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The inverse problem calculation of parabolic equation about the influence of solid particles in sewage on formation permeability

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ABSTRACT

In the water injection or the water treatment projects, the formation around the water injection well may be damaged due to solid particles. Therefore, in the process of the water injection, the inverse problem of the parabolic equation about the influence of solid particles in sewage on formation permeability is studied. When solid particles in sewage intrude into the formation, it will block the pores and reduce the formation permeability. The mathematical model is established, which can be divided into question 1 and question 2. Problem 1 is an inverse problem of partial differential equation with definite f, d_w, d_o and a. For problem 1, the homotopy continuation method is combined with the Newton method for the numerical solution. Problem 2 is an initial boundary value problem with definite s_p. On the basis of question 1, the solution can be realized by the particle swarm optimization algorithm, and the permeability of the reservoir can be obtained by s_p. The results show that the higher the concentration of solid particles in sewage is, the greater the damage to formation performance is.

Keywords: Solid particles in sewage; Formation permeability; Inverse problem of the parabolic equation

1. Introduction

Reinjection of wastewater is one of the most economical, effective and environmentally friendly reservoir development schemes. In the process of oilfield waterflooding development, the water injected into the formation always contains some solid particles or other chemical particles, which is not completely up to the standard. After the invasion of the formation, the water absorption capacity of the water injection layer is reduced and the oil layer is damaged, which can be divided into two types: In the first category, the particles infiltrated in the sewage layer caused various chemical and physical–chemical reactions, which changed the properties of rock and fluid in the strata and resulted in the decrease of formation permeability, such as the change of rock permeability, the deposition of salt, the loosening, migration and expansion of cemented particles, and the formation fluid was emulsified or formed into the foam. Second, the solid particles in the sewage invade the formation and block the pores to reduce the formation performance. After analysis, this kind of damage is considered to occur under the following mechanism: these solid particles form a layer of filter cake on the surface of the formation at the bottom of the well, making the well narrow. These solid particles intrude into the formation and form an inner filter cake, which makes the borehole narrow. These solid particles stay in the blast hole and cause blockage; these solid particles settle at the bottom of the well by gravity, which reduces the thickness of the production layer [1]. Often the occurrence of these four mechanisms is not single, sometimes two or more mechanisms occur at the same time. With the different water quality, the injury degree and proportion of various injury mechanisms will also change. Therefore, it is of great significance to study these damages for waterflood development.

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2. Materials and methods

In the process of oil field water injection development, the water quality of the formation does not fully meet the standard due to various reasons, and there is always a certain amount of solid particles. After these solid particles are injected into the formation with water, the formation pore channel will be blocked and the formation permeability will decrease [2], as shown in Fig. 1.

In this paper, the inverse problem model of a non-linear parabolic partial differential equation is discussed. Particle invasion is a complex but important mechanism, and also a typical example of the formation filtration process. In this process, the surface fluid velocity is a function of the location of the point in the reservoir. It is known that porosity a is the function of radial distance r and production time t, and permeability $b = b(a)$ is the function of a. The partial differential equation model with coefficients of a and $K(\phi)$ is established to describe the change rule of fluid pressure f, and porosity also changes with r and t. Therefore, this paper uses the inversion method to solve f, d_w, d_o, a, s_p [3] at the same time.

The inverse problem of mathematical equations has been applied in many important fields, such as geophysics, medicine, life science, material science, remote sensing technology and so on. The inverse problem of the mathematical equation is opposite to the positive problem. The positive problem refers to the classical problem of solving a differential equation, that is, the problem of solving a differential equation with a known differential equation and its definite solution conditions. The inverse problem refers to the problem that part of the information of the equation or the condition of the definite solution is unknown, but part of the known solution is regarded as the "additional condition", and the additional condition is used to find the unknown information [4]. Among them, there are many expressions of unknown information, such as the initial value, boundary value condition, solution region, some coefficients of the differential equation and so on. According to the unknown information to be solved and different additional conditions, the inverse problems of mathematical equations have different classifications. For some classical differential equation problems, such as the Cauchy problem and boundary problem of the hyperbolic equation and parallel equation, the Dirichlet problem of the elliptic equation and so on, the corresponding inverse problems can be proposed, and have a wide application background.

The solution of the positive problem of the mathematical equation is to give the coefficient and initial-boundary condition of a differential equation, and finally, get the solution of the differential equation. In the abstract, this process is a mapping from function (coefficient, initial boundary value condition) to function (solution of the equation); conversely, the inverse problem can also be seen as how to solve the inverse mapping of this mapping.

In the inverse problems of mathematical equations, additional conditions play an important role. A different formulation of additional conditions has different corresponding problems. For example, give the solution of differential equation with time-lapse T, or give the integral of the solution with respect to a certain variable, etc. Some of the methods for solving the inverse problems refer to the positive problems, such as the basic solution method, the finite difference method and the finite element method. Some also use the properties of the inverse problem itself, such as the Fourier moment method, Newton method, linear sampling method, homotopy method and so on [5].

Compared with the positive problem, most of the information given by the inverse problem is not necessarily a complete solution (or lack of information, or information redundancy), and the problem is usually non-linear. This brings more challenges, but because of its more practical application, the inverse problem has developed into a hot research topic in the cross fields of computing science, applied mathematics and industrial application. Because of this property of the inverse problem, we first introduce the concept of regularity. According to Hadamard's definition of the regularity of mathematical problems, it is generally believed that the solution of a mathematical problem should have the existence, uniqueness and stability, that is, the problem is well-defined, and only such a solution can have meaning of solution [6]. However, with the emergence of the inverse problem, most of the solutions of the inverse problem are ill-posed, that is, they do not satisfy the above three properties at the same time. At this point, we call the inverse problem ill-posed. Therefore, the main work of the inverse problem is to discuss the properties of the solution and to construct the numerical solution method.

So far, a lot of work has been done on the inverse problems of various mathematical equations, both in theory (to discuss the regularity of the solution of the inverse problem) and in numerical solutions (to construct a stable numerical solution).

The inverse problems of partial differential equations (PDEs) include differential equations, initial conditions, boundary conditions and additional conditions [7]. Their general forms are:

Differential equations: $\mathrm{Lu}(x,t) = f(x,t), x \in \Omega, t \in (0,\infty)$

Initial conditions: $\mathrm{Iu}(x,t) = \phi(x,t), x \in \Omega, t = 0$

Boundary conditions: $\mathrm{Bu}(x,t) = \phi(x,t), x \in \partial\Omega$

Additional conditions: $\mathrm{Au}(x,t) = k(x,t), x \in \partial\Omega$

where $u(x,t)$ is the solution of PDEs, $f(x,t)$ is the right-hand term, $\phi(x,t)$, $\phi(x,t)$ and $k(x,t)$ are initial boundary and additional conditions respectively. L, I, B and A are respectively partial differential operators, initial operators, boundary operators and additional operators. One of these quantities is unknown, which is the inverse problem of the partial differential equation. Therefore, inverse problems can be classified:

- When operator L is unknown, it is called the inverse problem of the operator recognition. Generally, the structure of operator L is known, but some parameters of operator are unknown, so this kind of inverse problem is often called a parameter identification problem.
- When the right term $f(x,t)$ is unknown, it is called the inverse problem of the source.

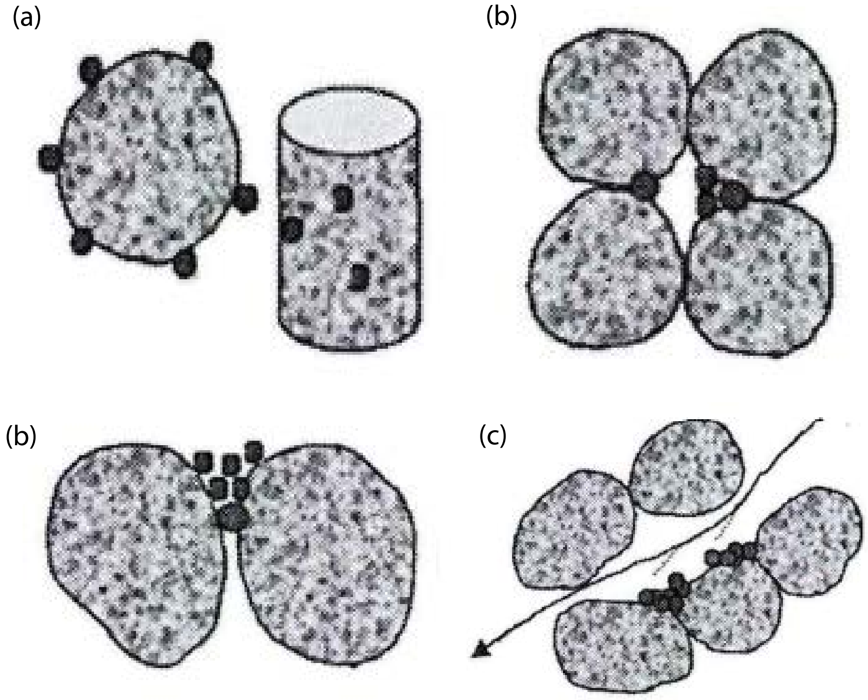

Fig. 1. Schematic diagram of the plugging position of solid particles: (a) surface, (b) crack, (c) throat, and (d) cave sand surface.

- When the initial condition $\phi(x,t)$ is unknown, the additional condition usually gives the state of the system at a certain time, so it is also called the inverse problem of the inverse time process.
- When boundary condition $\phi(x,t)$ is unknown, the inverse problem is called the inverse problem of boundary control in engineering.

2.1. Assumptions

Before studying the inverse problem of the parabolic equation about the effect of solid particles in sewage on formation permeability, we need to set up hypothesis conditions.

- Simulated formation is isotropic and slightly compressible;
- Oil and water are incompressible fluids, and they are permeating through the formation at constant temperature;
- Solid particles are water wet and exist in the water phase;
- Effect of particle retention on the conservation equation of the water phase is negligible;
- Influence of capillary force is not considered [8].

2.2. Mathematical model

2.2.1. Motion equation of oil–water two-phase

In the process of water injection, particles deposited in the pores and blocked the pore throat, resulting in a decrease of seepage area. It is assumed that the porosity $a = a(r,t)$ is a fully smooth unknown function with $\frac{\partial a}{\partial t} \le 0$, and the absolute permeability $b = b(a)$ is a known function of a, and $b'(a) \ge 0$. Relative permeability b_{ro} and viscosity c_0 of the oil phase are functions of saturation d_o, while relative permeability b_{rw} and viscosity c_w of water phase are functions of saturation d_w [9]. When the influence of the capillary force is not considered, the motion equation of oil–water two-phase is:

$$\begin{cases} \vec{e}_0 = \dfrac{b(a) b_{ro}(d_0)}{c_0(d_0)} \nabla f \\ \vec{e}_w = -\dfrac{b(a) b_{rw}(d_w)}{c_w(d_w)} \nabla f \\ d_0 + d_w = 1 \end{cases} \tag{1}$$

where $f = f(r,t)$ is the function of pressure.

2.2.2. State equation of oil–water two-phase

$$\begin{cases} g_o = g_{o0} h^{f_0} \\ g_w = g_{w0} h^{f_0} \end{cases} \tag{2}$$

where g_{o0} is the oil density when the fluid pressure is f_0; g_{w0} is the water density when the fluid pressure is f_0; h is the compression coefficient of the liquid.

2.2.3. Conservation law of matter for oil and water

$$-\nabla\left(g\vec{e}\right)=\frac{\partial\left(adg\right)}{\partial t} \tag{3}$$

$$\frac{\partial g}{\partial t}+j_i\left(d_i\right)\frac{\partial d_i}{\partial t}+k_i\left(a\right)\frac{\partial a}{\partial t}=l_i\left(a,d_i\right)\frac{\left[\frac{\partial}{\partial r}\left(\mathrm{rb}\left(a\right)\frac{b_{\mathrm{ri}}\left(d_i\right)}{c_i\left(d_i\right)}\frac{\partial f}{\partial r}\right)\right]}{r}+n_i\left(a,d_i\right)\left(\frac{\partial f}{\partial r}\right)^2 \tag{4}$$

where

$$i=o,w \tag{5}$$

$$j_i\left(d_i\right)=\frac{1}{l_i d_i} \tag{6}$$

$$k_i\left(a\right)=\frac{1}{l_i a} \tag{7}$$

$$l_i\left(a,d_i\right)=\frac{1}{l_i a d_i} \tag{8}$$

$$n_i\left(a,d_i\right)=\frac{1}{ad_i}\frac{b\left(a\right)b_{\mathrm{ri}}\left(d_i\right)}{c_i\left(d_i\right)} \tag{9}$$

2.2.4. Mathematical model

Under the hypothetical conditions, combined with the migration equation, retention rate equation of solid particles, the change of porosity and permeability after formation damage, the radial symmetric mathematical model with pressure f, particles concentration s_p, saturation d_i and porosity a after formation damage is derived [10].

$$\frac{\partial g}{\partial t}+j_o\left(d_o\right)\frac{\partial d_o}{\partial t}+k_o\left(a\right)\frac{\partial a}{\partial t}=l_o\left(a,d_o\right)\frac{\left[\frac{\partial}{\partial r}\left(\mathrm{rb}\left(a\right)\frac{b_{\mathrm{ro}}\left(d_o\right)}{c_o\left(d_o\right)}\frac{\partial f}{\partial r}\right)\right]}{r}+n_o\left(a,d_o\right)\left(\frac{\partial f}{\partial r}\right)^2 \tag{10}$$

$$\frac{\partial g}{\partial t}+j_w\left(d_w\right)\frac{\partial d_w}{\partial t}+k_w\left(a\right)\frac{\partial a}{\partial t}=l_w\left(a,d_w\right)\frac{\left[\frac{\partial}{\partial r}\left(\mathrm{rb}\left(a\right)\frac{b_{\mathrm{rw}}\left(d_w\right)}{c_w\left(d_w\right)}\frac{\partial f}{\partial r}\right)\right]}{r}+n_w\left(a,d_w\right)\left(\frac{\partial f}{\partial r}\right)^2 \tag{11}$$

$$s_p\frac{\partial d_w}{\partial t}+d_w\frac{\partial s_p}{\partial t}+d_w s_p=\frac{b\left(a\right)b_{\mathrm{rw}}\left(d_w\right)}{c_w\left(d_w\right)}\left(a,d_w\right)\left[s_p\frac{\partial^2 g}{\partial r^2}+\frac{\partial g}{\partial r}\frac{\partial s_p}{\partial r}\right] \tag{12}$$

$$d_0+d_w=1 \tag{13}$$

Conditions for the definite solution:

$$s_p\left(r,t\right)\big|_{t=0}=s_0,0<r<\infty \tag{14}$$

$$d_o\left(r,t\right)\big|_{t=0}=s_{o0},0<r<\infty \tag{15}$$

$$s_p\left(r,t\right)\big|_{r=r_w}=s_{p0}\left(t\right),t>0 \tag{16}$$

$$f\left(r,t\right)\big|_{t=0}=f_i,0<r<\infty \tag{17}$$

$$d_w\left(r,t\right)\big|_{t=0}=d_{w0},0<r<\infty \tag{18}$$

$$d_o\big|_{r=r_w}=d_o\left(t\right) \tag{19}$$

$$d_w\big|_{r=r_w}=d_w\left(t\right) \tag{20}$$

$$d_{w0}+d_{o0}=1 \tag{21}$$

$$f\left(\infty,t\right)=f,t>0 \tag{22}$$

The above problems are the initial boundary value problems of non-linear partial differential equations with unknown functions of pressure f, particle concentration s_p, saturation d_i and porosity a. In the formula, if the constants l, j, k and n are known, then the problem is a positive problem, which can be solved by using the initial and boundary data.

But now the l, j, k and n are functions with d_w, d_o and a, which depend on a large number of experimental results, so they cannot be measured at all. Therefore, the above model is reduced to question 1 and question 2 [11]. Problem 1 is an inverse problem with definite conditions for determining f, d_w, d_o and a, and problem 2 is an initial boundary value problem of s_p. Questions 1 and 2 are discussed in the following.

Question 1: determine the inverse problems of f, d_w, d_o and a.

Problem 1 is composed of Eqs. (10), (11), (13), (15), (17), (18) and (21) with a definite solution.

Problem 2: determine the initial boundary value problem of s_p.

$$d_w\frac{\partial s_p}{\partial t}=l_w\left(a,d_w\right)\frac{\partial f}{\partial r}\frac{\partial s_p}{\partial r}+\left[l_w\left(a,d_w\right)\frac{\partial^2 f}{\partial r^2}-\frac{\partial d_w}{\partial t}-d_w\right]s_p \tag{23}$$

Definite solution conditions: Eqs. (14) and (16).

When f, d_w, d_o and a are solved in problem 1, problem 2 is the initial boundary value problem of the first-order linear partial differential equation of s_p.

2.3. Numerical calculation

2.3.1. Numerical calculation of question 1

Problem 1 is the key and difficulty to solve this model. During production, the pressure drop funnel is formed near the bottom of the well, and the pressure shaving increases logarithmically to the wellbore. Therefore, when Eqs. (10) and (11) are discretized, it is advisable to divide ln$r$ equally according to the radial direction, and the step size is Δt. In order to maintain the flow conservation and volume conservation, the boundary of conductivity and mesh volume should be calculated respectively [12].

With regard to $i = o,w$, three non-linear equations with f, d_i and a of oil–water can be constructed respectively according to the definite solution conditions, namely, the non-linear equations about pressure f, saturation d_i and porosity a.

Because the problem needs to solve f, d_w, d_o and a at the same time, the jump interleaving method is used to solve it. The homotopy continuation method and the Newton method can be considered to solve the non-linear equations. The initial value of iteration is obtained by the homotopy continuation method, and the solution is refined by the Newton method [13]. To solve the current model, the adaptive continuation algorithm is proposed. In other words, the homotopy iteration with parameters is used to get a better approximate solution x^* of the true solution as the initial value, and Newton iteration is used to refine it.

2.3.1.1. Homotopy continuation method

Many inverse problems can be reduced to the problem of finding the zero point or the minimum value of a mapping. If $F{:}R^n \to R^n$ is introduced as the second-order continuous differentiable mapping, the most basic method is the Newton method. Given an initial value of x_0, its iteration formula is as follows:

$$F(x)=0 \tag{24}$$

$$x_{k+1}=x_k-F'(x_k)^{-1}F(x_k),\ k=0,1,... \tag{25}$$

The x_k converges to the solution x^* of $F(x) = 0$ under certain conditions. Newton's method has the advantage of second-order convergence, but it has the limitation of local convergence. With the development of science and technology, a large number of non-linear problems are put forward. It is very difficult to obtain the initial value which is enough to ensure convergence for these problems. Therefore, the study of large-scale convergence algorithms has become a meaningful topic [14].

Homotopy method is produced for solving Eq. (24) in a large range. The method is to construct a mapping Q which is easy to be solved, and then transform Q continuously to F, that is to construct homotopy $H{:}R^n \times R^1 \to R^n$:

$$H(x,\lambda)=\alpha(\lambda)Q(x)+\beta(\lambda)F(x) \tag{26}$$

$$H(x,0)=Q(x),H(x,1)=F(x) \tag{27}$$

If there is a C^1 is the dimensional manifold (homotopy path) $[x(s),\lambda(s)]$ that satisfy $H[x(s),\lambda(s)] = 0$, and the solution x^0 of $Q(x) = 0$ is connected with the solution x^* of $F(x) = 0$, then the curve can be traced from x^0 to x^*.

The predecessor of the homotopy method is the continuation method, however, the continuation method has a great weakness, it requires a monotonic change of homotopy parameters, which in essence requires $H_x'(x,\lambda)$ to be non-singular in a large range, which makes it far from achieving a large-scale solution [15].

2.3.1.2. Newton method

Newton's method, also known as the Newton–Raphson's method, is a method proposed by Newton in the 17th century to solve equations approximately in the real and complex fields [16]. How to construct the iterative function is very important when using the iterative method to solve the non-linear equation, so how to construct the iterative function to ensure the convergence of the iterative method? Newton's iterative method is one of the commonly used methods. There are about two ways to source or understand its iterative format:

- Let $f(x) = C^2[a,b]$, $f(x)$ is handed with Taylor expansion at point $x_0 \in [a,b]$:

$$f(x)=f(x_0)+f'(x_0)(x-x_0)+\frac{f''(\xi)(x-x_0)^2}{2} \tag{28}$$

By omitting the quadratic term, the linear approximation $f(x) \approx f(x_0) + f'(x_0)(x–x_0)$ of $f(x)$ is obtained. Thus, the approximate root $x=x_0-\frac{f(x_0)}{f'(x_0)}$ (assumption $f'(x_0) \neq 0$) of equation $f(x) = 0$ is obtained, and the iterative format $x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$ (assumed $f'(x_k) \neq 0$) can be constructed. This is Newton's iterative formula. If the resulting sequence $\{x_k\}$ converges to α, then α is the root of the non-linear equation [17].

- Newton iterative method is also known as Newton tangent method. Let ξ be the root of $f(x) = 0$, x_0 is chosen as the initial approximate value of ξ, and the tangent L of curve $y = f(x)$ through point $(x_0,f(x_0))$ is given. The equation of L is $y = f(x_0) + f'(x_0)(x - x_0)$. The abscissa $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ of the intersection of L and x-axis is obtained, and X_1 is the primary approximate value of ξ. Through point $(x_1,f(x_1))$, the tangent of curve $y = f(x)$ is made. The abscissa $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ of the intersection of tangent and x-axis is obtained, and x_2 is the quadratic approximation of ξ. Repeat the above process to obtain the approximation sequence of ξ, where $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$ is called the $n + 1$

approximation of ξ, and the above formula is called the Newton iteration formula, as shown in Fig. 2a. This is the geometric interpretation of the Newton tangent method [18]. In fact, the Newton iterative method can also be derived from the geometric sense. $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$. Using Newton's iterative formula, we can get x_{k+1} from x_k. From Fig. 2b, we can see that the intersection of the tangent L_k and the x-axis is x_{k+1}, so $x_{k+1} = \frac{f(x_k)}{x_k - x_{k+1}}$. After finishing, we can also get Newton's iterative formula:

2.3.2. Numerical calculation of question 2

On the basis of the numerical solution of problem 1, problem 2 becomes a linear first-order partial differential equation, which can be solved by the particle swarm optimization algorithm based on selection, thus s_p and the damage radius can be determined.

Combining the natural selection mechanism with particle swarm optimization, a particle swarm optimization algorithm based on selection is proposed. The basic idea is to sort the whole particle swarm according to the fitness value in each iteration, replace the position and speed of the worst half with the speed and position of the best half of the particles in the swarm, and keep the historical optimal value that each individual remembers [19]. It is an important direction to combine intelligent algorithms with a classical algorithm to construct some efficient new hybrid algorithms.

The basic idea of this paper is to combine the natural selection based hybrid particle swarm optimization (PSO) algorithm with the finite element method to solve the inverse problem of the parallel equation. The PSO algorithm and the finite element method are used circularly until the predetermined optimization target is met. The basic steps of the hybrid PSO algorithm based on the selection are as follows:

- *Step* 1: set the current parameters: number of particles, learning factor 1, learning factor 2, inertia weight, the maximum number of iterations, the dimension of the problem, range of positions, and position of particles.
- *Step* 2: randomly initialize the position and velocity of each particle in the population.
- *Step* 3: evaluate the fitness of each particle. The location and fitness of each particle are stored in the pbest of each particle, and the location and fitness of the individual with the best fitness are stored in the gbest.
- *Step* 4: update the speed and position of each particle.
- *Step* 5: for each particle, compare its fitness value with the best position it has experienced, and if it is better, take it as the best position currently.
- *Step* 6: compare and update the current values of all pbests and gbests.
- *Step* 7: rank the whole particle swarm according to the fitness value, replace the position and speed of the worst half with the speed and position of the best half of the particles in the swarm, and keep the pbest and gbest unchanged.

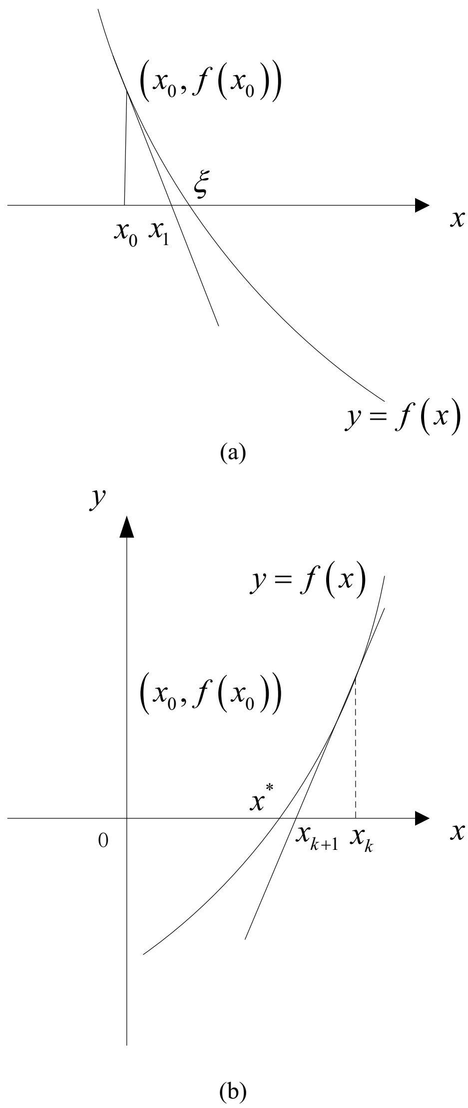

Fig. 2. Newton tangent method.

- *Step* 8: if the stop condition (usually the preset operation precision or iteration times) is met, the search stops and the result is output. Otherwise, return to (step 2), and continue the search.

PSO is a new optimization technology. Its idea comes from the theory of artificial life and evolutionary computation. PSO is optimized by particles following the best solution they find and the best solution for the whole group. PSO algorithm is simple and easy to implement with a few adjustable parameters, which have been widely studied and applied. A hybrid particle swarm optimization algorithm is proposed to solve the inverse problem of partial differential equation by combining the finite element method. It has obtained good results in the test of the inverse problem model of multiple parallel equations, which reflects the effectiveness, generality and robustness of the algorithm [20].

The pressure f, oil–water saturation $d_i(d_w, d_o)$ and porosity a of each point in the formation at any time are obtained by question 2. By substituting them into problem 2, the coefficients of the equation can be determined, and problem 2 becomes the initial and boundary value problem of the first-order linear differential equation. The particle concentration s_p can be calculated by the method of characteristic line. According to s_p and a, the permeability and damage radius of the formation can be calculated.

3. Results

In order to study the formation damage caused by solid particles in sewage to offshore sandstone reservoir, we will take the Shahejie Reservoir core as an example to study the indoor solid particle damage. Through the core displacement experiment, the influence of solid particles with different concentration and particle size on core permeability is determined.

3.1. Experimental core

The core of Shahejie Reservoir (gas permeability between) is selected as the test core. Before the experiment, the basic data of the core is obtained by vacuuming saturated formation water and other processes [21]. At the same time, the flow experiments under different confining pressures are carried out to eliminate the core lag effect. The specific core parameters are shown in Table 1.

3.2. Experimental sewage

3.2.1. Solid particles

The particles used are standard particles produced by Nawei Technology Co., Ltd.-, (No.2, Baichuan street, Suzhou Industrial Park). this standard particle is composed of highly cross-linked styrene diethylene glycol, with good stability, standard spherical structure and narrow particle size distribution. It can simulate the damage of different particle size suspended solids to the reservoir [22]. Different particle sizes were tested and checked on the laser particle size, which completely met the experimental requirements. Different particle sizes were prepared into different concentrations of suspended particle solutions. Samples were placed in beakers for precipitation, and the precipitation time was recorded. The precipitation time was long, which met the experimental requirements. During the experiment, manual stirring was carried out for 15 min.

3.2.2. Determination of particles size

The main throat radius distribution of Shahejie Reservoir in *B* oilfield is mainly 3–5 μm (average 4 μm), and the maximum is not more than 10 μm.

3.2.3. Determination of particles concentration

According to the current standard of water quality for injection in *B* oilfield, the concentration of solid particles

Table 2
Experimental factors and horizontal design

Project	Parameter level			
	Shahejie Reservoir			
Permeability k (md)	1–10	10–50	50–100	–
Particle sized (μm)	0.5	1	2	–
Particle concentration c_p (mg/L)	0.5	1	3	5

Table 1
Basic parameters of core

Number	Diameter (cm)	Length (cm)	Porosity (%)	Gas permeability (md)	Throat radius (μm)
Sand 1–1	2.5	5.36	11.26	7.14	1.36
Sand 1–2	2.5	5.12	11.42	7.36	1.85
Sand 1–3	2.5	5.69	11.20	8.60	1.65
Sand 1–4	2.5	5.55	11.36	6.69	1.89
Sand 1–5	2.5	5.10	12.35	7.14	2.25
Sand 1–6	2.5	5.23	11.36	7.20	1.25
Sand 1–9	2.5	5.45	10.60	6.85	2.01
Sand 1–10	2.5	5.23	11.69	6.32	2.58
Sand 1–11	2.5	5.02	11.58	7.12	1.58
Sand 1–12	2.5	5.46	12.03	6.32	1.25
Sand 1–13	2.5	5.66	10.47	7.14	1.45
Sand 1–14	2.5	5.50	10.58	6.58	1.69
Sand 1–15	2.5	5.32	11.54	9.53	1.97
Sand 1–16	2.5	5.58	11.69	8.64	1.89
Sand 1–17	2.5	5.56	11.75	7.69	2.13

Table 3
Damage degree of permeability

No.	Rate of flow (mL/min)	Particle radius (μm)	Particle concentration (mg/L)	Water permeability (md)	Core permeability damage rate (%)
Sand 1–1	1	0.17	0.5	2.37	10.36
Sand 1–2	1	0.25	0.5	3.10	11.26
Sand 1–3	1	0.29	0.5	2.10	12.36
Sand 1–4	1	1.30	0.5	2.80	14.54
Sand 1–5	1	1.25	1	2.97	15.47
Sand 1–6	1	1.53	1	3.21	16.36
Sand 1–9	1	2.36	1	3.10	17.14
Sand 1–10	1	2.50	1	3.02	18.36
Sand 1–11	1	2.62	3	2.54	20.14
Sand 1–12	1	2.96	3	2.69	22.36
Sand 1–13	1	3.27	3	3.58	23.65
Sand 1–14	1	3.73	3	3.42	25.40
Sand 1–15	1	4.03	5	4.02	28.52
Sand 1–16	1	4.18	5	4.25	30.14
Sand 1–17	1	4.32	5	4.44	32.11

in Shahejie Reservoir is required to be less than 10 mg/L. Combined with the water quality standard, we determined that the concentration levels of solid particles required for the experiment were 0.5, 1, 3 and 5 mg/L respectively. Particle size and concentration level of experimental particles and determination of experimental parameters were shown in Table 2.

3.3. Analysis of experimental results

The numerical calculation is carried out according to the above text method, and the results are shown in Table 3 below.

The results show that when the particle concentration in the injected water is constant, the core permeability damage rate of each point in the formation increases with the radius at the same time, that is, the formation near the wellbore is seriously damaged, and the formation farther away from the wellbore is less damaged [23]. In addition, with the increase of the particle concentration in the water injection, the damage degree of each point increases, and its amplitude decreases with the increase of the radius, and the damage range also increases. Therefore, controlling or reducing the particle concentration of injected water is an effective way to avoid or reduce formation damage and prevent water injection and water absorption [24–26].

4. Conclusions

Oilfield waterflooding is a commonly used development method in China. The quality of waterflooding water has an important influence on the protection of the oil layer and the realization of stable production. Injection of unqualified water will cause damage to the reservoir, especially for offshore sandstone reservoirs. The suspended particles contained in the injected water enter and block the internal throat of the reservoir, resulting in the decline of the permeability of the reservoir, which affects the development efficiency of the oilfield. Therefore, it is necessary to optimize the water quality of water injection. For this reason, this paper carries out the numerical calculation of the inverse problem of the parabolic equation about the affection of solid particles in sewage on the formation permeability and has achieved some results, but there are still many aspects of work to be further deepened and improved. For example, in the research process of this paper, it is found that there are many factors affecting the damage of particle migration in water injection wells, so the system model considering different factors is not the same. There are still many limitations in the research of the inverse problem of the parabolic equation about the affection of solid particles in sewage on formation permeability.

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References

[1] R.S. Volkov, S.V. Chvanov, D.D. Andriyanov, Diagnostics of the presence of solid particles in aqueous aerosol droplets by their interference pattern, Tech. Phys. Lett., 45 (2019) 605–608.
[2] Y. Litti, A. Nikitina, D. Kovalev, A. Ermoshin, R. Mahajan, G. Goel, A. Nozhevnikova, Influence of cationic polyacrilamide flocculant on high-solids’ anaerobic digestion of sewage sludge under thermophilic conditions, Environ. Technol., 40 (2019) 1146–1155.

[3] H.C. Du, M. Steinacher, C. Borca, T. Huthwelker, A. Murello, F. Stellacci, E. Amstad, Amorphous $CaCO_3$: influence of the formation time on its degree of hydration and stability, J. Am. Chem. Soc., 140 (2018) 14289–14299.
[4] P. Liang, J.-L. Zhu, Y.-Q. Zhang, X.-Z. Qin, X.-L. Lv, Study on the flow characteristics of mixed solid–solid particles, J. Chem. Eng. Jpn., 51 (2018) 477–483.
[5] E. Turibamwe, R. Wangalwa, A comparative study of two biological monitoring systems in assessing water quality: a case of River Birira, Sheema District, Uganda, Water Conserv. Manage., 4 (2020) 7–14.
[6] A. Maqbool, W. Hui, M.T. Sarwar, Nanotechnology development for in-situ remediation of heavy Metal(Loid)S contaminated soil, Environ. Ecosyst. Sci., 3 (2019) 9–11.
[7] A. Samsudin, M.A.A. Samah, M.Y. Ishak, Perception of government servants in Shah Alam on the utilization of food waste as a resource for biogas production in Malaysia, J. Clean WAS, 3 (2019) 16–20.
[8] K. Zaheri, M. Bayareh, A.A. Nadooshan, Numerical simulation of the motion of solid particles in a stirred tank, Int. J. Heat Technol., 37 (2019) 109–116.
[9] Y. Zhuang, X.J. Liu, H.Q. Xiong, L.X. Liang, Microscopic mechanism of clay minerals on reservoir damage during steam injection in unconsolidated sandstone, Energy Fuels, 32 (2018) 4671–4681.
[10] S.N. Antontsev, S.E. Aitzhanov, Inverse problem for an equation with a nonstandard growth condition, J. Appl. Mech. Tech. Phys., 60 (2019) 265–277.
[11] L. Govindarao, J. Mohapatra, Numerical analysis and simulation of delay parabolic partial differential equation involving a small parameter, Eng. Comput., 37 (2020) 289–312.
[12] S. Shekarpaz, H. Azari, Splitting method for an inverse source problem in parabolic differential equations: error analysis and applications, Numer. Methods Partial Differ. Equations, 36 (2020) 654–679.
[13] J. Vala, P. Jarošová, Optimization approaches to some problems of building design, Appl. Math., 63 (2018) 305–331.
[14] A.I. Korolkov, A.V. Shanin, A.A. Belous. Diffraction by an elongated body of revolution with impedance boundaries: the boundary integral parabolic equation method, Acoust. Phys., 65 (2019) 340–347.
[15] V.A. Permyakov, M.S. Mikhailov, E.S. Malevich, Analysis of propagation of electromagnetic waves in difficult conditions by the parabolic equation method, IEEE Trans. Antennas Propag., 67 (2019) 2167–2175.
[16] X.Q. Zhang, C.D. Sarris, Vector parabolic equation-based derivation of rectangular waveguide surrogate models of arched tunnels, IEEE Trans. Antennas Propag., 66 (2018) 1392–1403.
[17] M.B. Muhlestein, V.E. Ostashev, D.K. Wilson, A green's function parabolic equation description of infrasound propagation in an inhomogeneous and moving atmosphere, J. Acoust. Soc. Am., 143 (2018) 1740, https://doi.org/10.1121/1.5035677.
[18] Z. He, H.-C. Yin, R.-S. Chen, Fast analysis of electromagnetic scattering from a coated conductor with the parabolic equation, Appl. Comput. Electromagn. Soc. J., 33 (2018) 1293–1299.
[19] Y. Gao, Q. Shao, B.Z. Yan, Q.F. Li, S.X. Guo, Parabolic equation modeling of electromagnetic wave propagation over rough sea surfaces, Sensors, 19 (2019) 1252, doi: 10.3390/s19051252.
[20] R. Wang, Z. Li, G. Lu, Combination of the improved diffraction nonlocal boundary condition and three-dimensional parabolic equation decomposed model for predicting radio wave propagation, J. Electron. Inf. Technol., 40 (2018) 151–156.
[21] P.V. Konarev, F.N. Chukhovskii, V.V. Volkov, To the solution of the inverse problem of X-ray topo-tomography. Computer algorithms and 3D reconstruction on the example of a crystal with a point defect of coulomb type, Crystallogr. Rep., 64 (2019) 191–200.
[22] M.L. Li, L.F. Wang, Numerical solution of fractional partial differential equation of parabolic type using Chebyshev Wavelets method, Eng. Lett., 26 (2018) 224–227.
[23] L. Baudouin, A. Rondepierre, S. Neild, Robust control of a cable from a hyperbolic partial differential equation model, IEEE Trans. Control Syst. Technol., 27 (2019) 1343–1351.
[24] F. Gu, J.F. Guo, W.J. Zhang, P.A. Summers, P. Hall, From waste plastics to industrial raw materials: a life cycle assessment of mechanical plastic recycling practice based on a real-world case study, Sci. Total Environ., 601 (2017) 1192–1207.
[25] F. Gu, W.J. Zhang, J.F. Guo, P. Hall, Exploring 'Internet + Recycling': mass balance and life cycle assessment of a waste management system associated with a mobile application, Sci. Total Environ., 649 (2019) 172–185.
[26] J.K. Liu, Y.D. Liu, X.T. Wang, An environmental assessment model of construction and demolition waste based on system dynamics: a case study in Guangzhou, Environ. Sci. Pollut. Res. Int., 27 (2020) 37237–37259.