Desalination and Water Treatment
www.deswater.com

doi: 10.5004/dwt.2018.22887

126 (2018) 268–277
September

Integrated optimization of water and fertilizer coupling system under uncertainty

Xiao Liu[a,b], Mo Li[c], Ping Guo[a,*], Zhongxue Zhang[c,d]

[a]*Centre for Agricultural Water Research in China, China Agricultural University, Tsinghuadong Street No. 17, Beijing 100083, China, Tel. +86 1062738496; emails: guop@cau.edu.cn (P. Guo), liux1990@126.com (X. Liu)*
[b]*Heilongjiang Province Hydraulic Research Institute, Harbin 150080, China*
[c]*School of Water Conservancy and Civil Engineering, Northeast Agricultural University, Harbin 150030, China, email: limo0828@neau.edu.cn (M. Li)*
[d]*Key Laboratory of Agricultural Water Resources Use, Ministry of Agriculture, Harbin 150030, China, email: zhangzhongxue@163.com*

Received 27 November 2017; Accepted 2 August 2018

ABSTRACT

Water and fertilizer jointly affect the growth of crops. A nonlinear programming model for the optimal utilization of water and fertilizer under uncertainty was developed to maximize net benefits. The complexities in estimating crop yields, water requirements as well as economic profits were considered. The developed model improved upon existing management models through incorporating water-fertilizer production functions of crops, in which uncertainties in parameters and functions expressed as intervals were reflected. Moreover, it could improve irrigation efficiency, protect ecological environment, and thus promote the sustainable development of the agricultural system. The model was solved by interval quadratic programming method. Based on the experimental data from Qing'an experimental station as well as a representative experimental station in Heping irrigation district, Heilongjiang province, China, the developed model was applied to simultaneously optimize the utilization of water and fertilizer. The developed model and the corresponding solution method are beneficial to the optimization of fertigation schedule, and have broad prospects for addressing other issues that are related to agricultural water management.

Keywords: Interval quadratic regression; Interval quadratic programming; Water and fertilizer production function; Rice; Uncertainty

1. Introduction

Water and fertilizers are main limiting factors of the growth and yield of crops. Excessively increasing the amount of water and fertilizers to increase production while ignoring the actual needs of crops growth would result in low crop productivity from water and fertilizers [1–3]. Moreover, excessive nitrogen would cause a series of environmental problems through soil leakage, surface runoff, and evaporation [4,5]. The impacts of fertilizers and water on crop growth are interrelated. Increasing water productivity and fertilizer efficiency is important for water-saving agriculture. Therefore, it is important to optimize the integrated system of water and fertilizers to achieve maximum net returns.

Optimization techniques can be used to rationally distribute water and fertilizers to improve the comprehensive production efficiency of crops [6]. Many researches have been reported for the optimization of water and fertilizers for various crops [7–9], and many of which were based on water-fertilizer production function (WFPF) of various crops. WFPF, which reflects the relationship between yield and water and fertilizer of crops, is an important basis for the optimization of the coupled system of water and fertilizers. Various

* Corresponding author.

static and dynamic models of water-fertilizer production functions have been established, focusing on the whole period or the sub-periods of crops' growth. Most of these models were obtained through regression results based on field test data and many valuable results have been reported [10–12].

However, uncertainties exist in the estimation of crop WFPFs, diminishing its practicability [13,14]. For example, artificial errors and the variations of meteorological conditions result in the fluctuations of experimental data, which amplifies the uncertainties in the regression of WFPF of crops. Furthermore, varying market conditions, different food requirements and fluctuations of water supply lead to uncertainties as well in the comprehensive optimization of water and fertilizers. Optimization of the coupled system of water and fertilizers under uncertainty would provide practical guiding for field managements. However, limited researches have been reported to optimize water and fertilizers of individual crops based on WFPF under uncertainty.

Stochastic mathematical programming [15], fuzzy mathematical programming [16] and interval mathematical programming are uncertain optimization methods that are commonly used [17]. Among them, stochastic mathematical programming is a type of optimization models in which uncertain parameters in the objective or constraints are represented by probability distributions. Fuzzy mathematical programming could deal with vagueness in decision maker's aspirations (or preferences) and ambiguity in knowledge or information. Interval mathematical programming could deal with uncertainties approximated as the lower and upper boundaries. For the integrated management of water and fertilizer that is based on experimental data, interval mathematical programming can be adopted because it is useful when the available data are insufficient to get distributions of probabilities and membership degrees [15,16,18,19].

There are few studies on the integrated optimization of water and fertilizer system under uncertainty. In Tong and Guo's paper [20], intervals and functional intervals were introduced into crop water production functions for optimal allocation of water resources in the irrigation area, but fertilizers were not to be considered. Wang et al. [21–23] studied coupled water and fertilizer system to promote crop yields and agricultural net benefits without the consideration of uncertainty.

Therefore, the aim of this study is to develop a nonlinear programming model under uncertainty for the optimal allocation of water and fertilizers to maximize net benefits. Interval uncertainties in both functions and parameters are considered. The developed model is based on the interval WFPF that can be obtained by interval regression method. Based on the experimental data from Qing'an experimental station, located in the middle of Heilongjiang province, China, the developed model was applied to the integrated management of water and fertilizers of rice. As rice is the major crops of Heilongjiang province and Qing'an experimental station is one of the typical stations for planting rice, the results obtained are of practical guiding significance for efficient production of rice. The developed model and the corresponding results have important significance to save water and fertilizer resources, and further to improve the irrigation efficiency and maintain the reliable scientific basis for the sustainable development of the irrigation area.

2. Study systems

This paper verifies the accuracy and applicability of the integrated optimization model of water and fertilizers under uncertainty in Qing'an county, Heilongjiang province, China.

2.1. Study area

Rice is the main crop planted in the province of Heilongjiang. The data of crop water consumption throughout the growth period, nitrogen application rate and yields come from pilot experiments. The experiment for the growth of rice under different modes of irrigation and fertilization was conducted in the Heilongjiang Irrigation Experiment Center (Qing'an Station) from the beginning of May to the end of September in 2014. Geographical coordinates of Qing'an station are 125°44′ east longitude and 46°63′ north latitude. The experimental area belongs to the cold temperate continental monsoon climate. The average annual rainfall is 577 mm, the average temperature is 1.69°C, frost-free period is 128 d, sunshine hours is 2,600 h, and evaporation is 770 mm. The basic physical and chemical properties of the soil are as follows: organic matter content 41.4 g kg^{-1}, pH 6.40, total nitrogen 15.06 g kg^{-1}, total phosphorus 15.23 g kg^{-1}, total potassium 20.11 g kg^{-1}, alkali hydrolytic nitrogen 154.36 g kg^{-1}, available phosphorus 25.33 g kg^{-1}, and available potassium 157.25 g kg^{-1} [24].

2.2. Field experiment

The experiment used four types of water management modes, including controlled irrigation (C1), intermittent irrigation (C2), shallow wet irrigation (C3) and flood irrigation (C4) with the nitrogen application levels being 135 kg hm^{-2} (N1), 105 kg hm^{-2} (N2), 75 kg hm^{-2} (N3) and 0 kg hm^{-2} (N4, i.e., CK) treatments. Each treatment has times repetitions, a total of 48 plots, each plot area was 100 m^2. The tested type is Longqing Rice No. 3, which was transplanted on May 21th and yield predicted on September 18th.

Observation data included irrigation amount, soil moisture, height of water layer, water discharge, and yield. Using these data, we curved water and fertilizer production function (WFPF).

The irrigation situation is shown in Table 1.

2.3. Framework

This paper attempts to maximize the net benefit through allocating water and fertilizers to rice based on an quadratic programming model under uncertainty, in which the interval production function for the whole growth period is coupled. Interval regression was adopted to fit the interval WFPF based on which an interval quadratic programming model for optimizing water and fertilizers was established and solved. A brief description of this study is given in Fig. 1.

3. Interval water-fertilizer production function of rice

In this section, method of interval regression analysis was used to build water-fertilizer production function under uncertainty. The data of rice water consumption and nitrogen

Table 1
Water management modes of different irrigation management patterns

Growth stages		Control irrigation		Intermittent irrigation		Shallow wet irrigation		Flood irrigation	
		HL (mm)	LL (%)	HL (mm)	LL (%)	HL (mm)	LL (%)	HL (mm)	LL (%)
Reviving		30	100	30	100	30	100	30	100
Tillering	Front	100%	85	40	100	30	100	80	100
	Middle	100%	85	40	100	20	100	80	100
	Post	100%	60	Field drying	Field drying	Field drying	Field drying	Field drying	Field drying
Jointing-booting		100%	90	30	100	10	100	80	100
Heading and flowering		100%	85	40	100	20	100	80	100
Milk		100%	70%	40	100	20	100	80	100
Wax		Dry set	Dry set	Dry set	Dry set	Dry set	Dry set	Dry set	Dry set

Note: The "%" refers to the percentage of the saturated soil moisture content; the saturated soil moisture content is 54.72% in the table. HL means upper limit and LL means lower limit.

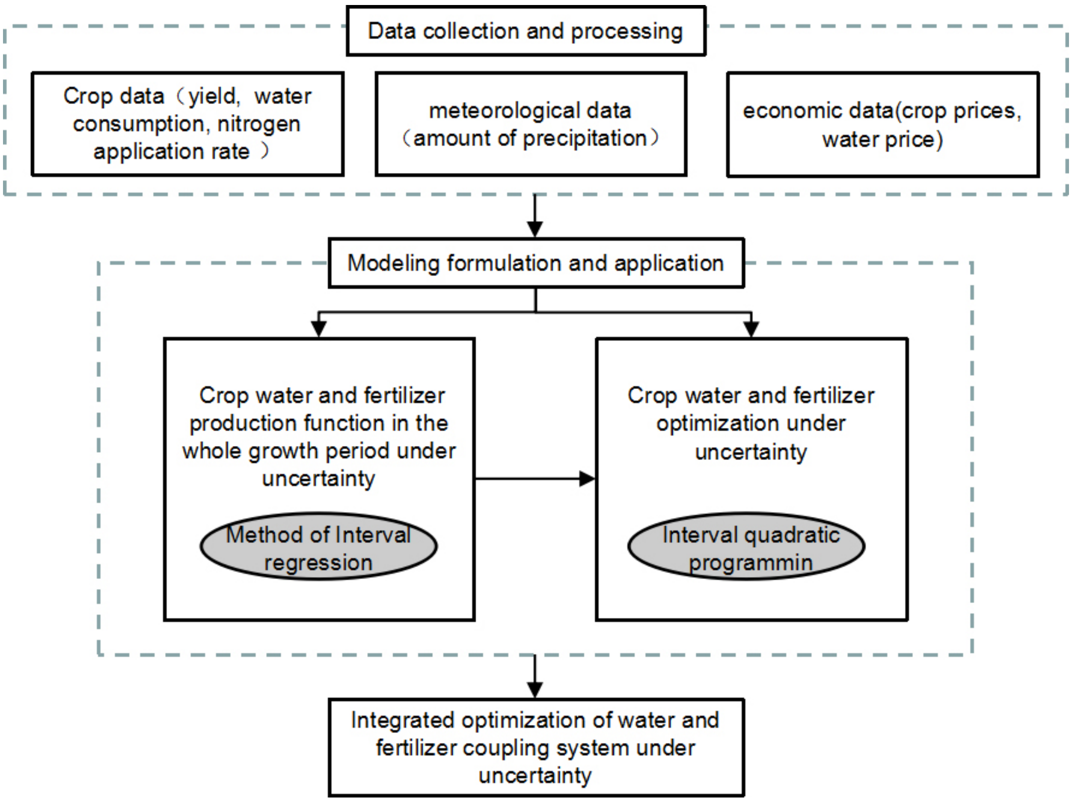

Fig. 1. System framework.

application rate in the whole growth period and yield came from the pilot study on water production functions and optimal irrigation schedule of rice in Qing'an experimental station.

3.1. Method of regression analysis

The purpose of this part is to fit the interval WFPF based on interval regresson method.

Because the information used for the estimation of WFPF are varying, the interval regression analysis is useful to address the collected information. An interval linear regression model can be written as:

$$Y(x) = B_0 + B_1x_1 + \ldots + B_nx_n = Bx \tag{1}$$

There are many approaches to achieve the interval regression analysis, such as translating into quadratic optimization problems, using neural networks, using support vector machines. This research used the relatively mature interval regression analysis method based on quadratic programming, and integrated the central tendency of least squares and possibilistic property of fuzzy regression. The proposed model can be represented as follows:

$$\underset{b,c}{\mathrm{Min}} J = m_1 \sum_{j=1}^{p} \left(y_i - b^t x_j \right)^2 + m_2 \sum_{j=1}^{p} c^t \left| x_j \right| \left| x_j \right|^t c \tag{2}$$

Subject to

$$b^t x_j + c^t \left| x_j \right| \geq y_j \tag{3}$$

$$b^t x_j + c^t \left| x_j \right| \geq y_i \tag{4}$$

$$b^t x_j - c^t \left| x_j \right| \leq y_i, \quad j = 1, \ldots, p \tag{5}$$

$$c_i \geq 0, i = 0, \ldots, n \tag{6}$$

where $\left| x_j \right| = \left(1, \left| x_{j1} \right|, \ldots, \left| x_{jn} \right| \right)^t$, $b = \left(b_0, \ldots, b_n \right)^t$, $c = \left(c_0, \ldots, c_n \right)^t$. m_1 and m_2 are weight coefficients. When the value of m_1 and m_2 are changed, the regression results may be different [25–27].

3.2. Calculation process and result analysis

The relationship among yield, water consumption, and nitrogen application rate in the whole growth period of rice was studied. Because the models of WFPF are usually quadratic in the higher yield area, the rice yield, the data of water and nitrogen application during the whole growth period of rice were analyzed by interval quadratic regression based on quadratic programming. We had five sets of values of the value of m_1:m_2, which equal to 1:0.0001, 1:0.5, 1:1, 0.5:1, 0.0001:1. LINGO software was used to solve the quadratic programming, and the results are shown in Tables 2 and 3 and WFPF can be written as:

$$\begin{aligned} Y^*(x) = &\left(b_0, c_0 \right) + \left(b_1, c_1 \right) x_1 + \left(b_2, c_2 \right) x_2 + \left(b_3, c_3 \right) x_1 x_2 \\ &+ \left(b_4, c_4 \right) x_1^2 + \left(b_5, c_5 \right) x_2^2 \end{aligned} \tag{7}$$

When $m_1 > m_2$, the central trend is more considered. The greater the value of m_1:m_2, the more the regression curve and the deterministic regression curve are consistent (i.e., only consider the central trend, regardless of interval). And when $m_2 > m_1$, the more concerned is how to completely include the data points in the obtained interval. The larger the value of m_2:m_1, the more the center trend is neglected. When the value of m_2:m_1 increases to a certain value, and then still increase the m_2:m_1 value, the obtained range will no longer change.

Table 2
Interval quadratic regression results of rice yield and water consumption and nitrogen application rate in whole growth period (determinate number results)

m_1	m_2	b_0	b_1	b_2	b_3	b_4	b_5
1	0.0001	–14,474.85	300.15	284.94	–0.43	–2.74	–1.03
1	0.5	–14,102.53	309.78	280.97	–0.43	–2.95	–1.01
1	1	–13,826.05	317.51	275.78	–0.37	–3.14	–1.00
0.5	1	–14,294.98	353.50	263.29	–0.30	–3.49	–0.96
0.0001	1	–19,608.97	678.07	171.77	–0.21	–6.15	–0.55

We selected the interval regression result of m_1 = 1, m_2 = 0.0001 of which the central trend is fully considered to express the uncertainties of WFPF in the whole growth period.

The optimized interval WFPF in the whole growth period of rice is:

$$\begin{aligned} Y^*(x) = &\left(-14474.85, 137.7179 \right) + \left(300.1521, 7.8261 \right) x_1 \\ &+ \left(284.9416, 1.1643 \right) x_2 + \left(-0.4318, 0 \right) x_1 x_2 \\ &+ \left(-2.7423, 0 \right) x_1^2 + \left(-1.0318, 0 \right) x_2^2 \end{aligned} \tag{8}$$

In this study, x_1 divided by 100 was taken in estimating the WFPF of rice using the interval regression method in order to ensure that the values of water consumption and nitrogen application rate are in the same order of magnitude.

The deterministic form of WFPF of rice in the whole growth period is expressed as follows:

y = –14474.47+300.1406x_1+284.9389x_2–0.4318$x_1 x_2$–2.7429x_1^2 –1.0317x_2^2. The comparison of the results of the measured values (test values), interval regression models and conventional regression models are shown in Table 3 and Fig. 2. Through Fig. 2 we can visually see the comparison of the above two WFPFs.

Table 3
Interval quadratic regression results of rice yield and water consumption and nitrogen application rate in whole growth period (interval results)

m_1	m_2	c_0	c_1	c_2	c_3	c_4	c_5
1	0.0001	137.72	7.83	1.16	0	0	0
1	0.5	398.92	4.55	0.00	0	0	0
1	1	345.75	4.29	0.26	0	0	0
0.5	1	357.38	4.03	0.25	0	0	0
0.0001	1	255.69	3.61	0.00	2.13E-02	0	0

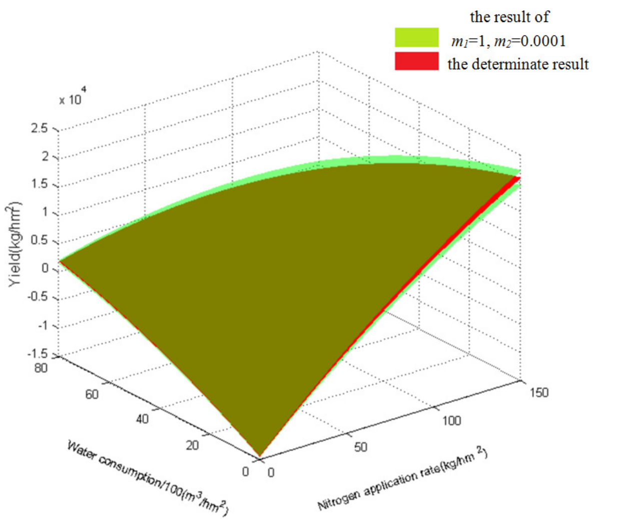

Fig. 2. Interval quadratic regression results of rice yield and water consumption and nitrogen application rate in the whole growth period.

It can be seen from Table 3 and Fig. 2 that whether it is interval regression model or conventional regression model, the forecast value and the measured value are fitted well. The values of the conventional regression model are contained in forecasting interval regression model. The interval regression model provides a central point and the forecast value of upper and lower limits, and forecast values of the general regression model was just a real value. Therefore, the interval regression model provides more information than the conventional regression model due to the reflection of uncertainty.

4. Interval quadratic programming model for water and fertilizer optimization under uncertainty

In this part, we developed the conventional optimal model (deterministic model) and then built the interval quadratic programming model (uncertainty model) of water fertilizer system to compare their results. The objective of the model is to maximize agricultural income, and decision variables are water consumption and nitrogen application rate. This model aims to optimize allocation of water and fertilizers.

4.1. Conventional optimal model of water and fertilizer coupling system

In this model, the deterministic WFPF is introduced into the optimization model of water and fertilizer system. The conventional method is used to optimize the objective function, in which the parameters are deterministic. The optimization model is formulated as follows [27]:

Objective function:

$$\begin{aligned} \max f &= GAY - CMA - DFA \\ &= BA\left[a_0 + a_1x_1 + a_2x_2 + a_3x_1x_2 + a_4\left(x_1\right)^2 + a_5\left(x_2\right)^2\right] \\ &\quad - C\left(100x_1 - P + s + c\right) - Dx_2A \end{aligned} \tag{9}$$

Subject to:

$$\mathrm{ET} \le \mathrm{ET}_{\max} \tag{10}$$

$$F \le F_{\max} \tag{11}$$

$$Y \ge Y_{\min} \tag{12}$$

Using LINGO to solve the problem, the calculation results are as follows:

x_1 = 42.95 10^2 m^3 hm^{-2} x_2 = 127.89 kg hm^{-2} the unit operating area ne f = 47,151.21 Yuan hm^{-2}.

4.2. Interval quadratic programming method

Since the interval WFPF of rice is quadratic, the optimization of irrigation water and fertilizers falls within the capacity of interval quadratic programming [28–30]. The basic principle of interval quadratic programming can be described as: if a number has "I" as the superscript, then it is an interval number. If it is not, then it is a real value. The left bound of the interval is indicated by the superscript "L", and the right bound is indicated by the superscript "R".

Then the interval quadratic programming can be written as follows [31–33]:

$$\min \quad z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} x_i x_j \tag{13}$$

$$s.t. \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,\ldots,m \tag{14}$$

$$x_j \ge 0, j = 1,\ldots,n \tag{15}$$

where $c^I = \left(c_1^I,\ldots,c_n^I\right)$, $Q^I = \left(q_{ij}^I\right)_{n\times n}$ and $b^I = \left(b_1^I,\ldots,b_m^I\right)$.

For an inequality constraint "≥", it can be transformed into "≤" by multiplying the "–1" in both sides of the equation. If the objective function is "max", then it can be converted to the "min" form to match the above model.

The interval matrix $Q^I = \left(q_{ij}^I\right)_{n\times n}$ is a symmetric positive definite matrix, $Q = \left(q_{ij}\right)_{n\times n} \in Q^I$, $q_{ij} = q_{ji}$. So the problem can be written as:

$$\begin{aligned} &\min \quad z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} x_i x_j \\ &s.t. \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,\ldots,m \\ &\qquad x_j \ge 0, j = 1,\ldots,n \end{aligned} \tag{16}$$

where $c_j \in c_j^I, b_i \in b_i^I, a_{ij} \in a_{ij}^I, q_{ij} \in q_{ij}^I, q_{ij} = q_{ji}$.

Definitely $S = \{\left(c_j, b_j, a_{ij}, q_{ij}\right) \middle| c_j \in c_j^I \middle|, b_i \in b_i^I, a_{ij} \in a_{ij}^I, i = 1,\ldots,m,$ $j = 1,\ldots,n, q_{ij} \in q_{ij}^I, q_{ji} = q_{ji}, i, j = 1,\ldots,n\}$ Values of z, c_j, b_i, a_{ij} and q_{ij} can be obtained from the numerical programming problem:

$$z^L = \min_{c_j, b_i, a_{ij}, q_{ij} \in S} \min_{x} \quad z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} x_i x_j \tag{17}$$

$$s.t. \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,\ldots m \tag{18}$$

$$x_j \ge 0, \quad j = 1,\ldots n \tag{19}$$

and

$$z^U = \max_{c_j, b_i, a_{ij}, q_{ij} \in S} \min_{x} \quad z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} x_i x_j \tag{20}$$

$$s.t.\quad \sum_{j=1}^{n} a_{ij}x_j \le b_i, i=1,...m \tag{21}$$

$$x_j \ge 0,\quad j=1,...n \tag{22}$$

Respectively, this pair of numerical values expresses the boundaries of the target value. The Lagrangian dual method can be used to convert the interval quadratic programming model into a deterministic quadratic programming problem:

$$z^L = \min_x \quad z = \sum_{j=1}^{n} c_j^L x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}^L x_i x_j \tag{23}$$

$$s.t.\quad \sum_{j=1}^{n} a_{ij}^L x_j \le b_i^R,\quad i=1,...,m \tag{24}$$

$$x_j \ge 0,\quad j=1,...,n \tag{25}$$

$$Z^U = \max_{x,\lambda,\delta} \quad z = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}^R x_i x_j - \sum_{i=1}^{m} b_i^L \lambda_i \tag{26}$$

$$s.t.\quad \sum_{i=1}^{n} q_{ij}^R x_i + \sum_{i=1}^{m} r_{ij} - \delta_j = -c_j^R,\quad j=1,...n \tag{27}$$

$$a_{ij}^L \lambda_i \le r_{ij} \le a_{ij}^R \lambda_i,\quad i=1,...,m,\quad j=1,...,n \tag{28}$$

$$\lambda_i, \delta_j \ge 0,\quad i=1,...,m,\quad j=1,...,n \tag{29}$$

where $r_{ij} = a_{ij}\lambda_i, a_{ij} \in a_{ij}^I$.

4.3. Optimization of coupled water-fertilizer system of rice under uncertainty

4.3.1. Model development

The interval WFPF was introduced into the interval optimization model to optimize the coupled water-fertilizer system. Interval quadratic programming method is used to solve this problem. Assuming that the variation of soil water storage throughout the growth period is zero, the amount of groundwater recharge in the irrigation area is negligible. Considering the high level of water management (i.e., the amount of evapotranspiration during the whole growth period is equal to the effective irrigation quota plus the effective precipitation minus the leakage), according to the principle of maximum net return, the optimization model for allocating water and fertilizers for rice under uncertainty can be formulated as follows:

$$\begin{aligned}\max f^{\pm} &= (BAY^{\pm} - CM^{\pm}A - DF^{\pm}A)\\ &= BA\left[a_0^{\pm} + a_1^{\pm}x_1^{\pm} + a_2^{\pm}x_2^{\pm} + a_3x_1^{\pm}x_2^{\pm} + a_4\left(x_1^{\pm}\right)^2 + a_5\left(x_2^{\pm}\right)\right]\\ &\quad - CA\left(100x_1^{\pm} - P + s + c\right) - Dx_2^{\pm}A\end{aligned} \tag{30}$$

Subject to:

$$\mathrm{ET}^{\pm} \le \mathrm{ET}_{\max} \tag{31}$$

$$F^{\pm} \le F_{\max} \tag{32}$$

$$Y^{\pm} \ge Y_{\min} \tag{33}$$

4.3.2. Method of solution

According to the experimental data from the Qing'an experimental station in 2014, the original model is as shown in Eqs. (24)–(27). The original objective function is "max", and it can be converted to the "min" form through multiplying "–1" and reversing the upper and lower bounds.

It can be transformed into the following form:

$$\begin{aligned}\min(-f) &= -BAY^{\pm} + CM^{\pm}A + DF^{\pm}A\\ &= -BA\left[a_0^{\pm} + a_1^{\pm}x_1^{\pm} + a_2^{\pm}x_2^{\pm} + a_3x_1^{\pm}x_2^{\pm} + a_4\left(x_1^{\pm}\right)^2 + a_5\left(x_2^{\pm}\right)^2\right]\\ &\quad + C\left(100x_1^{\pm} - P + s + c\right)A + Dx_2^{\pm}A\end{aligned} \tag{34}$$

$$\begin{aligned}\min(-f) &= \sum_{j=1}^{n} c_j^I x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}^I x_i x_j\\ &= [-BAa_0^{\pm} - CA(P-s-c) + (-BAa_1^{\pm} - 100C)x_1^{\pm}\\ &\quad + (-BAa_2^{\pm} + DA)x_2^{\pm}] + \frac{1}{2}\times 2[-a_3x_1^{\pm}x_2^{\pm} - a_4(x_1^{\pm})^2 - a_5(x_2^{\pm})^2]\end{aligned} \tag{35}$$

$$z = \min f' = \min(-f) \tag{36}$$

Subject to:

$$x_1^{\pm} + 0x_2^{\pm} \le \mathrm{ET}_{\max} \tag{37}$$

$$0x_1^{\pm} + x_1^{\pm} \le F_{\max} \tag{38}$$

$$Y \ge Y_{\max} \tag{39}$$

The lower bound:

$$z^U = \max_{x,\lambda,\delta} \quad z = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}^R x_i x_j - \sum_{i=1}^{m} b_i^L \lambda_i - \frac{1}{2}\left[2BAa_4\left(x_1\right)^2 + 2BAa_3 x_1 x_2 + 2BAa_5\left(x_2\right)^2\right] - (\mathrm{ET}_{\max}\lambda_1 + F_{\max}\lambda_2) \quad (40)$$

Subject to:

$$2BAa_4 x_1 + BAa_3 x_1 x_2 + r_{11} + r_{21} - \delta_1 = -\left(-BAa_1^{\pm} + 100CA\right)^R \quad (41)$$

$$BAa_3 x_1 + 2BAa_5 x_2 + r_{12} + r_{22} - \delta_2 = -\left(-BAa_2^{\pm} + DA\right)^R \quad (42)$$

$$\left(a_0^{\pm} + a_1^{\pm} x_1 + a_2^{\pm} x_2 + a_3 x_1 x_2 + a_4 x_1^2 + a_5 x_2^2\right)^L \ge Y_{\min} \quad (43)$$

$$r_{11} = \lambda_1 \quad (44)$$

$$r_{12} = r_{21} = 0 \quad (45)$$

$$r_{22} = \lambda_2 \quad (46)$$

$$\lambda_1, \lambda_2, \delta_1, \delta_2 \ge 0 \quad (47)$$

The calculation results are as follows:
x_1 = 40.23 m³ hm⁻², x_2 = 126.45 kg hm⁻², and the lower bound of the operating area benefit per unit area is f = 44,177.09 Yuan hm⁻².

The upper bound:

$$z^L = \min_{x} \quad z = \min(-f) = \sum_{j=1}^{n} c_j^L x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}^L x_i x_j$$
$$[-BAa_0^{\pm} - CA(P-s-c) + (-BAa_1^{\pm} - 100C)x_1 + (-BAa_2^{\pm} + DA)x_2] + \frac{1}{2}\times 2[-a_3 x_1 x_2 - a_4(x_1)^2 - a_5(x_2)^2] \quad (48)$$

Subject to:

$$x_1 + 0x_2 \le \mathrm{ET}_{\max} \quad (49)$$

$$0x_1 + x_2 \le F_{\max} \quad (50)$$

$$\left(a_0^{\pm} + a_1^{\pm} x_1 + a_2^{\pm} x_2 + a_3 x_1 x_2 + a_4 x_1^2 + a_5 x_2^2\right)^L \ge Y_{\min} \quad (51)$$

The calculation results are as follows:
x_1 = 44.43 10² m³ hm⁻², x_2 = 128.67 kg hm⁻², and the upper bound of the operating area benefit per unit area is f = 50,100.59 Yuan hm⁻².

4.4. Result analysis and discussion

The results are shown in Table 4 and Fig. 3.

As shown in Table 4 and Fig. 3, x_1 changed from 42.95 to [40.23;44.43] (m³ hm⁻²), x_2 changed from 127.9 to [126.5;128.7], f changed from 47,151 to [44,177;50,100] (Yuan hm⁻²), and the predicted yield changed from 10,511 to [9,899;11,194] (kg hm⁻²). That is to say, the results from the interval and conventional optimization models are very close. Furthermore, the results of the conventional optimization model fell within the windows of interval solutions. On the other hand, the interval results of water consumption have wider ranges compared with those of nitrogen application rates (Fig. 3). That is to say, the interval results of nitrogen application rate are more similar with conventional optimization results. This

Table 4
Comparison of the measured value of rice y and the estimated value of the interval regression model $Y^*(x)$, the comparison of the conventional regression model forecast value $Y'(x)$

x_1 (10² m³ hm⁻²)	x_2 (kg hm⁻²)	Y (kg hm⁻²)	$Y^*(x)$ (kg hm⁻²)	Center value (kg hm⁻²)	$Y'(x)$ (kg hm⁻²)
50.85	135	10,110	[10,006;11,090]	10,422	10,424
50.35	105	10,220	[9,595;10,601]	9,973	9,974
48.60	75	7,348	[7,325;8,234]	7,655	7,655
59.71	135	10,081	[9,393;10,615]	9,879	9,880
59.18	105	10,267	[9,123;10,268]	9,571	9,571
57.22	75	7,489	[7,064;8,108]	7,462	7,462
68.30	135	8,566	[8,388;9,745]	8,941	8,942
67.76	105	7,953	[8,256;9,534]	8,770	8,771
66.14	75	6,806	[6,365;7,548]	6,833	6,833
76.54	135	8,039	[7,043;8,529]	7,661	7,662
76.17	105	7,360	[7,013;8,423]	7,594	7,594
74.71	75	6,015	[5,283;6,600]	5,817	5,817

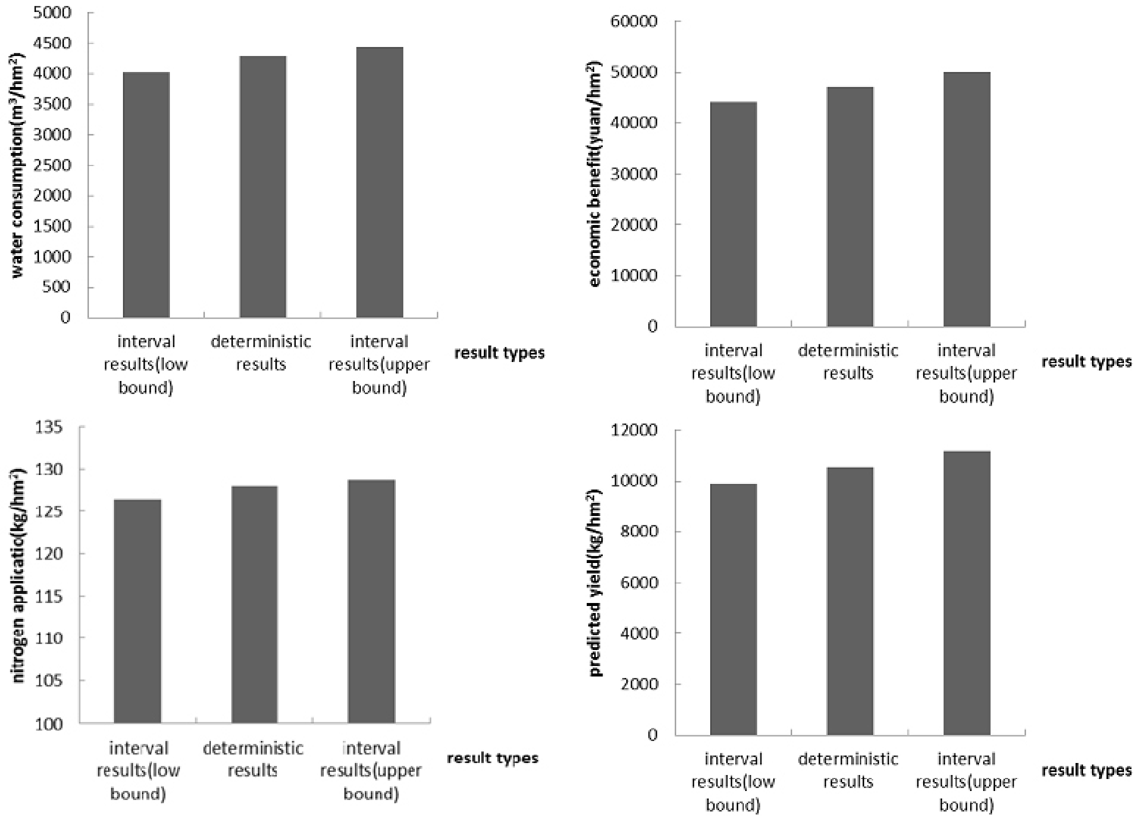

Fig. 3. Results of optimal integrated optimization of water and fertilizer coupling system.

Table 5
Results of optimal integrated optimization of water and fertilizer coupling system

	Water consumption (m^3 hm^{-2})	Nitrogen application rate (kg hm^{-2})	Economic benefit (Yuan hm^{-2})	Predicted yield (kg hm^{-2})
Conventional model	4,295	127. 9	47,151	10,551
Interval quadratic programming model	[4,023;4,443]	[126.5;128.7]	[44,177;50,100]	[9,899;11,194]

suggests that water consumption has a more flexible impact on the net return.

We compared our results with previous research that the optimal application rate of nitrogen during irrigation is 105 kg hm^{-2}. The above results are basically the same. The difference in nitrogen application rate maybe because their criteria is WUE (water use efficiency), while ours is the highest net benefit, which is also affected by water and fertilizer prices. Moreover, we can get a higher net return by controlling irrigation and nitrogen application rates. However, adopting controlled irrigation under proper nitrogen application rates cannot lead to the highest yields, because of lower agricultural water use compared with other irrigation methods. It reminds us why net benefits would not increase in some cases when crop yields increase.

Through the analyses of the results, we can deduce that in the range of optimized interval results, the net return increases with the nitrogen application rate and water consumption. Therefore, in the range of interval value, it is possible to select different irrigation quantities and fertilizer amounts according to the water quantity, water demand, fertilizer price and yield requirement, which is one of the advantages of inexact optimization. This study not only promotes the practical application of the inexact methods in the study of coupled water and fertilizer application, but also can reflect the complexity and uncertainty in the actual situation. It is of great theoretical and practical value to save water resources, improve irrigation efficiency and maintain sustainable development of irrigation areas, and has very broad research and development prospects. The regression model and the optimization model were suitable for rice cultivation in the southwestern part of Heilongjiang province, China.

5. Conclusions

According to experiment data from Qing'an experimental station, the relationship among rice yield, water consumption and nitrogen application rate was analyzed by interval quadratic programming method, and thus the interval WFPF of rice was obtained. Based on the interval

WFPF, an optimization model under uncertainty for the integrated allocation of water and fertilizer throughout the whole growth period of rice was established. The developed model was transformed into a deterministic model by solving the numerical solution of interval quadratic programming. Results from the conventional deterministic model and the developed interval quadratic programming model were compared. Results showed that the optimization method under uncertainty extended the ranges of results from traditional optimization method, but the differences between the results of the two models were not significant. Therefore, both of the two models can be applied in the study area and can be extended to other areas.

The research results of this paper provide decision makers with information under uncertainty, with the following advantages: (a) it effectively communicates uncertainties into the WFPF and optimization model, (b) it introduces WFPF to the optimization model, and applies the optimization model to the coupled water-fertilizer system; (c) it is applicable to practical problems through addressing the needs of achieving higher net returns by optimally applying water and nitrogen fertilizer. This study only introduces interval and functional interval methods. Fuzzy, random, and other types of uncertainties should also be considered in future studies.

Acknowledgements

This research was supported by the National Key Research and Development Plan (No. 2016YFC0400107), National Natural Science Foundation of China (51621061).

Symbols

x	—	Real input vector, $x = (1, x_1, \ldots, x_n)^t$
B	—	Interval coefficient vector, $B = (B_0, \ldots, B_n)$
$Y(x)$	—	Corresponding estimated interval
B_i	—	Interval coefficient, $B_i = (b_i, c_i)$
b_i	—	Center
c_i	—	Radius.
m_1, m_2	—	Weight coefficients
$Y^*(x)$	—	Yield of rice, kg hm^{-2}
x_1	—	Water consumption, 10^2 m^3 hm^{-2}
x_2	—	Nitrogen application rate, kg hm^{-2}
f	—	Objective function (in monetary currency, say Chinese yuan)
G	—	Price of rice, Yuan kg^{-1}
C	—	Cost of per unit of irrigation water, Yuan m^{-3}
D	—	Price of fertilizer, Yuan kg^{-1}
A	—	Planting area of rice, hm^2
Y	—	Yield, kg hm^{-2}
M	—	Decision variable, representing the irrigation quota per unit area in the whole growth period of rice, m^3 hm^{-2}
F	—	Decision variable, representing the nitrogen application rate per unit area in the whole growth period, kg hm^{-2}
P	—	Effective precipitation in the whole growth period of rice, m^3 hm^{-2}
s	—	Seepage in the whole growth ($s = 0$), m^3 hm^{-2}
c	—	Drainage in the whole growth, m^3 hm^{-2}
ET	—	Water consumption, m^3 hm^{-2}
$\mathrm{ET}_{\max}$	—	Upper limit value of water consumption, m^3 hm^{-2}
$F_{\max}$	—	Upper limit value of nitrogen application rate, kg hm^{-2}
$Y_{\min}$	—	Lower limit value of yield, kg hm^{-2}
$f^{\pm}$	—	Interval number of objective function (in monetary currency, say Chinese yuan)
$Y^{\pm}$	—	Interval number of the yield, kg hm^{-2}
$M\pm$	—	Interval number of decision variable, representing the irrigation quota per unit area in the whole growth period of rice, m^3 hm^{-2}
$F^{\pm}$	—	Interval number of decision variable, representing the nitrogen application rate per unit area in the whole growth period of rice, kg hm^{-2}
$a_i^{\pm}$	—	Interval number of interval regression coefficient, $i = 0, 1, \ldots, 5$
ET±	—	Interval number of water consumption with interval value, m^3 hm^{-2}

References

[1] D. Piw, J.H. Sa, K.Y. Kim, E.C. Jeon, Effect of fertilizer application on ammonia emission and concentration levels of ammonium, nitrate, and nitrite ions in a rice field, Environ. Monit. Assess., 154 (2009) 275–282.

[2] K.P. Woli, M.B. David, R.A. Cooke, G.F. Mcisaac, C.A. Mitchell, L.A. Schipper, A.J. Gold, E.A. Davidson, Nitrogen balance in and export from agricultural fields associated with controlled drainage systems and denitrifying bioreactors, Ecol. Eng., 36 (2010) 1558–1566.

[3] C. Li, F. Zhang, T. Zhu, T. Feng, P.L. An, Evaluation and correlation analysis of land use performance based on entropy-weight TOPSIS method, Trans. Chin. Soc. Agric. Eng., 29 (2013) 217–227 (in Chinese).

[4] P. Belder, B.A.M. Bouman, R. Cabangon, G.A. Lu, E.J.P. Quilang, Y.H. Li, J.H.J. Spiertz, T.P. Tuong, Effect of water-saving irrigation on rice yield and water use in typical lowland conditions in Asia, Agric. Water Manage., 65 (2014) 193–210.

[5] C.V. Kessel, A.J. Eagle, J.A. Bird, W.R. Horwath, B.A. Linquist, S.M. Brouder, J.E. Hill, Rice yield and nitrogen utilization efficiency under alternative straw management practices, Agron. J., 92 (2000) 1096–1103.

[6] A.H. Sun, S.J. Zhu, Y.F. Guo, Z.X. Zhang, Jensen model and modified Morgan Model for rice water-fertilizer production function, Procedia Eng., 28 (2012) 264–269.

[7] Z.K. Hu, C. Liu, W.H. Zhuang, T.T. Liu, T.G. Li, Z. Li, Optimization of multi-objective crop pattern based on water and fertilizer resources allocation, Appl. Mech. Mater., 522–524 (2014) 1034–1038.

[8] G. Vijayakumar, D. Tamilmani, P.K. Selvaraj, Maximizing Water and Fertilizer Use Efficiencies under Drip Irrigation in Chili Crop, J. Manage. Public Policy, 2010.

[9] R.D. Singh, S.K. Sikka, Shivani, K. Rajan, Effect of irrigation and nitrogen on yield, water use efficiency and nutrient balance in rice (*Oryza sativa*)-based cropping system, Indian J. Agric. Sci., 78 (2008) 21–26.

[10] P. Guo, G.H. Huang, L. He, B.W. Sun, ITSSIP, Interval-parameter two-stage stochastic semi-infinite programming for environmental management under uncertainty, Environ. Modell. Software, 23 (2008) 1422–1437.

[11] Q.F. Hu, S. Shang, S.G. Weng, B.Q. Meng, Modeling and analysis by partial least squares for water-fertilizer production function of winter wheat in Xiaohe irrigation district, Water Saving Irrigation, 1 (2006).1–4, 8 (in Chinese).

[12] Y.Q. Cao, L. Liu, L. Jiang, W.N. Zhang, Modeling and analysis of water-fertilizer production function of winter wheat by use of least-square regression, Adv. Sci. Technol. Water Resour., 2 (2010) 14–18 (in Chinese).

[13] Y.P. Li, G.H. Huang, S.L. Nie, X.S. Qin, ITCLP: an inexact two-stage chance-constrained program for planning waste management systems, Resour. Conserv. Recycl., 49 (2007) 284–307.
[14] P. Guo, G.H. Huang, Y.P. Li, An inexact fuzzy-chance-constrained two-stage mixed-integer linear programming approach for flood diversion planning under multiple uncertainties, Adv. Water Resour., 33 (2010) 81–91.
[15] G.H. Huang, D.P. Loucks, An inexact two-stage stochastic programming model for water resources management under uncertainty, Civ. Eng. Environ. Syst., 17 (2000) 95–118.
[16] B.W. Baetz, A grey fuzzy linear programming approach for municipal solid waste management planning under uncertainty, Civ. Eng. Environ. Syst., 10 (1993) 123–146.
[17] P.G. Guo, G.H. Huang, Y.P. Li, Interval stochastic quadratic programming approach for municipal solid waste management, Environ. Eng. Sci., 7 (2008) 569–579.
[18] G.H. Huang, Grey Mathematical Programming and Its Application to Municipal Solid Waste Management Planning, Civil Engineering, 1994.
[19] G.H. Huang, N. Sae-Lim, L. Liu, Z. Chen, An interval-parameter fuzzy-stochastic programming approach for municipal solid waste management and planning, Environ. Model. Assess., 6 (2001) 271–283.
[20] F.F. Tong, P. Guo, Simulation and optimization for crop water allocation based on crop water production functions and climate factor under uncertainty, Appl. Math. Model., 37 (2013) 7708–7716.
[21] H.D. Wang, L.F. Wu, M.H. Cheng, J.L. Fan, F.C. Zhang, Y.F. Zou, H.W. Chau, Z.J. Gao, X.K. Wang, Coupling effects of water and fertilizer on yield, water and fertilizer use efficiency of drip-fertigated cotton in northern Xinjiang, China, Field Crop Res., 219 (2018) 169–179.
[22] J.F. Pan, Y.Z. Liu, X.H. Zhong, R.M. Lampayan, G.R. Singleton, N.R. Huang, K.M. Liang, B. Peng, K. Tian, Grain yield, water productivity and nitrogen use efficiency of rice under different water management and fertilizer-N inputs in South China, Agric. Water Manage., 184 (2017) 191–200.
[23] H, Chen, C.Q. Yu, C.S. Li, Q.C. Xin, X. Huang, J. Zhang, Y. Yue, G.R. Huang, X.C. Li, W. Wang, Modeling the impacts of water and fertilizer management on the ecosystem service of rice rotated cropping systems in China, Agric. Ecosyst. Environ., 219 (2016) 49–57.
[24] Z. Zhang, Y. Zhang, M. Wang, Experimental study on coupling effect of water and nitrogen of rice in cold black soil region, J. Northeast Agric. Univ., 11 (2015) 49–55 (In Chinese).
[25] S. Donoso, N. Marín, M.A. Vila, Fuzzy Regression with Quadratic Programming: An Application to Financial Data, International Conference on Intelligent Data Engineering and Automated Learning, 2006, pp. 1304–1311.
[26] H. Tanaka, K. Koyama, H. Lee, Interval Regression Analysis Based on Quadratic Programming, IEEE International Conference on Fuzzy Systems, 1996, pp. 325–329.
[27] H. Tanaka, H. Lee, Interval regression analysis by quadratic programming approach, IEEE Trans. Fuzzy Syst., 6 (1998) 473–481.
[28] W. Li, An improving Procedure of the Numerical Solution Method for Interval Quadratic Programming, International Conference on Intelligent Computation Technology and Automation, 2009, pp. 165–168.
[29] W. Li, Algorithm for Interval Quadratic Programming with Equality Constraints, International Conference on Fuzzy Systems and Knowledge Discovery, 2009, pp. 78–81.
[31] S. Liu, R. Wang, A numerical solution method to interval quadratic programming, Appl. Math. Comput., 189 (2007) 1274–1281.
[32] P. Guo, G.H. Huang, Interval-parameter semi-infinite fuzzy-stochastic mixed-integer programming approach for environmental management under multiple uncertainties, Waste Manage., 30 (2010) 521–531.
[33] W. Li, X. Tian, Numerical solution method for general interval quadratic programming, Appl. Math. Comput., 202 (2008) 589–595.