Desalination and Water Treatment
www.deswater.com
doi: 10.5004/dwt.2018.22499

121 (2018) 244–249
July

Application of set pair analysis model based on maximum similarity rules in groundwater level prediction

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Received 24 February 2018; accepted 11 May 2018

ABSTRACT

Influenced by combined effects of climate change and human activities, groundwater system shows varieties of uncertain properties, such as stochastic, fuzzy, gray, unascertained and chaotic characteristics. Based on shortages of distributed parameter model and lumped parameter model in groundwater level prediction, considering current researches of time series application with set pair analysis, groundwater level prediction model was established with set pair analysis based on rules of maximum similarity forecast (SPA-MSF). On the basis of theory of similarity forecast, maximum connection degree is used to measure the similarities among historical samples of groundwater level quantitatively with time series consistency analysis. Steps of modeling and solving a five-element connection degree model for groundwater level prediction was applied in monthly and inter-annual depth forecast at Shandanqiao observation well, Zhangye Heihe River basin. The results indicated that goodness of fit and trend between predictive value and measured value were optimal. Besides, the SPA-MSF model was proved to be of high prediction accuracy and better generalization with posterior variance test.

Keywords: Set pair analysis; Five-element connection number model; Maximum similarity rules; Groundwater level prediction; Posterior variance test

1. Introduction

The dynamic prediction of groundwater level refers to forecasting of any possible changes of water level in a specific period in the future based on the change of measured data and the evolution of groundwater system and combined with theories and methods of hydrology and hydrogeology, in a bid to enable the system decisions more scientific and proactive and to have important theoretical and practical values. Currently there are many methods to predict the dynamic changes of groundwater level, and they are generally divided into the distributed model based on finite difference method, finite element method and other numerical methods and the lumped model based on nonlinear time series prediction theories including artificial neural networks model, threshold autoregressive model and chaos theory. It should be noted that, the distributed model breaks the restrictions of isotropic homogeneous aquifer, structural rule and infinite distribution required in analytic method to some extent, but the model building needs to be supported by a large amount of hydrogeological data, and the model is strongly sensitive to initial conditions and boundary conditions, adding uncertainty to the predicting results of the

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Presented at the 3rd International Conference on Recent Advancements in Chemical, Environmental and Energy Engineering, 15–16 February, Chennai, India, 2018

1944-3994/1944-3986

model; while as for the nonlinear method for dynamic prediction of groundwater level with time as univariate element, the determination of time delay τ and embedded dimension m in phase-space reconstruction [1], the optimal design of network parameters and network structure in BP neural network model [2], and the selection of a priori knowledge and suitable kernel function in support vector machines [3] also affect the prediction accuracy of the model.

The set pair analysis (SPA) theory, as a new method for analysis of uncertainties of hydrology and water resources, provides a new approach for dynamic prediction of groundwater level. However, less literature is about the application of SPA to time series model prediction, and traditional researches on the application of SPA to time series prediction mainly concentrate on the coupling of neural network model, autoregressive model and other non-linear theories [4–10]. In addition, the model structure and computational process are relatively complicated, and impose certain limitations to the time series that is short of long-term measured data and that the impact factor relation is complicated and uncertain, and the annual dynamic prediction of groundwater level has not been involved in any research. This paper attempts to introduce the SPA theory to the dynamic prediction of groundwater level and explores the groundwater level prediction based on SPA and maximum similarity forecast (SPA-MSF), to provide new research ideas for dynamic prediction of groundwater level.

2. Basic principles of set pair analysis

The SPA method, first put forward by Chinese Scholar Zhao Keqin on the basis of unity of opposites and universal relations in philosophy at the National Conference on Systems Theory and Regional Planning in Baotou, Inner Mongolia in 1989, is a new method to analyze uncertainty relations.

The core of the SPA: construct a set pair for two related sets in an uncertain system, analyze the identity, difference and contrariety of a particular attribute of the set pair, and describe the identical discrepancy contrary (IDC) relations of the set pair with the connection degree. Supposed that the related sets A and B form a set pair $H(A,B)$, the connection degree used to describe $H(A,B)$ relations is defined as follows:

$$\mu_{A\sim B} = \frac{S}{N} + \frac{F}{N}I + \frac{P}{N}J = a + bI + cJ \quad (1)$$

where S is the number of identity, S/N is the identical degree, abbreviated as a; F is the number of difference, F/N is the different degree, abbreviated as b; P is the number of contrariety, P/N is the contrary degree, abbreviated as c; $S + F + P = N$; I is the difference coefficient, and can be assigned to a value within [–1, 1]; J is contrariety coefficient, $J \equiv -1$.

From the certain–uncertain system theory, the connection degree has a hierarchical and deployable structure. Therefore, the extensibility of a, bI, cJ can derive K element connection degree on the basis of commonly-used connection degree:

$$\mu_{A\sim B} = a_1 + a_2 + b_1 I_1 + b_2 I_2 + \ldots + b_{K-4} I_{K-4} + c_1 J_1 + c_2 J_2 \quad (2)$$

Generally the above formula can be simplified as the following when the identity and contrariety structures are not refined:

$$\mu_{A\sim B} = a + b_1 I_1 + b_2 I_2 + \ldots + b_{K-2} I_{K-2} + cJ \quad (3)$$

where $a + b_1 + b_2 + \ldots + b_{K-2} + c = 1$; $b_1, b_2, \ldots, b_{K-2}$ represent difference components; $I_1, I_2, \ldots, I_{K-2}$ represent component factors with uncertain difference.

3. SPA-MSF modeling for dynamic prediction of groundwater level

Similar prediction method is a kind of non-parameter prediction method that aims to find one or more samples that are most similar to the present sample as the pre-set forecasting results under the principle of "like causes produce like results". Use the IDC of SPA to measure the similarity among historical samples of groundwater level time series, and make the weighted average of groundwater level in several most similar historical samples as the pre-set predicted value of groundwater level; thus, the similar prediction model of groundwater level based on SPA is built.

Supposed that the groundwater level time series is $Y = \{x(t)\}$ ($t = 1, 2, \ldots, n$); considering the delayed response of dynamic evolution of groundwater level in aquifer system, the groundwater level $x(t)$ in the t year depends on its front m sample values $x(t-1)$, $x(t-2)$, …, $x(t-m+1)$, $x(t-m)$, and the predicted value of the defined set $A_t = (x(t), x(t+1), \ldots, x(t+m-1))$ ($t = 1, 2, \ldots, n-m$) is the subsequent value $x(t+m)$ of A_t. It can be simply interpreted as: the predicted value $x(t+m)$ of groundwater level in the $(t+m)$ year, as a dependent variable, is the function of m independent variables (impact factor) in the set A_t:

$$x(t+m) = f(x(t),\ x(t+1), \ldots,\ x(t+m), x(t+m-1)) \quad (4)$$

While for the subsequent predicted value of groundwater level $x(n+1)$ in the $(n+1)$ year, the SPA based on the maximum similarity for dynamic prediction of groundwater level can be adopted to analyze the identical degree, different degree and contrary degree between the set $A_{n+1} = (x(n-m+1), x(n-m+2), \ldots, x(n-1), x(n))$ and A_t. Here are the modeling processes:

Step 1: Build sets A_1, A_2, A_{n-m} and A_{n+1} and their subsequent values $x(m+1)$, $x(m+2)$, …, $x(n)$ and $x(n+1)$, as shown in Table 1. m value cannot be made too small, otherwise it cannot objectively reflect the lag effect of dynamic changes of groundwater level; m value cannot be made too large either, otherwise, the non-linear relation of groundwater level time series cannot be accurately expressed, and meanwhile it can lead to strong dependency among variables. m value is recommended to be within 4–6.

Step 2: Convert all elements in sets $A_1, A_2, \ldots, A_{n-m}$, and A_{n+1} into corresponding symbols, respectively, according to a certain classification criterion. This paper adopts mean-standard deviation method to calculate the mean $\bar{X}_j$ and standard deviation S_j of the same element (impact factor) $x_j(t)$ $j = (1, 2, \ldots, m)$ in a set, and then classifies the elements in the set A_t into I, II, III, IV and V which correspond to intervals $[0, \bar{X}_j - 0.9S_j]$, $[\bar{X}_j - 0.9S_j, \bar{X}_j - 0.3S_j]$, $[\bar{X}_j - 0.3S_j, \bar{X}_j + 0.3S_j]$, $[\bar{X}_j + 0.3S_j, \bar{X}_j + 0.9S_j]$ and $[\bar{X}_j + 0.9S_j, +\infty]$, respectively.

Step 3: Quantize with symbol the set $A_{n+1} = (x(n-m+1), x(n-m+2), \ldots x(n-1), x(n))$ that the predicted groundwater level $x(n+1)$ corresponds to in the $(n+1)$ year according to the established classification criterion. Use set A_{n+1} and $(n-m)$ sets A_t to build set pair $H(A_{n+1}, A_t)$ $t = (1, 2, \ldots, n-m)$, respectively, and analyze the IDC of elements in sets $A_{n+1} - A_t$. Count the number of identical symbols (identity) as S; count the number of different symbols with one level (partial difference) as F_1; count the number of different symbols with two levels (whole difference) as F_2; count the number of different symbols with three levels (inverse difference) as F_3; count the number of different symbols with four levels (contrariety) as P. Calculate the connection degree $\mu_t(A_{n+1}, A_t)$ of set pairs $H(A_{n+1}, A_t)$.

Step 4: Determine the correlate $\mu_t(A_{n+1}, A_t)$ of set pair $H(A_{n+1}, A_t)$. Assign suitable values to factors I_1, I_2, I_3, with uncertain difference. In this paper, different factors are assigned to –0.5, 0, 0.5 by using particular value method; the contrariety factor is $J = -1$.

Step 5: Arrange correlates $\mu_t(A_{n+1}, A_t)$ in descending order. Agreed as follows: the larger the correlate is, the higher the similarity between set A_{n+1} and set $A_t = (x(t), x(t+1), \ldots, x(t+m-1))$ $(t = 1, 2, \ldots, n-m)$ is; and vice versa. Select the weighted average of subsequent values of K similar sets as the predicted value of groundwater level $x(n+1)$ in the $(n+1)$ year.

$$x(n+1) = \frac{1}{K}\sum_{k=k_1}^{k_K} \omega_k x(m+k) \tag{5}$$

where ω_k is the weight of K similar sets to the set A_{n+1}, and equals to the ratio of the mean value of elements in set A_{n+1} to the mean value of elements in set A_t.

4. Case study

4.1. Overview of study area

Heihe River, originating from the middle part of Qilian Mountain, is the second largest inland river in arid and semi-arid region of Northwest China. It is about 821 km and flows into east and west Juyan Lake basins [11]. Shandanqiao observation well is located in Zhangye basin, nearby the junction of Heihe River (main stream) and Shandanhe River, major tributary of Heihe River. The groundwater is low in buried depth with great withdrawal; the groundwater level is largely affected by human exploration and dynamically changes upon runoff and mining quantity. To fully demonstrate the effectiveness of SPA-MSF model in predicting groundwater level, this thesis chooses monthly data of shallow groundwater level of Shandanqiao observation well during 1981–2004 from Zhangye Hydrographic and Water Resources Survey Bureau to build time series, and uses time-series data from 1981 to 2002 to build SPA-MSF model and from 2003 to 2004 to verify SPA-MSF model.

4.2, Dynamic prediction of groundwater depth

4.2.1. Consistency analysis of time series

The premise of similarity forecast model of groundwater depth based on SPA is that the similar condition will generate similar result. Thus, the SPA-MSF model must be built with consistency analysis of time series data. The overall trend of buried depth over time is shown in Fig. 1 and the trend line slanting rate is -6.7×10^{-4} according to linear regression analysis of time series of monthly groundwater depth at Shandanqiao observation well during 1981–2002. The result shows that groundwater depth series at Shandanqiao observation well achieves a dynamic balance with good consistency.

In the meanwhile, change-point locations at Yingluoxia Station, a major hydrologic control station dividing the upstream and midstream of Heihe River trunk, during long-series runoff process can be a basis data of dynamic groundwater depth in Zhangye basin along Heihe River [12]. A study by Zou et al. [12] adopts both residual mass curve and rank test to identify the changing points of groundwater depth along Heihe River during 1947–2006 and then verify their veracity by Brown-Forsythe method, sequential clustering method and Slide-F-Test. The mathematical statistics shows that 1959 and 1979 are the time points when hydrological run-off of Heihe River changed.

Therefore, it keeps a consistency in time series when choosing monthly shallow groundwater depth data during 1981–2002 to build SPA-MSF model and to predict the monthly groundwater depth during 2003–2004. It meets the premise of similarity forecast and is feasible in theory.

4.2.2. Intra-annual dynamic prediction of groundwater depth

SPA-MSF model for monthly groundwater depth values in 2003 is built on monthly time series of 1981–2002; SPA-MSF model for monthly groundwater depth values 2004 is built on monthly time series of 1981–2003 and particularly, the monthly groundwater depth data in 2003 should be the measured values so as to maximize the effectiveness of measured value. Limited by the article length, this thesis takes the groundwater depth in January 2004 as an example to specify the prediction process of groundwater level based on SPA and maximum similarity forecast.

1. Use January data during 1981–2003 to build 18 sets (A_1~A_{18}) and current Set A_{n+1} (A_{19}) (Table 1). For example,

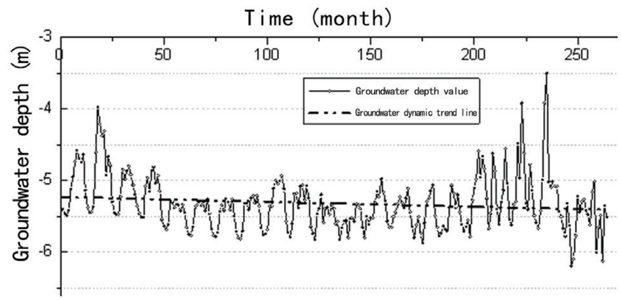

Fig. 1. Time series of groundwater depth at Shandanqiao observation well (1981–2002).

Set A_1 = (5.40, 5.37, 5.30, 5.36, 5.54) corresponds to January data during 1981–1985, and the subsequent value $x(6)$ = 5.66 corresponds to groundwater depth value of January in 1986, and so on.

2. Classify and symbolize the same element $x_j(t)$ upon level in Set A_t(t = 1, 2, …, 18) through mean-standard deviation method. See Table 2 for the calculation results of mean value $\bar{X}_j$ and standard deviation S_j of $x_j(t)$.
3. Symbolize the current Set $A_{n+1}(A_{19})$ = (5.54, 5.49, 5.43, 5.26, 5.36) to A_{19} = (4, 3, 3, 1, 2) according to the classification criteria established based on the table. Then, respectively, set pair of $A_{19}$with A_1, A_1, …, A_{18} and calculate the connection number. For example, the detailed calculation process of connection number of Set A_{19} and Set A_{12} is: A_{12} = (5.60, 5.39, 5.29, 5.44, 5.48) and symbolize it to A_{12} = (4, 2, 2, 3, 3). Compare and analyze the similarities and differences of matching elements in set pair $H(A_{19}, A_{12})$, and then the number of identical symbols (identity), symbols with one-level difference (partial difference), symbols with two-level difference (whole difference), symbols with three-level difference (inverse difference) and symbols with four-level difference (contrariety) are, respectively, 1, 3, 1, 0 and 0. Finally calculate the connection degree of set pair $H(A_{19}, A_{12})$ and the result is $\mu_t(A_{19}, A_{12}) = \frac{1}{5} + \frac{3}{5}I_1 + \frac{1}{5}I_2 + 0 \cdot I_3 + 0 \cdot J$.
4. Set I_1 = 0.5, I_2 = 0, I_3 = –0.5 and J = –1 and then calculate connection degree $\mu_t(A_{19}, A_t)$ of each set pair and get Set A_{10}, A_{11} and A_{15} which are the most similar (with the largest connection number) with current Set $A_{n+1}(A_{19})$ (Table 3).
5. According to formula (1), the predictive value $x(24)$ of January groundwater depth in 2004 based on SPA-MSF model is easily calculated to be 5.44 m. And, the predictive values of groundwater depth in other months during 2003–2004 could also be worked out in a similar way. The results are shown in Table 4. Figs. 2 and 3 visually show the monthly predictive value and measured value of groundwater depth during 2003–2004 based on SPA-MSF model.

Table 4 shows that in the fitting sequence of the predicted and measured values of groundwater depth in the total 24 months of 2003 ~ 2004, the relative error $|\delta|$ is all within 20%, which complies with the requirements of the Standard for Hydrological Information and Hydrological Forecasting (SL 250-2000) [13]. The number of $|\delta| \le 5\%$ is 12, accounting for 50%; the number of $5\% < |\delta| \le 10\%$ is 7, accounting for 29.2%; the number of $10\% < |\delta| \le 15\%$ is 3, accounting for 12.5%; the number of $15\% < |\delta| \le 20\%$ is 2, accounting for 8.3%.

Moreover, it can be seen from Figs. 2 and 3 that the monthly predicted value and measured value of the groundwater depth at Shandanqiao in 2003 ~ 2004 using dynamic SPA-MSF model have good goodness of fit and both show a strong tendency, that is, the groundwater depth reaches the maximum in July each year, which is dominated by the artificial mining, and the groundwater depth reaches the minimum in September when the surface water supplies the shallow groundwater. The average absolute error of groundwater depth is 0.26 m; the minimum absolute error was 0.04 m only while the maximum absolute error reached 0.70 m in August of 2003 and 0.87 m in August of 2004, which was because August is between the peak (July and September), and in this period, the groundwater recharge-runoff-discharge system shows more complicated properties under the influence of human activities, which correspond to the sudden change of the random factor caused by the peak shift. The model thus produces a large systematic error.

4.2.3. SPA-MSF model verification

The validity and reliability of the model can be verified through the analysis of the absolute error, relative error and percentage of eligible points of the predicted value and measured value from the established groundwater level time series model. In this paper, the posterior variance test method is used to verify the validity and reliability of the model [14]. The posterior variance test is performed with the monthly measured data of the groundwater depth of Shandanqiao observation well in 2003 ~ 2004 and the predicted data of PA-MSF model as the examples, and the test result is shown in Table 5.

It can be obtained from Table 5 that the value of C is smaller than 0.35 and the small error probability P is greater than 0.9 when the SPA-MSF method is used in the prediction of groundwater depth of Shandanqiao observation well in 2003 ~ 2004. The model has good generalization ability and so the prediction accuracy is high.

Table 2
Prediction accuracy grades by mean-standard deviation method

Prediction accuracy grades	$\bar{X}_j$	S_j
x_1	5.463	0.209
x_2	5.471	0.209
x_3	5.477	0.207
x_4	5.484	0.203
x_5	5.479	0.208

Table 1
Table of sets constituted by groundwater level time series

Set	Set element					Subsequent value
A_1	$x(1)$	$x(2)$	$x(3)$	…	$x(m)$	$x(m+1)$
A_2	$x(2)$	$x(3)$	$x(4)$	…	$x(m+1)$	$x(m+2)$
…	…	…	…	…	…	…
$A_{(n-m)}$	$x(n-m)$	$x(n-m+1)$	$x(n-m+1)$	…	$x(n-1)$	$x(n)$
A_{n+1}	$x(n-m+1)$	$x(n-m+2)$	$x(n-m+3)$	…	$x(n)$	$x(n+1)$

Table 3
Similar set and connection degree (number) of Set A_{19}

Set	x_1	x_2	x_3	x_4	x_5	Mean value within the Set	Subsequent value	Connection Degree Vector	Connection Number
A_{10}	5.56	5.64	5.60	5.39	5.29	5.49	5.44	(0.4, 0.6, 0, 0, 0)	0.7
A_{11}	5.64	5.60	5.39	5.29	5.44	5.47	5.48	(0.4, 0.6, 0, 0, 0)	0.7
A_{15}	5.44	5.48	5.44	4.83	5.54	5.35	5.49	(0.6, 0.2, 0,2, 0, 0)	0.7
A_{19}	5.54	5.49	5.43	5.26	5.36	5.41	5.12	–	–

Table 4
Monthly predictive value vs. measured value of groundwater depth during 2003–2004 at Shandanqiao observation well based on SPA-MSF model

Year	Project	Month											
		1	2	3	4	5	6	7	8	9	10	11	12
2003	Measured value (m)	5.36	5.43	5.47	5.58	5.47	5.55	5.99	4.60	4.01	4.90	4.53	5.17
	Predictive value (m)	5.54	5.62	5.66	5.64	5.43	5.35	5.52	5.30	4.16	5.06	4.96	5.49
	Absolute error (m)	0.18	0.19	0.19	0.06	–0.04	–0.20	–0.47	0.70	0.15	0.16	0.43	0.32
	Relative error (%)	3.38	3.58	3.53	1.17	–0.64	–3.52	–7.83	15.19	3.82	3.22	9.54	6.21
2004	Measured value (m)	5.12	5.32	5.38	5.49	4.82	5.48	6.46	4.45	3.82	3.95	3.87	4.62
	Predictive value (m)	5.44	5.54	5.67	5.64	5.38	5.34	5.99	5.32	3.86	4.26	4.36	5.15
	Absolute error (m)	0.32	0.22	0.29	0.15	0.56	–0.14	–0.47	0.87	0.04	0.31	0.49	0.53
	Relative error (%)	6.23	4.19	5.35	2.81	11.66	–2.54	–7.26	19.66	1.05	7.93	12.75	11.50

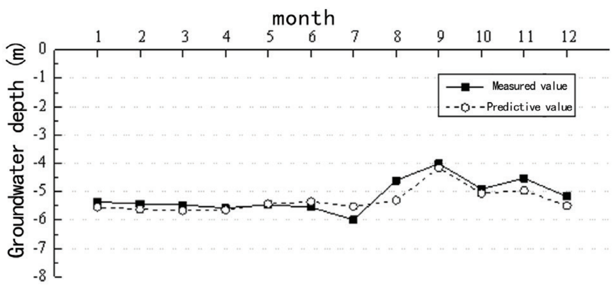

Fig. 2. Comparison of the predicted value and the measured value of groundwater depth of Shandanqiao observation well in 2003 month by month.

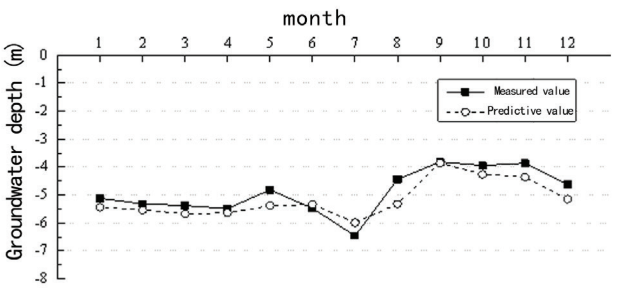

Fig. 3. Comparison of the predicted value and the measured value of groundwater depth of Shandanqiao observation well in 2004 month by month.

4.3. Result analysis

(1) The five-element connection degree model proposed in this paper characterizes the complex nonlinear relationship of the identical degree, different degree and contrary degree of $H(A_{n+1}, A_t)$. Uncertainties can be quantitatively expressed. For example, in the groundwater depth prediction for Shandanqiao observation well in January of 2004 (Table 4), the IDC connection degree vector (0.6, 0.2, 0.2, 0, 0) of the set $A_{n+1}(A_{19})$ and the set A_{15} indicates under the level of the standard deviation of the mean of the corresponding elements, the identical degree of the elements with the same level in the set $H(A_{19}, A_{15})$ is 0.6, the different degree of the elements with the difference level of 1 is 0.2, the different degree of the elements with the difference level of 2 is 0.2, and the different degree of the elements with the difference level of 3 and the contrary degree of the elements with the difference level of 4 are both 0.

(2) The SPA-MSF model measures the similarity between historical samples of time series of groundwater level from the perspective of IDC connection degree and takes the groundwater level of the most similar historical samples as the predicted value of the groundwater level in the set month (year), providing a novel research idea for the dynamic prediction of groundwater level. In addition, the groundwater depth SPA-MSF model improves the traditional statistical forecasting model, that is, the form of forecasting function changes with the change of historical data. Therefore, the prediction model

Table 5
Posterior variance test result of SPA-MSF

Year	2003	2004
S_1	0.5332	0.7636
S_2	0.1825	0.2207
C	0.3422	0.2891
P	0.9167	1

based on the maximum similarity between the samples meets the actual evolution of the groundwater system. Table 4 shows the application of SPA-MSF model has high precision in the prediction of groundwater depth from the perspective of absolute error – relative error and meets requirements of relevant specifications. Figs. 2 and 3 show that the predicted value and the measured value have a good goodness of fit and the tendency of the month of the maximum depth and the month of the minimum depth are basically consistent, meeting the characteristics of the recharge-runoff-discharge regime of groundwater and dynamic changes. Generally speaking, the theoretical basis of SPA-MSF method for predicting groundwater level is reasonable, and the calculation process is simple and effective, so it has a good application prospect in time series analysis and prediction.

5. Conclusion

The application of the SPA-MSF in the dynamic prediction of groundwater level proposed in this paper describes the similarity between samples in quantitative form. The key to improving the precision of dynamic prediction of groundwater level is how to quantify the connection degree more accurately and objectively. However, the value of the coefficient I with uncertain difference is worth exploring in depth. This chapter focuses on the application study of the SPA in the dynamic prediction of groundwater level. For convenience, the values of the coefficients I_1, I_2, and I_3 with uncertain difference are, respectively, –0.5, 0 and 0.5 using the special value method. But the special value method may cause the relative error higher and the prediction accuracy lower in the dynamic forecast of groundwater level. Taking the prediction of the depth value of Shandanqiao observation well in August of 2004 as the example, the absolute error is 0.87 m and the relative error reaches 19.66%. The maximum similarity (correlate) between the historical samples is 0.4, but it is debatable whether the correlate 0.4 is the maximum similarity. Therefore, the triangular fuzzy number method [15], the trapezoidal fuzzy number method [16] and Markov theory of optimization of coefficients with uncertain difference and other methods proposed in a study by Liu et al. [4] provide theoretical guidance for further research on the reasonable value of I. At the same time, this chapter classifies and symbolizes each element in set A_t based on mean standard deviation method, and the identical degree, different degree and contrary degree of the set pair $H(A_{n+1}, A_t)$ are defined on this basis. However, there is a lack of a solid scientific foundation and an in-depth exploration of the principle of set pair analysis.

Acknowledgments

This study was supported by the University Key Scientific Research Projects of Henan Province (15A570004), the University Young Backbone Teacher Training Program of Henan Province (2016GGJS-075) and the High Level Talents Scientific Research Project of NCWU.

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