et al. [9] explained the time to failure between pipe breaks by a hazard function and defined an optimal replacement criterion involving hazard functions. Minimizing the cost function with conditional probabilities to estimate the expected future costs leads to the replacement criterion. Tanaka et al. [10] proposed a mathematical model to estimate optimal renewal time based on the Weibull hazard function and least LCC estimation approach.

These previous studies predict deterioration using a model but without classifying the failure types, and determine an optimal rehabilitation timing. However, in real pipeline systems, diverse types of pipe failures can occur because a pipeline system consists of many components and pipe deterioration can proceed by various factors. Establishing an optimal strategy based on pipe failure types is the building block of asset management for a pipeline system. The maintenance strategies of aging pipeline depend on the deterioration type and level of pipeline condition. Therefore, in this study, the authors briefly classify the pipe failure type due to deterioration into destruction, which requires replacement, break, and repair; these types are assumed to be in competition and the probability of occurrence of each failure type is estimated using a competing deterioration hazard model. In addition, the optimal replacement interval is determined through optimal replacement model. The optimal replacement model demonstrates how to determine an optimal replacement interval for the pipeline with least LCC approach. Moreover, the expected repair cost for pipe breaks occurring during the life cycle of the pipeline was formulated with a great deal of mathematical and stochastic method. The Weibull deterioration hazard model and exponential hazard model are used to address the time to destruction and break of each pipeline, and the model takes into account the nature of the competition between several types of failure by using a competing deterioration hazard model. The competing deterioration hazard model is estimated by a Bayesian technique based on the Metropolis–Hasting method (M–H method), and a Markov chain Monte Carlo method for obtaining a sequence of random samples from a probability distribution for which direct sampling is difficult. The optimal maintenance model proposed in this study builds on a recursive structure, which was proposed by Tamura and Kobayashi [11] and estimated through the least LCC approach.

2. Preassumptions of the model

In this study, we estimate the optimal renewal interval which minimizes the expected LCC in the infinite time base. The maintenance scheme of aging pipelines is set as that whenever pipe break is detected, the damaged pipeline will be repaired, or if destruction is detected, the destroyed pipeline will be immediately replaced. In addition, an aging pipeline which has completed a certain operating time is replaced proactively regardless of whether complete failure occurs. In a pipeline system, we classify the state of a pipeline as being one of the three distinct levels of deterioration, denoted as E_i (i = 0, 1, 2). Level E_0 reflects the pipe is in healthy condition. Level E_1 denotes a state in which leakage due to break is found and immediate repairs are required. Level E_2 reflects that a pipeline has lost its function of water supply because of complete failure. Thus, whenever the condition level E_2 is detected, the damaged pipeline will be immediately replaced by a new one. The repairs for breaks are not regarded as a structural reinforcement of whole pipeline; it is assumed that the repairs are applied to the damaged part. In addition, the breaks may not occur even once or may occur many times during the life time of a pipeline.

3. Pipeline deterioration model

3.1. Modeling strategy

What is important for the maintenance of the infrastructure is to predict the procedure of the deterioration. This plays an important role in estimating the expected social cost and rehabilitation costs over the life cycle of the infrastructure [12]. For this purpose, it is necessary to predict the probability of pipe failure on the basis of available data [13]. In this study, to predict the deterioration of pipe failure types which are in competition, a competing deterioration hazard model [14] is used.

Pipe failures, break, and destruction depend largely on the duration of use of the pipeline, and the hazard function should therefore consider the elapsed time. In this study, the times to break and destruction are used as random variables described by probability density functions and explained by using the exponential hazard model and Weibull hazard model, respectively. The probability density functions correspond to the probability of occurrence of break and destruction. The exponential hazard model and Weibull hazard model, which are suitable for addressing this process, are applied with the assumption that the probability of pipe break and destruction increases with time, respectively, as follows:

$$\lambda_b(\tau) = \gamma_b \tag{1}$$

$$\lambda_d(\tau) = \gamma_d m \tau^{m-1} \tag{2}$$

where m is the acceleration parameter that represents the time dependency of the hazard function and γ_j ($j = b, d$) is the parameter expressing the arrival rate of pipe failure, break, and destruction. It is assumed that γ_j depends on the characteristics of the pipeline, and that it can be expressed as follows:

$$\gamma_j = \exp\left(\mathbf{x}_\mathbf{i} \boldsymbol{\beta}_\mathbf{j}{}'\right) \tag{3}$$

where $\mathbf{x}_\mathbf{i} = (x_i^1, \cdots, x_i^k)$ is the characteristic vector that represents the observed value for pipeline i and $\boldsymbol{\beta}_\mathbf{j} = (\beta_j^1, \cdots, \beta_j^k)$ represents the unknown parameter vectors. In addition, k is total number of covariates and the apostrophe (′) denotes transposition. By using the exponential hazard model and Weibull hazard model, the probability-density function $f_j(\tau)$ and survival function $\tilde{F}_j(\tau)$ can be expressed as shown in Table 1.

3.2. Competing deterioration hazard model

As long as a pipeline is in operation, there is a chance of a pipe failure, break, or destruction. However, it is difficult to distinguish between break and destruction in actual pipe accident cases. Therefore, in this study, it is assumed that the accident that the repair is performed is break, and

Table 1
Equations of probability density, survival, and hazard functions of the exponential and Weibull deterioration models

	Probability density function	Survival function	Hazard function
Exponential	$f_b(\tau) = \gamma_b \exp(-\gamma_b \tau)$	$\tilde{F}_b(\tau) = \exp(-\gamma_b \tau)$	$\lambda_b(\tau) = \gamma_b$
Weibull	$f_d(\tau) = \gamma_d m \tau^{m-1} \exp(-\gamma_d \tau^m)$	$\tilde{F}_d(\tau) = \exp(-\gamma_d \tau^m)$	$\lambda_d(\tau) = \gamma_d m \tau^{m-1}$

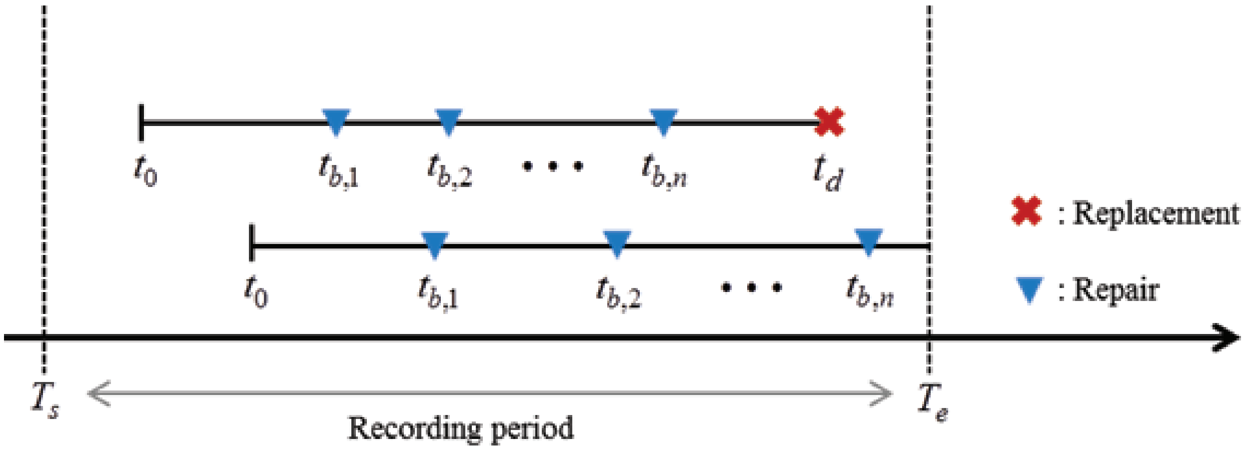

Fig. 1. Pipe failure information of completely observed data scheme.

the accident in which the replacement is performed is the destruction in the recorded pipeline accident history data.

For a single pipe, the time point of pipe installation is denoted by t_0, the time point of the nth break is denoted by $t_{b,n}$, (n = 0, 1,…N) and the point time of the pipe destruction is denoted by t_d. Assuming that the accident history information of the pipeline is completely observed, the data on the break and destruction of the pipeline can be expressed as shown in Fig. 1. We define T_s as the time at the starting of the recording period and T_e as the end of the recording period.

Based on competing risk theory, because pipe replacement due to destruction blocks the occurrence of break, a pipe destruction can be regarded as a competing event of break. Considering this competition, the conditional probability that the observed information, $n(n$ = 0, 1,…$N)$ breaks and destruction, occurs in pipeline i can be represented by the following equation:

$$\ell(d_i,\varepsilon_i,t_i \mid \mathbf{x}_i,\theta) = \left[\left\{\prod_{n=0}^{N} f_b(t^i_{b,n+1}-t^i_{b,n} \mid \mathbf{x}_i,\theta)\tilde{F}_d\left(t^i_{b,n+1} \mid \mathbf{x}_i,\theta\right)\right\}^{\omega_i} \cdot\left\{\prod_{n=0}^{N} f_b(t^i_{b,n+1}-t^i_{b,n} \mid \mathbf{x}_i,\theta)\tilde{F}_d\left(t^i_{b,n+1} \mid \mathbf{x}_i,\theta\right) f_d(t^i_d \mid \mathbf{x}_i,\theta)\right\}^{1-\omega_i}\right]^{\varepsilon_i} \cdot\left[\tilde{F}_b\left(T_e-t_0 \mid \mathbf{x}_i,\theta\right)\tilde{F}_d\left(T_e-t_0 \mid \mathbf{x}_i,\theta\right)\right]^{1-\varepsilon_i} \quad (4)$$

where the ε_i and ω_i are dummy variables. The ε_i receives a value of 1 when pipe failure was encountered and 0 otherwise. In addition, the reported pipe failure type can be represented by the dummy variable ω_i. This variable is 0 when pipe destruction has occurred and 1 otherwise. And, $t_{b,0}$ is equal to t_0. Here, we define the unknown parameter vector for the competing deterioration hazard model as $\theta = (\beta, m), (\beta = (\beta_b, \beta_d))$. Pipe failure of each of the I pipelines is supposed to be mutually independent of the other parts of the pipeline systems. If the observed information of pipeline i is $\xi_i = (\omega_i, \varepsilon_i, t^i_{b,n}, t^i_d, x_i)$, the simultaneous probability density of the pipe deterioration can therefore be expressed by the following likelihood function:

$$L\left(\theta \middle| \xi\right) = \prod_{i=1}^{I} \ell\left(\omega_i, \varepsilon_i, t^i_{b,n}, t^i_d \mid \mathbf{x}_i, \theta\right) \quad (5)$$

where ξ represents $\xi = (\xi_1, \cdots, \xi_n)$. The unknown parameter θ of the proposed model is estimated using the Bayesian estimation method based on the Metropolis–Hastings algorithm. It is assumed that the prior probability density function of unknown parameter, m and β follow a gamma distribution ($m \sim g(m_0, k_0)$) and a conjugate multidimensional normal distribution ($\beta \sim \mathcal{N}_K(\mu_o, \Sigma_o)$), respectively. With this assumption, the probability density function can be further expressed as follows:

$$f(m \mid m_0, k_0) = \frac{1}{\Gamma(m_0)} k_0^{m_0} m^{m_0 - 1} e^{-k_0 m} \quad (6)$$

$$g(\beta \mid \mu_o, \Sigma_o) = \frac{1}{\sqrt{(2\pi)^K |\Sigma_o|}} \cdot \exp\left\{-\frac{1}{2}(\beta-\mu_o)\Sigma_o^{-1}(\beta-\mu_o)'\right\} \quad (7)$$

where $\Gamma(m_0)$ denotes the gamma function and μ_o and Σ_o represent the prior expectation vector and the prior variance–covariance matrix of $\mathcal{N}_K(\mu_o, \Sigma_o)$, respectively. According to

Bayesian theory, the posterior probability density can be expressed as:

$$\pi(\theta \mid \xi) \propto L(\theta \mid \xi) f(m \mid m_0, k_0) g(\beta \mid \mu_o, \Sigma_o) \tag{8}$$

The M–H method is used to carry out sampling from an empirical distribution that is similar to posterior distribution $\pi(\theta \mid \xi)$ and accordingly obtains samples from the original distribution [15]. In addition, a random walk is used to improve the sampling efficiency.

4. Optimal replacement model

With the estimated occurrence probability of each pipe failure type over time, the optimal replacement model is established by considering expected social cost and maintenance cost. The occurrence of a pipe failure $j(j = b(break), d(destruction))$ causes social cost which is denoted by C_j and is assumed to be a constant value. When the predetermined time interval of replacement is set by z, the expected social cost of failure type j is followed in the probabilistic manner via the probability density function $f_j(t)$, as presented in Table 1. Thus, the discounting present value of expected social cost $EC_j(z)$ calculated during replacement period $[0, z)$ can be expressed by the integral form as follows:

$$EC_j(z) = \int_0^z C_j f_j(t) \exp(-\rho t) dt \tag{9}$$

where the coefficient ρ is an instantaneous discounted rate of money over time.

Meanwhile, the cost of pipe replacement activities is denoted by R_d and assumed to be a constant value. It is assumed that a replacement is carried out in case of the occurrence of a pipe destruction during $[0, z)$ or when the age of pipeline reaches time z. The expected replacement cost follows the probabilistic manner via probability density function $f_d(t)$ and the survival probability function $\tilde{F}_d(t)$ (Table 1), when pipe age reaches z. Thus, the present discounted cost of the expected replacement cost for the next predetermined replacement time $EM(z)$ can be expressed as follows:

$$EM(z) = \int_0^z R_d f_d(t) \exp(-\rho t) dt + R_d \tilde{F}_d(z) \exp(-\rho t) \tag{10}$$

The cost of repair activities for pipe break is denoted by R_b and assumed to be a constant value. The repair scheme for break is on a need basis; in other words, it is assumed that when a break occurs, it will be repaired. Suppose that the repair for break is carried out in the arbitrary time y. The present discounted cost of the accumulated expected cost due to pipe break (social and repair cost) from time y to z is denoted by $L(y)$ as shown in Fig. 2. If we consider the possibility of occurrence of next other breaks until time z, when the next repair time is denoted as $y + t$, the $L(y)$ can be calculated by considering $L(y + t)$ caused by next repair as follows:

$$L(y:\tau) = \int_0^{\tau - y} \left\{ C_b + R_b + L(y+t) \right\} f_b(t) \exp(-\rho t) dt \tag{11}$$

where τ is a stochastic variable of replacement time ($0 \le y \le \tau \le z$).

Integral Eq. (11) can be rearranged as follows through a complex solving process which is further explained in Appendix A.

$$L(y:\tau) = \frac{\gamma_b (c_b + R_b)}{\rho} \left[1 - e^{\rho(y-\tau)} \right] \tag{12}$$

Consequently, the present discounted cost of the accumulated expected cost due to pipe break from buried time $t = 0$ to stochastic replacement time τ is denoted as $\tilde{L}(\tau)$ and can be calculated in the following form:

$$\tilde{L}(\tau) = L(0:\tau) = \frac{\gamma_b (c_b + R_b)}{\rho} \left[1 + e^{\rho \tau} \right] \tag{13}$$

Under the strategy of proactive pipeline replacement in the time interval z, it is assumed that whenever a pipe failure j is detected, it will be repaired or replaced immediately. The expected LCC after the next replacement time is estimated as the net present value of social costs and rehabilitation (repair and replacement) costs.

As the social costs and rehabilitation costs are a constant value, the expected LCC takes equal value for every replacement time. In order words, the expected LCC estimated at the next replacement time is equal to the expected LCC estimated at the present replacement. The expected LCC denoted as $LCC(0:z)$ can be regulated through the regression estimation expressed as follows:

$$LCC(0:z) = \int_0^z \left\{ \tilde{L}(\tau) + C_d + R_d + LCC(0:z) \right\} f_d(\tau) \exp(-\rho \tau) d\tau + \left\{ \tilde{L}(z) + R_d + LCC(0:z) \right\} \tilde{F}_d(z) \exp(-\rho z) \tag{14}$$

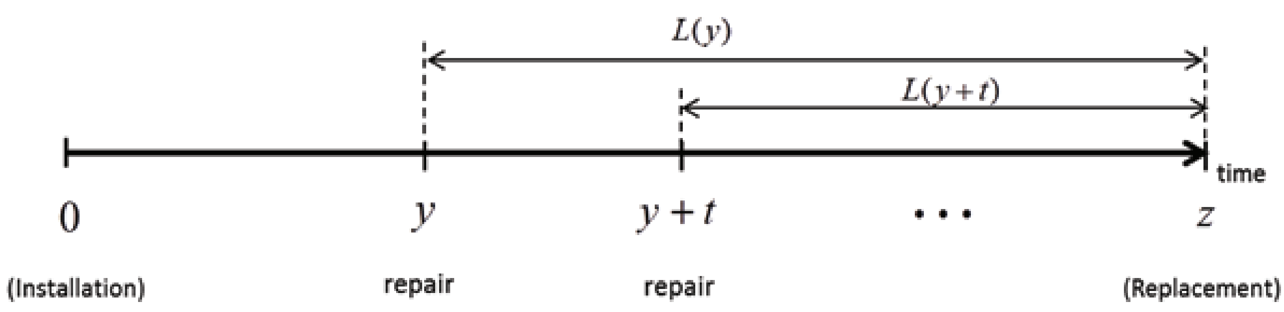

Fig. 2. The accumulated expected cost due to pipe break from time y to z, $L(y)$.

The following two functions $\Lambda_d(z)$ and $\Gamma_d(z)$ are defined as:

$$\Lambda_d(z) = \tilde{F}_d(z)\exp(-\rho z) \tag{15}$$

$$\begin{aligned}\Gamma_d(z) &= \int_0^z f_d(\tau)\exp(-\rho\tau)d\tau \\ &= -\int_0^z \exp(-\gamma_d\tau^m - \rho\tau)d(\gamma_d\tau^m - \rho\tau) - \rho\int_0^z \exp(-\gamma_d\tau^m - \rho\tau)d\tau \\ &= 1 - \Lambda_d(z) - \rho\int_0^z \Lambda_d(\tau)d\tau\end{aligned} \tag{16}$$

With the functions $\Lambda_d(z)$ and $\Gamma_d(z)$, the integral equation part about $\tilde{L}(\tau)$ can be simply rearranged as follows:

$$\begin{aligned}&\int_0^z \tilde{L}(\tau)f_d(\tau)\exp(-\rho\tau)d\tau \\ &= \frac{\gamma_b(c_b + R_b)}{\rho}\left\{1+\exp(\rho z)\right\}\Gamma_d(z) = \Omega\cdot\Gamma_d(z), \\ &\Omega = \frac{\gamma_b(c_b+R_b)}{\rho}\left\{1+\exp(\rho z)\right\}\end{aligned} \tag{17}$$

Substituting Eqs. (15)–(17) into Eq. (14), the following explicit expression for the expected LCC is obtained:

$$LCC(0:z) = \frac{\left(C_d + R_d\right)\Gamma_d(z) + \Omega\cdot\Gamma_d(z) + (\tilde{L}(z) + I_d)\Lambda_d(z)}{\rho\int_0^z \Lambda_d(t)dt} \tag{18}$$

Therefore, the optimal replacement model can be formulated as follows:

$$\Phi(0) = \min_z\left\{LCC(0:z)\right\} \tag{19}$$

where the optimal value function $\Phi(0)$ is denoted as the minimum expected LCC estimated at the initial time.

5. Empirical study

5.1. Overview of the empirical study

In order to evaluate the applicability of the proposed model in this study, the authors carried out an empirical analysis with the actual data of the water distribution system of S city in South Korea. The total length of the distributing pipeline with a diameter of 80 mm or more is approximately 1,000 km. The entire distribution pipeline system is composed of a variety of pipe types, cast iron pipe (CIP), ductile cast iron pipe (DCIP), polyethylene (PE) pipe, polyvinyl chloride (PVC) pipe, steel pipe (SP), and so on. In this study, we focus on the CIP and DCIP to obtain a statistically significant number of data sets. In Korea, the CIP was used mainly as a distributing pipe until the 1990s, and since then, DCIP has been mainly used. Actually, in S city, now, about 90% of the water distributing pipes are DCIP and CIP. In addition, the CIP is not being used since 2003 in S city. Table 2 presents the basic information of the pipe types used in this study.

In this empirical study, we estimate the optimal replacement interval of CIP and DCIP using the optimal replacement model and compared the economic efficiency of the two pipe types. To determine optimal rehabilitation strategy, we should consider not only replacement and repair costs of pipeline but also the costs of damage caused by pipe failure. The social cost C, rehabilitation cost R, and discounted rate ρ play a major role in establishing the optimal replacement strategy in least LCC analysis. The unit cost of pipe replacement is the construction cost required to replace 1 m length of pipe. Table 3 provides the standard construction cost of pipe replacement presented by K-water [16]. Because repair costs for breaks are greatly influenced by location, repair method, and so on, it is difficult to generalize. Therefore, in this study, it is assumed that the repair cost is proportional to the pipe replacement cost. The social cost is the cost of traffic disruption and damage to third parties due to pipe failure. The social costs of break and destruction are calculated by multiplying the repair and replacement costs by the indirect cost factor (ICF). The ICF is assigned to each pipe based on the overlying land use [13]. The discount rate is used by average real discount rate (3.87%) which was calculated by measuring the inflation rate and the interest rate on commercial banks of Korea from 1996 to 2016. In this study, the sensitivity of the optimal replacement model to repair cost, discount rate and social cost assumptions is analyzed.

5.2. Estimation results from pipe deterioration model

The Weibull hazard model used for the Bayesian estimation is specified as follows:

$$\text{Break: } \lambda_b\left(t_i\right) = \exp(\beta_{b0} + \beta_{b1}x_{i1} + \beta_{b2}x_{i2}) \quad (i = 1,\cdots,I) \tag{20}$$

Destruction:

$$\lambda_d\left(t_i\right) = \exp(\beta_{d0} + \beta_{d1}x_{i1} + \beta_{d2}x_{i2})m\cdot t_i^{m-1} \quad (i = 1,\cdots,I) \tag{21}$$

Table 2
Basic information of target pipes

Features	Value					
Material	Ductile cast iron			Cast iron		
Years laid (average age)	From 1957 to 2010 (13 years)			From 1944 to 2003 (27 years)		
Diameter (mm)	80~900			80–800		
Number of pipes	26,577			4,057		
Total length (km)	848.1			72.1		
Number of failures	1,405	Break	833	403	Break	297
		Destruction	572		Destruction	106

Table 3
The unit cost of pipe replacement (K-water [16])

Diameter (mm)	Replacement cost ($/m)	Diameter (mm)	Replacement cost ($/m)
Below 300	315	600	590
300	335	700	716
350	360	800	926
400	399	900	1,094
450	446	1,000	1,317
500	493	1,200	1,818

Table 4
Results of estimation of parameters for the competing deterioration hazard model

	CIP		DCIP	
	Estimated value	Geweke statistics	Estimated value	Geweke statistics
β_{b0}	–3.789 (–4.157, –3.368)	0.057	–4.201 (–4.647, –3.862)	0.069
β_{b1}	–1.393 (–2.103, –0.917)	0.0448	–1.068 (–1.542, –0.554)	0.021
β_{b2}	2.041 (1.362, 2.808)	0.130	2.325 (1.807, 2.851)	0.224
β_{d0}	–10.917 (–11.217, –10.593)	0.061	–11.177 (–11.431, –10.832)	0.116
β_{d1}	–2.527 (–2.906, –2.094)	0.067	–2.501 (–3.106, –2.014)	0.007
β_{d2}	3.299 (2.786, 3.814)	0.169	3.243 (2.712, 3.726)	0.074
m	2.610 (2.490, 2.754)	0.078	2.524 (2.441, 2.580)	0.011

Notes: Values in parenthesis show 95% credible intervals.

The unknown parameter β_{j0} is a constant term, and β_{j1} and β_{j2} represent the pipe diameter and pipe length, respectively. In this study, other characteristic variables that reflect the influence of outer and inner rust, soil unit weight, top traffic volume, and so on were neglected, because data were unavailable. The unknown parameters can be expressed as follows:

$$\boldsymbol{\theta} = (\beta_{b0}, \beta_{b1}, \beta_{b2}, \beta_{d0}, \beta_{d1}, \beta_{d2}, m) \tag{22}$$

To carry out the M–H method, the number of iterations required to reach a steady state was set by $\underline{N} = 10{,}000$ and the number of iterations for parameter sampling was set by $\bar{N} = 20{,}000$. The 10,000 burn-in samples were omitted and the remaining 10,000 parameter samples were used to conduct the estimation.

Table 4 shows the results of estimations by the M–H method. The estimated values are the sample average of parameters, and the values in parentheses refer to 95% credible intervals. Because all the 95% confidence intervals do not contain zero, the estimated results will be significant at the 5% level [17]. The absolute value of the Geweke test statistics are all less than 1.96, so the convergent hypothesis cannot be dismissed at a significance level of 5%. As shown in Table 4, the parameters corresponding to the diameter were obtained as negative and the parameters corresponding to the pipe length as positive values in both CIP and DCIP. It means that small pipe diameter pipes have a higher risk of pipe failure compare with large diameter pipes and long pipes have a higher risk of pipe failure compare with short pipes. These results are in agreement with previous studies. With the estimation results for the competing deterioration hazard model, it is possible to formulate the probability density for each type of pipe failure, break, or destruction.

Figs. 3 and 4 show the cumulative failure probability of the CIP and DCIP types to destruction and break, respectively. The cumulative failure probability curves of the Bayesian estimates and the 95% credible intervals are shown in these figures. The failure probabilities for both break and destruction increase over time. In this study, the time to break and destruction are described by exponential distribution and Weibull distribution, respectively. The cumulative distribution curves shown in Figs. 3 and 4 are obtained from the integration of the probability density function, and equal to the difference between 1 and the survival probability. As can be inferred from these figures, break shows higher failure probability than destruction in both pipe materials. In addition, we could confirm that the failure probabilities of break and destruction in CIP increase more rapidly than those in DCIP over time.

5.3. Optimal replacement interval and expected life cycle cost

Estimation for optimal replacement interval and expected LCC are carried out in the second phase after estimating the competing deterioration hazard model. The occurrence probability of pipe break and destruction are predicted, and the least LCC analysis is subsequently conducted on the basis of a maintenance strategy that repairs of breaks and pipe replacements due to destruction are carried out as needed. In addition, an aging pipeline which has completed a certain operating time is replaced proactively regardless of whether pipe destruction occurs. Minimization problem to seek for

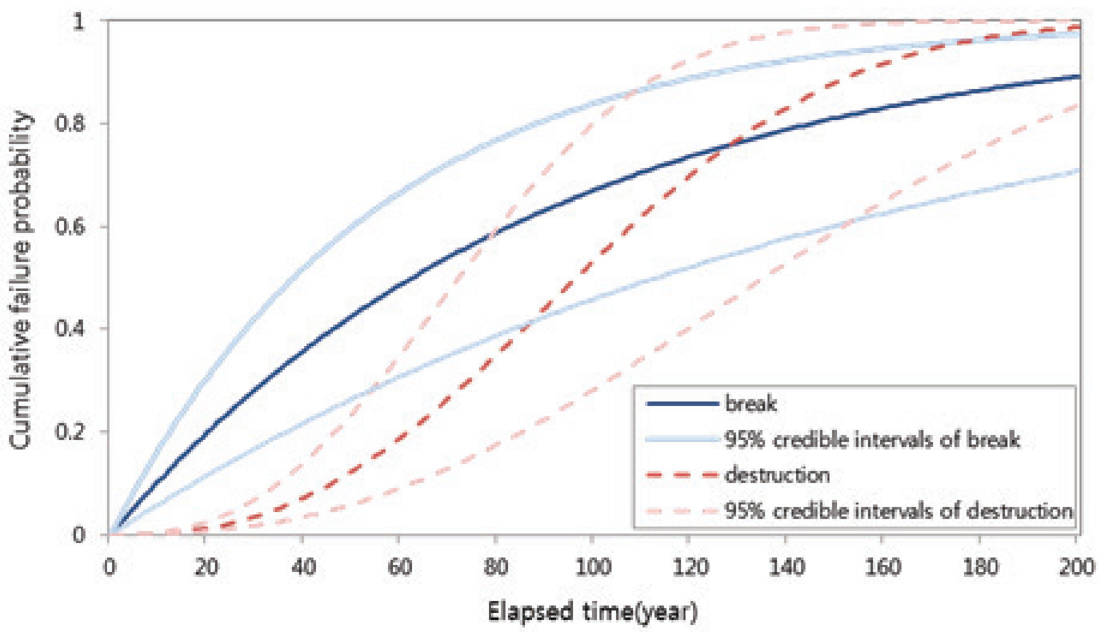

Fig. 3. Cumulative failure probability in DCIP: break and destruction.

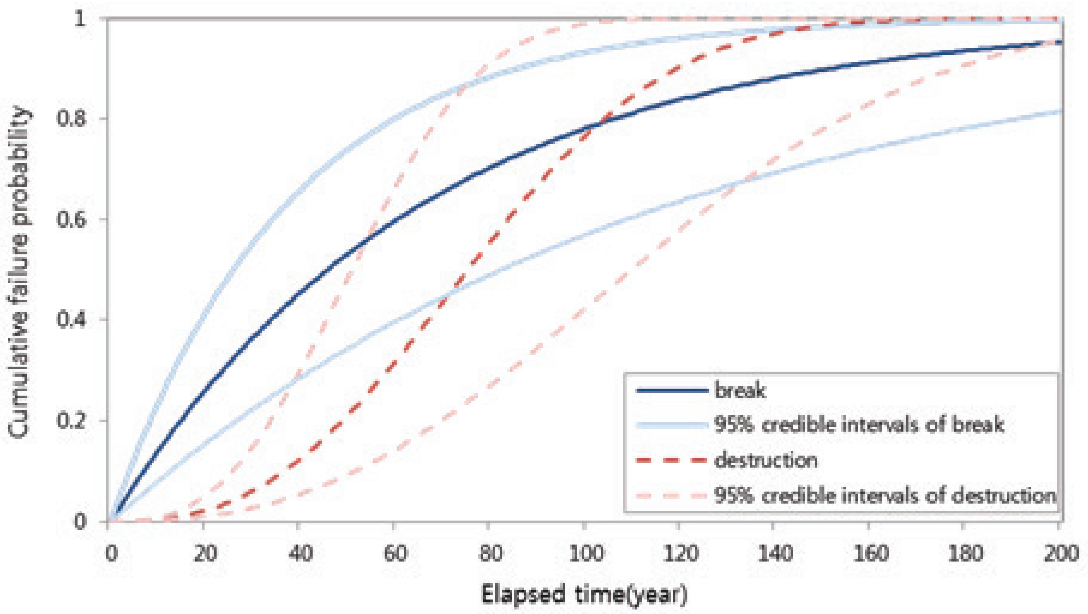

Fig. 4. Cumulative failure probability in CIP: break and destruction.

the optimal replacement timing z is empirically analyzed using Eq. (19).

In this study, it is assumed that the repair cost of the pipe break is proportional to the pipe replacement cost and that the social cost due to pipe failure depends on the overlying land use. Figs. 5 and 6 show the sensitivity of the model to repair cost, discount rate, and social cost assumptions. Fig. 5 shows the change of optimal replacement interval according to the change of ratio of unit repair cost to unit replacement cost (R_b/R_d) of DCIP (diameter: 800 mm and length: 1 m). As can be seen from Fig. 5, the optimal replacement interval is shorter as the repair cost increases. This is because it is more economical to perform the replacement than to repeat the repair with high repair costs.

Since interest rates and inflation rates are not constant every year, the discount rate also fluctuates. Therefore, the change in the optimal replacement interval due to the change in the discount rate is shown in Fig. 5. As shown in this

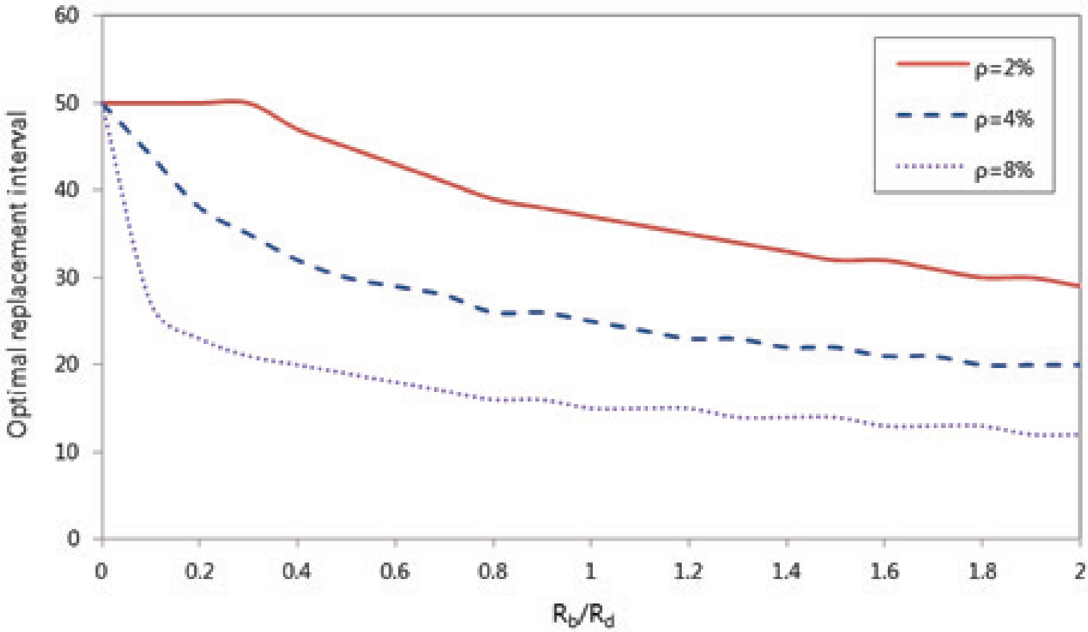

Fig. 5. Optimal replacement interval according to R_b/R_d for different values of the discount rate ρ.

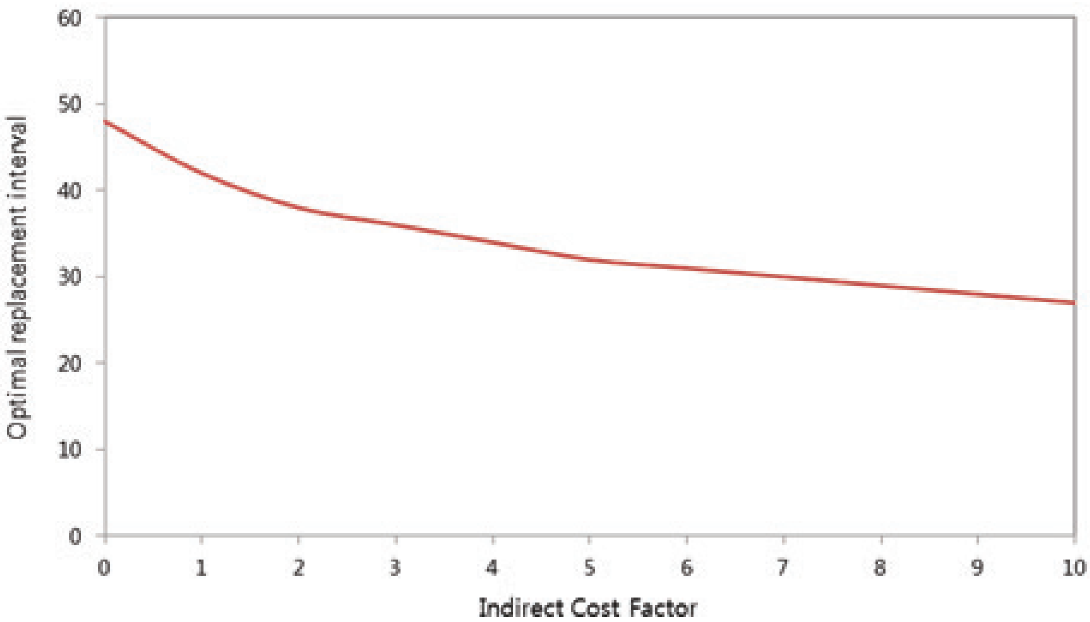

Fig. 6. Optimal replacement interval according to the change of social cost due to pipe break.

figure, the optimal replacement interval increases as the discount rate decreases.

Pipe failure causes direct costs from repair or replacement, as well as indirect costs (social costs) such as traffic disruption and damage to third parties. In this study, the social cost of break and destruction was calculated by multiplying the repair cost and replacement cost by the indirect cost index, respectively. The social cost of pipe failure depends on the type of land in which the pipe is buried. Fig. 6 shows the change of optimal replacement interval according to the change of social cost due to pipe break. As can be seen in Fig. 6, the larger the indirect cost index (i.e., the higher the social cost), the shorter the pipe replacement interval. In other words, because the social cost caused by repeated pipe breaks is large, it is economical to carry out pipe replacement early.

Figs. 7 and 8 show the change of LCC by the diameter according to the replacement interval of the unit length DCIP

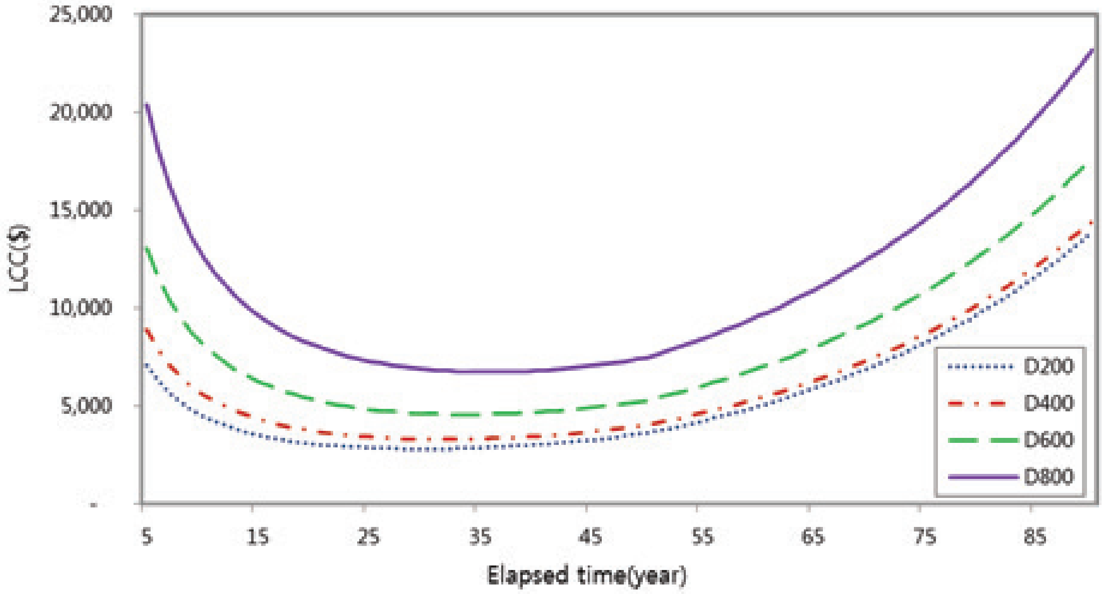

Fig. 7. Expected life cycle cost comparison of pipe diameter: DCIP.

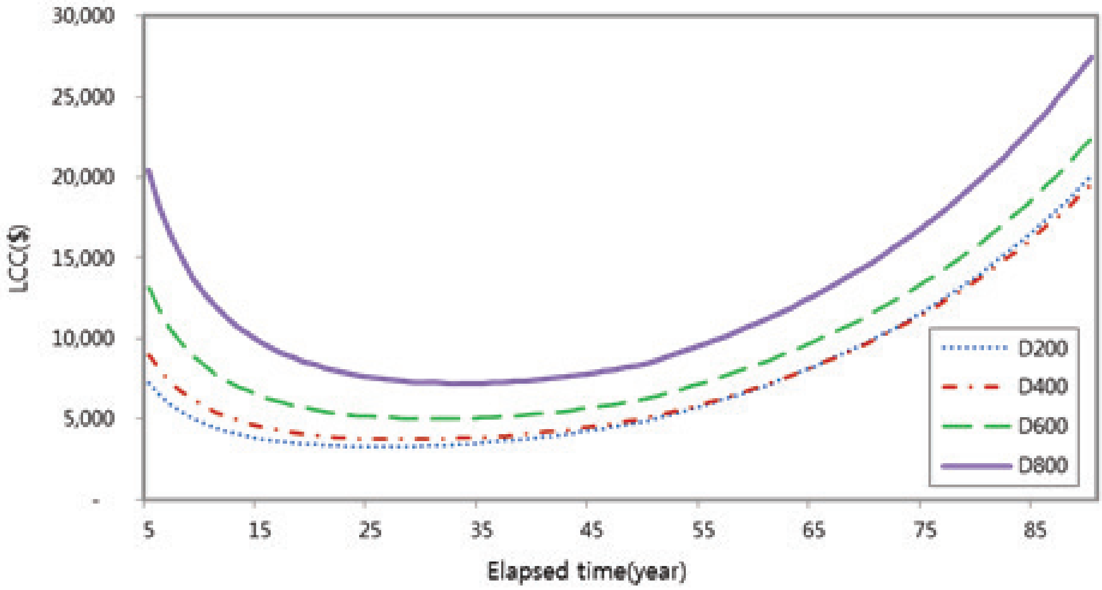

Fig. 8. Expected life cycle cost comparison of pipe diameter: CIP.

and CIP, based on the optimum replacement model. Here, the ICF for social cost calculation is assumed to be 3. In addition, the discount rate and R_b/R_d are assumed to be 3.87% and 0.3, respectively.The expected failure costs tend to increase over time due to the increase in failure probability. On the other hand, the expected maintenance costs tend to decrease with time due to discounting. Thus, the total expected LCC forms a convex curve over time as shown in Figs. 7 and 8.

As shown in Figs. 7 and 8, pipes of larger diameters have a high LCC and a long optimal replacement interval *z*. This is because larger pipes show low failure probability and are more expensive to replace and repair. In addition, on comparing Figs. 7 and 8, it is possible to verify that DCIP shows a low LCC for the same diameter as the CIP and the optimal replacement interval of DCIP is longer than CIP. The results of least life cycle analysis for each pipe diameter and material are presented in Table 5. From the results, we can confirm that the DCIP is more economical than CIP.

6. Conclusions

Pipe failures cause social and economic loses as well as inconvenience to consumers. Pipeline systems are underground, and it is difficult to inspect and monitor

Table 5
Results of least life cycle cost analysis and optimal replacement interval

	CIP		DCIP	
	Optimal replacement interval (z, year)	Optimal LCC ($/m)	Optimal replacement interval (z, year)	Optimal LCC ($/m)
D200	26	3,274	30	2,829
D400	28	3,721	32	3,320
D600	31	4,995	34	4,580
D800	33	7,198	36	6,748

pipe condition. Thus, in many cases, the maintenance of pipeline systems depends on a manager's empirical judgment. Therefore, it is required to predict pipe deterioration by accumulated inspection data and to establish optimal maintenance strategy which minimizes expected LCC. In real pipeline systems, there are various types of failure and maintenance strategies depending on the failure type. In this study, pipe failure is briefly classified into destruction, which requires replacement, and break, which requires repair. The deterioration procedures of destruction and break are forecasted by using a competing deterioration hazard model. In addition, we proposed an optimal replacement model which minimizes expected LCC that considers repair of breaks and pipeline replacement during life time of pipeline.

The empirical application of the proposed model was carried out on a real pipeline system in S city in Korea. The occurrence probabilities of break and destruction in CIP and DCIP were estimated using competing deterioration hazard model. From the estimation results, we could confirm that break shows higher occurrence probability than destruction in both pipe materials. In addition, break and destruction in CIP occur at a higher rate than those in DCIP over time. With these estimation results of competing deterioration hazard model, the optimal replacement interval and expected LCC can be calculated using optimal replacement model. However, considering that the optimal replacement model depends largely on the accuracy of the deterioration prediction, competing deterioration hazard model established in this study needs to be further advanced considering the environment and operational factors that affect the deterioration of the pipeline.

The sensitivity of the optimal replacement model to repair cost, discount rate, and social cost assumptions is analyzed. It was confirm that as the ratio of unit repair cost to unit replacement cost decreases, the optimal replacement interval increases. This suggests that the improvement of pipe repair techniques such as non-excavation repair methods can make the pipe longer to use as the repair cost is lowered. In addition, it was confirmed that the higher increase social costs of pipe break reduced the optimal replacement interval. This indicates that in areas such as commercial areas and major roads where social costs are high due to pipe break, the replacement interval needs to be shorter than other areas in view of the risk of pipe break.

In the empirical study, we could obtain the optimal LCC and optimal replacement interval of each pipe type and diameter using optimal replacement model. The results demonstrated that the DCIP is more a beneficial type of pipe than CIP in asset management of the pipeline system. However, it is considered that these results can be reliably secured if the exact unit repair cost, social cost, and appropriate discount rate considering discount rate change are properly calculated.

Although in-depth analysis is required, this study successfully explains how to determine the optimal replacement strategy for the pipeline. Moreover, it is noteworthy that mathematical and stochastic expressions of expected repair costs for breaks occurring during the life cycle of the pipeline. From the application view point, we believe that our model could be extended to other components of infrastructure and would contribute to advancing asset management.

Our proposed model has not discussed the following points, which are considered for a future extension of our study:

- It is required to establish optimal replacement strategy which considers pipe material and diameter change. Furthermore, the optimal strategy has to not only be economical but also has to meet hydraulic condition of pipeline.
- Water supply is performed with not a single pipe but pipe networks. Because the budget for pipeline system management is limited, it is necessary to determine the pipe replacement priority. In addition, the replacement priority strategy has to consider the significance of pipe and network properties.

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Appendix A

The steps for solving the integral Eq. (11) are described here.

Let, $z+t=\tau$ then, $dt=d\tau$ and $t=\tau-z$. (A1)

The Eq. (11) can be arranged as follows:

$$L(z)=\int_0^{T-z}\{c_b+R_b+L(z+t)\}\cdot\gamma_b\exp(-\gamma_b t)\cdot\exp(-\rho t)dt$$
$$=-\int_T^z\{c_b+R_b+L(\tau)\}\cdot\frac{d}{d\tau}\left[-\exp(\gamma_b(z-\tau))\right]\cdot\exp(\rho(z-\tau))d\tau \quad (A2)$$

Let $\mu(z)=c_b+R_b+L(z)$, $\mu(\tau)=c_b+R_b+L(\tau)$ and $K(z,\tau)=\frac{d}{d\tau}\left[-\exp(\gamma_b(z-\tau))\right]\cdot\exp(\rho(z-\tau))$ as a kernel function. Then, the $\mu(z)$ can be expressed as follows:

$$\mu(z)=c_b+R_b-\int_T^z\mu(\tau)k(z,\tau)d\tau=c_b+R_b+\left[\int_0^T-\int_0^z\right]\mu(\tau)k(z,\tau)d\tau \quad (A3)$$

Let, $g(z)=c_b+R_b+\int_0^T\mu(\tau)k(z,\tau)d\tau$ then, $\mu(z)$ can be simplified as follows:

$$\mu(z)=g(z)-\int_0^z\mu(\tau)k(z,\tau)d\tau \quad (A4)$$

Because $k(z,\tau)$ has the form $k(z-\tau)$, the integral form can be transformed by the convolution of $\mu(\tau)$ and $k(z)$.

$$\int_0^z\mu(\tau)k(z,\tau)d\tau=\int_0^z\mu(\tau)k(z-\tau)d\tau=\mu(z)\times k(z) \quad (A5)$$

Thus, $\mu(z)$ can be expressed as follows:

$$\mu(z)=g(z)-\mu(z)\times k(z) \quad (A6)$$

On taking the Laplace transform of Eq. (A6), the following expression can be obtained:

$$La\left[\mu(z)\right]=La\left[g(z)-\mu(z)\times k(z)\right]=\frac{La\left[g(z)\right]}{1+La\left[k(z)\right]} \quad (A7)$$

Since, $\mu(z)=c_b+R_b+L(z)$

$$L(z)=La^{-1}\left[\frac{La\left[g(z)\right]}{1+La\left[k(z)\right]}\right]-c_b-R_b=\frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty}\frac{g(s)}{1+k(s)}e^{zs}ds-c_b-R_b \ :$$

Bromwich integral (A8)

where $g(s)=La\left[g(z)\right]$ and $k(s)=La\left[k(z)\right]$.

Here, the $g(z)$ and $k(z)$ are represented again as follows:

$$g(z)=c_b+R_b+\int_0^T\mu(\tau)k(z,\tau)d\tau=c_b+R_b+\int_0^T[c_b+R_b+L(\tau)]\gamma_b\exp((z-\tau)(\gamma_b+\rho))d\tau$$

$$k(z)=\gamma_b\exp(z(\gamma_b+\rho))$$

With Laplace transform, $k(s)$ can be expressed as follows:

$$k(s)=La[k(z)]=\int_0^\infty\gamma_b\,e^{z(\gamma_b+\rho)}\,e^{-sz}dz=\left[\frac{\gamma_b}{\gamma_b+\rho-s}e^{z(\gamma_b+\rho-s)}\right]_0^\infty$$
$$=-\frac{\gamma_b}{\gamma_b+\rho-s} \quad (A9)$$

From the definition of Laplace transform, the condition $s>\gamma_b+\rho$ is satisfied, and hence Eq. (A9) can be arranged as follows:

$$1+La(k(z))=\frac{\rho-s}{\gamma_b+\rho-s}=\frac{s-\rho}{s-(\gamma_b+\rho)} \quad (A10)$$

Meanwhile, $g(s)$ can be expressed as follows:

$$g(s)=La(g(z))=\int_0^\infty e^{-sz}\left[c_b+R_b+\int_0^T(c_b+R_b+L(\tau))\gamma_b\,e^{(\gamma_b+\rho)(z-\tau)}d\tau\right]dz$$
$$=\int_0^\infty(c_b+R_b)e^{-sz}dz+\int_{\tau=0}^T\int_{z=0}^\infty(c_b+R_b+L(\tau))\gamma_b e^{(\gamma_b+\rho-s)z}e^{-\tau(\gamma_b+\rho)}dzd\tau \quad (A11)$$

Interchanging the integration order $d\tau dz$ to $dzd\tau$, Eq. (A11) can be arranged as follows:

$$La(g(z))=\left[\frac{-(c_b+R_b)}{s}e^{-sz}\right]_{z=0}^\infty$$
$$+\int_{\tau=0}^T\left[(c_b+R_b+L(\tau))\frac{\gamma_b}{\gamma_b+\rho-s}e^{(\gamma_b+\rho-s)z}\right]_{z=0}^\infty e^{-\tau(\gamma_b+\rho)}d\tau$$
$$=\frac{(c_b+R_b)}{s}+\int_0^T(c_b+R_b+L(\tau))\frac{-\gamma_b}{\gamma_b+\rho-s}e^{-\tau(\gamma_b+\rho)}d\tau \quad (A12)$$

Because $g(0)=c_b+R_b+\int_0^T(c_b+R_b+L(\tau))\gamma_b e^{-\tau(\gamma_b+\rho)}d\tau$, Eq. (A12) can be arranged as follows:

$$La(g(z))=\frac{(c_b+R_b)}{s}-\left(\frac{g(0)-(c_b+R_b)}{\gamma_b+\rho-s}\right) \quad (A13)$$

Therefore, with Eqs. (A10) and (A13),

$$\frac{La(g(z))}{1+La(k(z))}e^{sz}=\left(\frac{s-\rho-\gamma_b}{s-\rho}\right)\left[\frac{(c_b+R_b)}{s}-\left(\frac{g(0)-(c_b+R_b)}{\gamma_b+\rho-s}\right)\right]e^{sz} \quad (A14)$$

Since, $\mu(z)=c_b+R_b+L(z)=g(z)-\int_0^z\mu(\tau)k(z-\tau)d\tau$ the following condition is satisfied.

$$\mu(0)=c_b+R_b+L(0)=g(0)-0$$

$$\therefore g(0)-(c_b+R_b)=L(0) \quad (A15)$$

Substituting Eq. (A15) in Eq. (A14), the following result is obtained.

$$\frac{La(g(z))}{1+La(k(z))}e^{sz}=\left(1-\frac{\gamma_b}{s-\rho}\right)\left[\frac{(c_b+R_b)}{s}+\frac{L(0)}{s-(\gamma_b+\rho)}\right]e^{sz}$$
$$=e^{sz}\left[\frac{(c_b+R_b)+\frac{\gamma_b(c_b+R_b)}{\rho}}{s}+\frac{L(0)-\frac{\gamma_b(c_b+R_b)}{\rho}}{s-\rho}\right] \quad (A16)$$

Let, $h(s)=\dfrac{La(g(z))e^{sz}}{1+La(k(z))}$ and let, $h_1(s)=e^{sz}\left(\dfrac{c_b+R_b+\frac{\gamma_b(c_b+R_b)}{\rho}}{s}\right)$, $h_2(s)=e^{sz}\left(\dfrac{L(0)-\frac{\gamma_b(c_b+R_b)}{\rho}}{s-\rho}\right)$.

Since,
$h(s)=h_1(s)+h_2(s)$, $\sum\mathrm{Res}(h(s))=\sum\mathrm{Res}(h_1(s))+\sum\mathrm{Res}(h_2(s))$

$$\sum\mathrm{Res}(h_1(s))=c_b+R_b+\frac{\gamma_b(c_b+R_b)}{\rho} \tag{A17a}$$

$$\sum\mathrm{Res}(h_2(s))=e^{\rho z}\left(L(0)-\tfrac{\gamma_b(c_b+R_b)}{\rho}\right) \tag{A17b}$$

Eq. (A8) can be arranged by Jordan's lemma:

$$\begin{aligned}L(z)&=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}h(s)ds-c_b-R_b\\&=\left[\frac{1}{2\pi i}(2\pi i)\sum\mathrm{Res}(h(s))\right]-c_b-R_b\\&=\frac{\gamma_b(c_b+R_b)}{\rho}(1-e^{\rho z})+L(0)e^{\rho z}\end{aligned} \tag{A18}$$

Here, since, $L(T)=0=\dfrac{\gamma_b(c_b+R_b)}{\rho}(1-e^{\rho T})+L(0)e^{\rho T}$ the $L(0)$ can be obtained by:

$$L(0)=\frac{\gamma_b(c_b+R_b)}{\rho}(1-e^{-\rho T}) \tag{A19}$$

Therefore, substituting Eq. (A19) in Eq. (A18), we get $L(z)$ as follows:

$$\begin{aligned}L(z)&=\frac{\gamma_b(c_b+R_b)}{\rho}\left[1-e^{\rho z}+e^{\rho z}-e^{\rho(z-T)}\right]\\&=\frac{\gamma_b(c_b+R_b)}{\rho}\left[1-e^{\rho(z-T)}\right]\end{aligned} \tag{A20}$$