



## Marangoni convection flow of two immiscible fluids in an open cavity

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### ABSTRACT

This paper concerns the numerical study of the flow characteristics of Marangoni convection in a square cavity filled with two layers of immiscible fluids. The enclosure has an adiabatic bottom wall and a free surface subjected to a thermocapillary effect, while the vertical walls are differentially heated. The governing equations are discretized using the finite volume method. The system of algebraic equations obtained is solved using the line by line method, based on the SIMPLER algorithm to treat the pressure-velocity coupling. The explanation of the influence of various dimensionless parameters such as the dynamic viscosity and thermal diffusivity ratios on both hydrodynamic and thermal behavior of the flow is provided taking into account the interaction of the two fluids within the cavity. The results show that a high Marangoni number ( $Ma = 1,000$ ) generates a movement of the fluid from low values of interfacial tension to large values. Heat transfer is reinforced when the upper phase in contact with air is less viscous and more conductive of heat.

*Keywords:* Thermocapillary convection; Marangoni convection; Finite volume method; Two immiscible fluids; Square cavity

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### 1. Introduction

Due to its applications, the convective heat transfer in a system composed of different superposed fluid layers has received much consideration over the past several years in engineering design of advanced technology. These applications cover such diverse fields as solar energy collectors, storage tanks, crystal growth, and many other geological, chemical, and astrophysical systems.

In the literature, there are few works that have studied heat transfer phenomena in the presence of two immiscible fluids [1–4]. Alsabery et al. [5] analyzed the natural convection in a cavity filled with two immiscible fluids: a layer of non-Newtonian fluid in contact with a porous layer of the nanofluid. The authors studied the effect of different control parameters on heat transfer such as: Rayleigh and Darcy numbers, volume fraction of nanoparticles, power law index, thickness of the porous layer and the inclination angle of the cavity. Their results showed that heat transfer

is affected by changing the inclination angle of the cavity and by the variation of the quoted parameters. Koster and Nguyen [6] have numerically investigated the natural convection in a cavity composed of two superposed phases; each phase is occupied by one fluid with the lower phase presents a density inversion. The existence of thermal transport between the upper layer and the density inversion layer across the interface has been proven by their study. Natural convection with entropy generation in a cubic enclosure partially filled with air and nanofluid has been studied numerically and experimentally by Malekshah and Salari [7]. The effect of several parameters including the interface aspect ratio, the Rayleigh number and the volume fraction of solid particles has been examined. The results showed that the generation of entropy and the average Nusselt number are reduced by increasing the interface aspect ratio. Prakash and Koster [8] have realized an experimental and numerical study on the thermal convection of two layers of liquids in a system heated from below. It has

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been shown that the thermal coupling is best when the buoyancy forces in the two layers have similar resistance.

The thermocapillary convection, also known as Marangoni convection, in layers of immiscible fluids has recently attracted the attention of researchers due to its several industrial applications including elimination of evaporation of volatile components, surface melting and alloying techniques, processing of ceramics and semi-conductors that can involve two phases (molten and gaseous). When two immiscible layers are superposed in a differentially heated cavity, one has not been interested only in convection problems induced by the density variations, but also by the surface tension gradients that may exist along the interface separating the two immiscible liquids and the free surface in contact with air. The works involve Marangoni convection can be found in [9,10]. Wang and Kahawita [11] studied the natural convection of Marangoni in a system with two layers of stratified fluids, without taking into account mass transfer at the interface. Their results were given in terms of velocity, temperature distribution and variation of the Nusselt number over a wide range of control parameters. These results showed the existence of different aspects of the complex interaction between the mechanisms of buoyancy and surface tension. Golia and Viviani [12] studied the natural convection of Marangoni with two immiscible liquids superposed and enclosed in a rectangular cavity heated by its sidewalls. The problem was solved via a numerical computer code based on the finite difference method. Their results showed that for the small aspect ratios of the enclosure, the numerical results are in agreement with the analytical solution obtained according to the approximation of the shallow cavities. Liu et al. [13] have studied numerically the thermocapillary and thermogravitational convection of two immiscible fluids in a cavity heated and cooled by its sidewalls. These two layers of fluids are superposed not only because of the density differences in the gravitational field, but also because of the interfacial gradients. The results marked the presence of an interfacial cell either in the upper or lower layer, its size is a function of different parameters such as the coefficient of expansion, the viscosity and the thickness of each layer of fluid. The role of this cell is to guarantee the continuity of speed and balance of stresses. Marangoni thermal-solutal convection in a shallow rectangular free-surface cavity subject to mutually perpendicular temperature and concentration gradients has been examined by Zhang et al. [14]. The results showed that with the increase of the ambient temperature in the case of a constant flow, the intensity of the thermal Marangoni effect decreases, then increases; moreover, the flow structure, temperature and concentration fields in the cavity are significantly affected by the radiative heat transfer on the free surface.

Jory and Anbalagan [15] realized an axisymmetric two-dimensional cylindrical geometry to numerically evaluate the effect of Marangoni convection in shallow liquid layers. They used oil as a test fluid. Their study showed that Marangoni convection is further influenced by several parameters, including temperature gradients (9.0, 11.5 and 14.0 K), thickness of the two layers (4.0, 4.5 and 5.0 mm) and microgravity conditions ( $0.5 \times 10^{-3}$  g). The results also revealed that the thermo-capillary force is less important. Moreover, microgravity studies have guaranteed that Marangoni convection dominates natural convection.

In order to enrich research in this field, we intend to undertake this work which consists in the numerical study of the thermocapillary convection in an open square cavity partially filled with two immiscible fluids. The interface separating the two immiscible fluids as well as the free surface of the enclosure is subjected to surface tension gradients. The study focuses on the effect of two pertinent parameters on the flow pattern and heat transfer. It is about the dynamic viscosity and the thermal diffusivity ratios of the two liquids.

## 2. Statement of the problem and mathematical formulation

The present study aims to analyze the thermocapillary flow entrained in an open two-dimensional (2D) cavity filled with two immiscible and incompressible viscous liquids: liquid 1 (lower phase) and liquid 2 (upper phase). The aspect ratio of each individual phase is taken equal to 1/2.

We have not identified the nature of the two liquids since we are going to vary their ratio of dynamic viscosity and thermal diffusivity in order to study all the possibilities (different combinations of the two liquids). Therefore, the variation of these two parameters implies the change of their type.

The enclosure presented in Fig. 1 has an adiabatic bottom wall and a free surface subjected to the thermocapillary effect, while the vertical walls are heated by deference. The two immiscible liquids are superimposed in this cavity and they are in contact with the air via the free surface. In order to simplify the resolution of the problem, we consider both the upper free surface and the planar liquid-liquid interface as non-deformable (Sen and Davis [16]).

The industrial application associated with this type of configuration is the crystal growth technique. The latter is one of the engineering applications involving the study of heat transfer in systems containing layers of dissimilar (immiscible) fluids. This process aims to make higher quality single crystals from the polycrystalline materials melted in a differentially heated crucible.

At the interface between the two immiscible layers, the surface tension is supposed to vary linearly with the temperature according to the following expression:

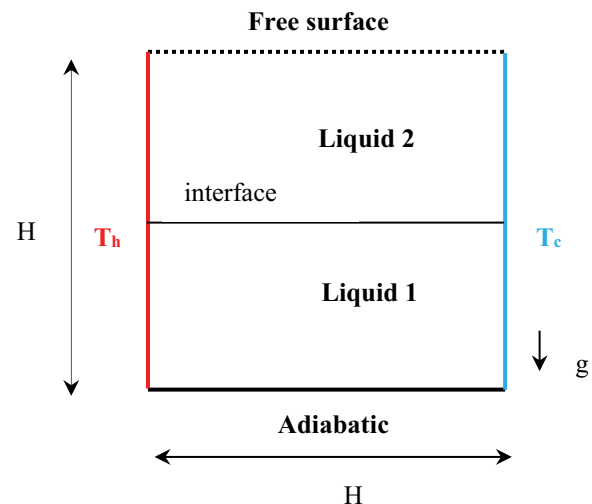


Fig. 1. Geometry of the physical problem.

$$\sigma_{1-2} = \sigma_{(1-2)0} - \gamma_{1-2}(T_1 - T_0) \quad (1)$$

where  $T_0 = (T_h + T_c)/2$  is the temperature coefficient of interfacial tension. The subscript 0 denotes a reference state.  $\gamma_{1-2} = -\partial\sigma_{1-2}/\partial T$  is the temperature coefficient of surface tension between liquid 1 and liquid 2.

On the free top surface, a second-surface tension effect exists and is given by:

$$\sigma_{2-air} = \sigma_{(2-air)0} - \gamma_{2-air}(T_2 - T_0) \quad (2)$$

$$\text{With } \gamma_{2-air} = \frac{\partial\sigma_{2-air}}{\partial T} \quad (3)$$

$\gamma_{2-air}$  is the temperature coefficient of surface tension between liquid 2 and air.

By considering the following dimensionless parameters:

$$X = \frac{x}{H}, Y = \frac{y}{H}, (U, V)_i = \frac{(U, V)_i H}{\alpha_1}, \quad (4)$$

$$P_i = \frac{P_i H^2}{\rho_i \alpha_1^2}, \theta_i = \frac{(T_i - T_c)}{\Delta T}, Pr = \frac{\nu_1}{\alpha_1}$$

where  $\Delta T = T_h - T_c$ ;  $i$  represents the fluid ( $i = 1, 2$ ).

The non-dimensional form of continuity, momentum and energy equations can be written as follows:

For the lower layer:

$$\frac{\partial U_1}{\partial X} + \frac{\partial V_1}{\partial Y} = 0 \quad (5)$$

$$U_1 \frac{\partial U_1}{\partial X} + V_1 \frac{\partial U_1}{\partial Y} = -\frac{\partial P_1}{\partial X} + Pr \left( \frac{\partial^2 U_1}{\partial X^2} + \frac{\partial^2 U_1}{\partial Y^2} \right) \quad (6)$$

$$U_1 \frac{\partial V_1}{\partial X} + V_1 \frac{\partial V_1}{\partial Y} = -\frac{\partial P_1}{\partial Y} + Pr \left( \frac{\partial^2 V_1}{\partial X^2} + \frac{\partial^2 V_1}{\partial Y^2} \right) \quad (7)$$

$$U_1 \frac{\partial \theta_1}{\partial X} + V_1 \frac{\partial \theta_1}{\partial Y} = \left( \frac{\partial^2 \theta_1}{\partial X^2} + \frac{\partial^2 \theta_1}{\partial Y^2} \right) \quad (8)$$

For the upper layer:

$$\frac{\partial U_2}{\partial X} + \frac{\partial V_2}{\partial Y} = 0 \quad (9)$$

$$U_2 \frac{\partial U_2}{\partial X} + V_2 \frac{\partial U_2}{\partial Y} = -\frac{1}{\rho^*} \frac{\partial P_2}{\partial X} + \nu^* Pr \left( \frac{\partial^2 U_2}{\partial X^2} + \frac{\partial^2 U_2}{\partial Y^2} \right) \quad (10)$$

$$U_2 \frac{\partial V_2}{\partial X} + V_2 \frac{\partial V_2}{\partial Y} = -\frac{1}{\rho^*} \frac{\partial P_2}{\partial Y} + \nu^* Pr \left( \frac{\partial^2 V_2}{\partial X^2} + \frac{\partial^2 V_2}{\partial Y^2} \right) \quad (11)$$

$$U_2 \frac{\partial \theta_2}{\partial X} + V_2 \frac{\partial \theta_2}{\partial Y} = \alpha^* \left( \frac{\partial^2 \theta_2}{\partial X^2} + \frac{\partial^2 \theta_2}{\partial Y^2} \right) \quad (12)$$

The ratios of physical properties of the two fluid layers are defined as:

$$\rho^* = \frac{\rho_2}{\rho_1}, \nu^* = \frac{\nu_2}{\nu_1}, \mu^* = \frac{\mu_2}{\mu_1}, k^* = \frac{k_2}{k_1}, \alpha^* = \frac{\alpha_2}{\alpha_1} \quad (13)$$

where  $\rho$ ,  $\nu$ ,  $\mu$ ,  $\kappa$  and  $\alpha$  are the density, the kinematic viscosity, the dynamic viscosity, the thermal conductivity and the thermal diffusivity of fluids.

The boundary conditions associated to the governing equations cited above are as follows:

$$\text{At } X=0: U_i = V_i = 0, \theta_i = 1 \quad (14)$$

$$\text{At } X=1: U_i = V_i = 0, \theta_i = 1 \quad (15)$$

$$\text{At } Y=1: U_i = V_i = 0, \frac{\partial \theta_i}{\partial Y} = 1 \quad (16)$$

In order to facilitate the resolution of the problem and ensure the coupling between the two layers of fluid, the continuity of both velocity and temperature, heat flux balance and shear stress balance are performed at the liquid-liquid interface and the upper surface:

### 2.1. At the interface between the two fluids

$$U_1 = U_2 \quad (17)$$

$$V_1 = V_2 = 0 \quad (18)$$

$$\theta_1 = \theta_2 \quad (19)$$

$$\frac{\partial \theta_1}{\partial Y} = k^* \frac{\partial \theta_2}{\partial Y} \quad (20)$$

$$\frac{\partial U_1}{\partial Y} = \mu^* \frac{\partial U_2}{\partial Y} - Ma \frac{\partial \theta_2}{\partial X} \quad (21)$$

Eqs. (20) and (21) express, respectively, the continuity of heat flux and the viscous stresses between the two liquids 1 and 2.

### 2.2. At the free surface

$$V_2 = 0 \quad (22)$$

$$\frac{\partial \theta_2}{\partial Y} = -Bi(\theta_2 - \theta_{air}) \quad (23)$$

$$\mu^* \frac{\partial U_2}{\partial Y} = -\sigma^* Ma \frac{\partial \theta_2}{\partial X} \quad (24)$$

Eq. (24) expresses the continuity of the viscous stresses existing between the air and the liquid 2 of the upper phase. The integration of this equation, along the free surface, leads to determine the expression of the velocity  $U_2$  at this location.

where  $Ma = \frac{\gamma_{1-2}\Delta TH}{\mu_1\alpha_1}$  is the Marangoni number which represents the intensity of the thermocapillary forces with respect to diffusion forces and  $Bi = \frac{H/k}{1/h_{air}}$  is the Biot number which is used to control the upper free surface and expresses the ratio between the effects of thermal convection at the free surface and the conduction effects of the fluid in contact with this limit [11].

Eqs. (22)–(24) are used as a boundary condition on this open surface. The local Nusselt number is evaluated along the heated wall by the following expression:

$$Nu = -\frac{\partial\theta}{\partial X}\Big|_{X=0} \tag{25}$$

The average Nusselt number is determined from:

$$Nu_{avg} = \int_0^1 Nu \cdot dy \tag{26}$$

### 3. Numerical modeling and validation

The numerical resolution of the governing dimensionless Eqs. (5)–(12) as well as the associated boundary conditions Eqs. (14)–(16) is based on the finite volume method given by Patankar [17]. The SIMPLER algorithm has been adapted to perform the velocity-pressure coupling. The domain of computation is composed of two Newtonian fluids zones separated by an interface while the upper surface is open to the atmosphere. The set of Eqs. (17)–(24) is integrated on a control volume, leading to equilibrium equations for fluxes at the interface and the free surface.

In order to verify the credibility of our computer code, it is necessary to compare our results with those published in the literature. Thus, the present numerical results are compared to those of Liu and Roux [18] who analyzed the thermocapillary convection in a shallow cavity (with an aspect ratio  $A = 4$ ) opened by its upper surface. The system is composed of two layers of immiscible liquids subjected to a temperature gradient along their interface and their free surface. The comparison represented in Fig. 2 shows a very good agreement between both results.

The subsequent numerical simulations were performed using an uniform grid of  $100 \times 100$  nodes, after analyzing the sensitivity of the results to the grid. When the residuals of the maximum mass of grid control volume are less than  $10^{-5}$ , the convergence criteria are satisfied.

### 4. Results and discussions

In this part, we present the results regarding the influence of both dynamic viscosity and thermal diffusivity ratios on the hydrodynamic and thermal fields of the thermocapillary convection within the enclosure. The calculations

were made by fixing the Marangoni and Biot numbers at 1,000 and 1, respectively.

#### 4.1. Dynamic viscosity ratio

In order to analyze the effect of the dynamic viscosity ratio, we consider, for the latter, the range of  $0.1 \leq \mu^* \leq 10$  with an increment of 10. This range is obtained by varying the upper fluid’s viscosity while the fluid 1’s (the reference fluid) viscosity is kept constant. The simulations have been performed for  $Ma = 1,000$ ,  $Bi = 1$  and  $\alpha^* = 1$ .

Fig. 3 presents the contours of the streamlines and isotherms in the two immiscible regions. For  $\mu^* = 0.1$ , which means that the upper phase is less viscous than the lower phase, we can distinguish for the fluid 1, a large main clockwise recirculation cell as well as a small secondary anticlockwise one located along the left wall of the lower compartment of the cavity, especially at the corner. Indeed The upward movement occurs within a large region which characterizes the principal cell. The rest of the fluid undergoes, as for it, a counter-clockwise rotation because of the main cell and the presence of fluid 2 located above. On the other hand, in the upper compartment, occupied by fluid 2, the flow structure is described by two large main cells and a small anticlockwise recirculation cell developed in the right corner above the interface. The main cells are counter-rotative, they rotate in opposite directions; the left one is clockwise. In this case, the particles move at low velocity because the fluid occupying the lower compartment is more viscous than the one in the upper phase. At the free surface where the surface tension gradients are applied, the boundary conditions of velocity and temperature are essential to solve the physical problem [Eqs. (19)–(21)]. The air particles pull the fluid 2’s particles to the left, in this case the free surface acts as a lid driven.

For  $\mu^* = 1$ , that is, as the two fluids share the same thermophysical properties, the lower domain is occupied by a Single clockwise cell structure (negative values of streamlines), corresponding to the ascension of the fluid along the left hot wall and its descension along the right cold wall. However, the upper domain is occupied by a two

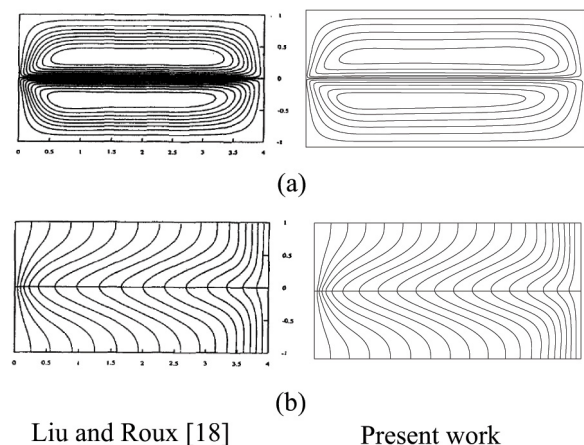


Fig. 2. Streamlines (a) and isotherms (b).  $Pr = h^* = \kappa^* = \rho^* = \lambda^* = 1$ ;  $A = 4$ ;  $\mu^* = 10$ ;  $Ma = 1,000$ .

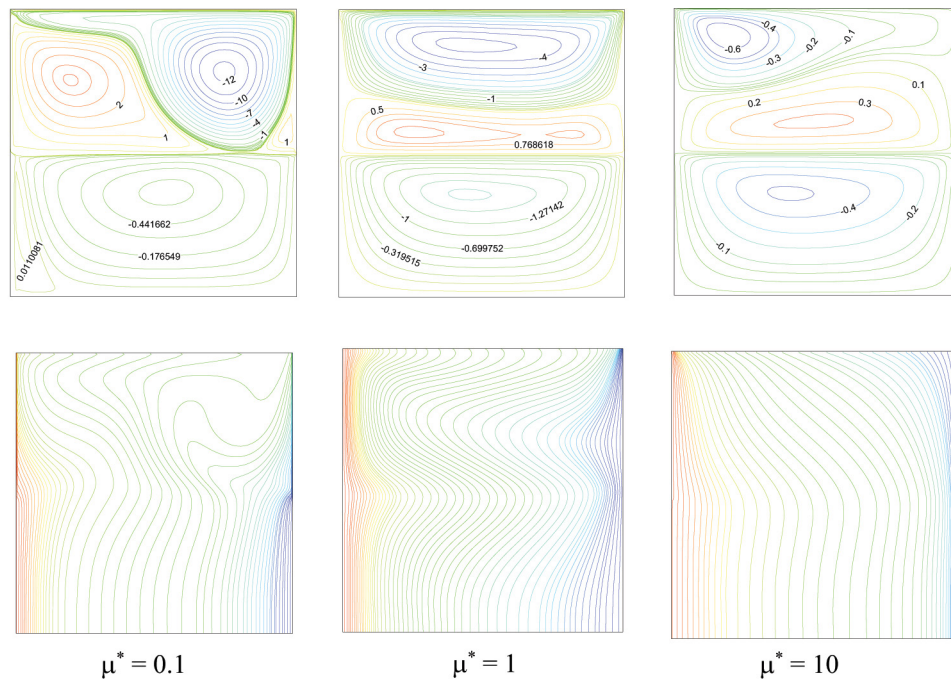


Fig. 3. Streamlines and isotherms in the two immiscible regions for different dynamic viscosity ratios.  $Ma = 1,000$ ;  $Bi = \alpha^* = \rho^* = C_p^* = 1$ .

counter-rotative cell structure: a main clockwise cell and a second intermediate anticlockwise cell set in motion by the previous one. The intermediate cell located near the interface aims to ensure the continuity of heat flux and shear stress between the two immiscible systems.

For  $\mu^* = 10$ , the intermediate anticlockwise cell occupies more space, by flattening the vortex created in the vicinity of the free surface.

Regarding the isotherms, for low values of the viscosity ratio ( $\mu^* = 0.1$ ), the heat transfer takes place by conduction in the lower compartment given the vertical stratification of the isotherms and since that the fluid in this region is more viscous. The convective regime is installed within the fluid 2 which occupies the upper compartment; the isotherms are gradually deformed, which suggests that there is an active motion of the fluid particles and a strong penetration of the flow from the free surface to the second-phase, caused by the presence of strong interfacial tension gradients on the free limit and along the interface. In this case, the formation of thermal boundary layers in the two end walls is apparent only in the lower half of the cavity. However, for a symmetrical system ( $\mu^* = 1$ ), the isotherms are considerably deformed by the flow in both layers, the heat exchange takes place by convection in both phases as well as along the interface. The development of the thermal boundary layers is observed along the side walls. The structure of the flow is almost identical in the two phases since the fluid molecules move in the two zones with the same velocity. When the upper phase is more viscous than lower one ( $\mu^* = 10$ ), the isotherms are inclined towards the cold wall because the fluid is entrained on the free surface, from the hot wall towards the cold wall. The thermal boundary layer thickness near the left hot wall is more intense in this case compared to the previous cases.

We present in Fig. 4 the vertical velocity component profile along the midlines of each zone, according to the variation of the dynamic viscosity ratio.

As it can be seen, the amplitude of the velocity in both phases is improved (or reduced) when the viscosity ratio is higher (or lower) than the unit. Indeed, the fluid particles move at high velocity in the lower phase when the viscosity of the fluid prevailing in this zone is very low compared to that of the fluid occupying the upper phase ( $\mu^* = 1$ ). On the other hand, for  $\mu^* = 0.1$ , the maximum velocity of the fluid particles in the lower zone decreases by decreasing the viscosity of the upper fluid (the lower phase is more viscous compared to the second-phase). It is observed from this figure that, the particles are more active in the phase presenting a low viscosity.

#### 4.2. Thermal diffusivity ratio

We present in this section, the results regarding the characteristics of the flow within the enclosure under the influence of the thermal diffusivity ratio. To do so, we consider a range of  $0.1 \leq \alpha^* \leq 10$ , by taking  $Ma = 1,000$ ,  $Bi = 1$  and  $Bi = \mu^* = \rho^* = C_p^* = 1$ .

It should be noted that the thermal diffusivity ratio ( $\alpha^*$ ) is proportional to the thermal conductivity ratio ( $\kappa^*$ ) of the two immiscible liquids, because in the present study, we set all physical properties to a value equal to 1 ( $\mu^* = \rho^* = C_p^* = 1$ ). Therefore, by varying the ratio of thermal conductivity of the two liquids ( $\kappa^*$ ) in the Eq. (20), we come to determine the thermal diffusivity ratio ( $\alpha^*$ ). Therefore, it is to note that a high thermal diffusivity ratio expresses a high thermal conductivity.

Fig. 5 shows the effect of the thermal diffusivity ratio on the hydrodynamic and thermal structure of the flow.



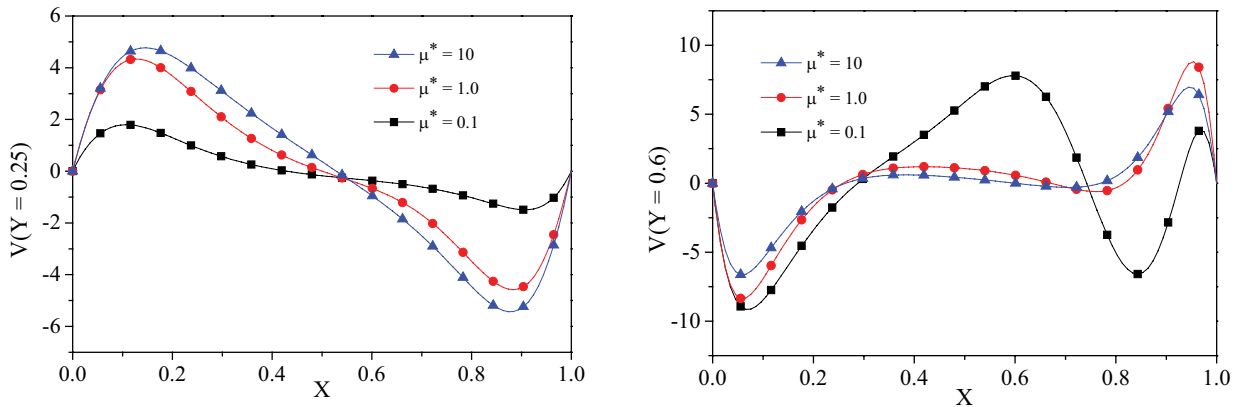


Fig. 4. Vertical velocity component profile at the midlines of each zone, according to the viscosity ratios.  $Ma = 1,000$ ;  $Bi = \alpha^* = \rho^* = C_p^* = 1$ .

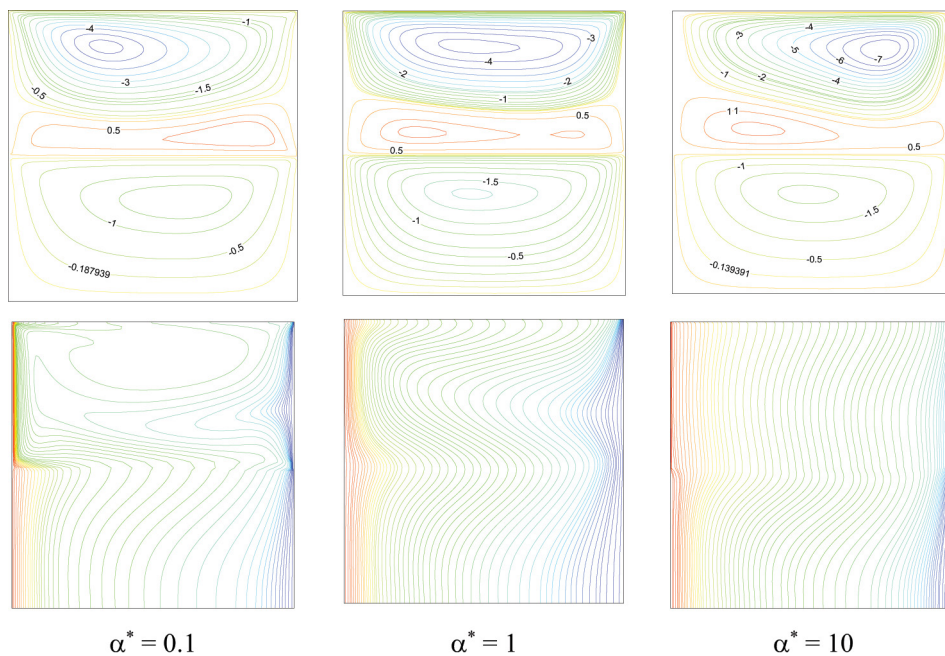


Fig. 5. Streamlines and isotherms in the two immiscible regions for different thermal diffusivity ratios.  $Ma = 1,000$  and  $Bi = \mu^* = \rho^* = C_p^* = 1$ .

We can notice, for all values of  $\alpha^*$ , that the lower compartment is formed of a single clockwise cell which occupies the entire layer. In the upper phase, the flow included a large clockwise cell located in the upper part of the cavity, at the vicinity of the free surface. The core of this vortex tends to move towards the cold wall as  $\alpha^*$  increases. Along the interface, an interfacial anticlockwise cell develops in the upper layer to satisfy the conditions of continuity along this separation, that is, the two layers are mechanically coupled via this cell. Its thickness depends on the dimension of the main upper cell and becomes more important near the hot wall as  $\alpha^*$  increases. It is to note that the flow is affected by thermocapillary stresses along the free surface and along the interface. For a positive Marangoni number, the temperature coefficient due to the surface tension must also be positive. For this, the interfacial tension must depend inversely on the temperature, that is,  $\sigma_s/\sigma T$  must

be negative: the hot regions of the cavity have low values of surface tension and vice versa. A very high Marangoni number ( $Ma = 1,000$ ) implies a strong temperature gradient and hence, a strong surface tension gradient, which generates a movement of the fluid from low values of interfacial tension to large values. This is why the fluid particles located below the free surface move from the hot regions to colder regions.

As for the isotherms, they are described, for  $\alpha^* = 0.1$  by the presence of thin thermal boundary layers along the side walls in the upper compartment of the cavity. Since the lower phase is more conductive of heat. Therefore, the thermal energy created by the movement of particles in the lower phase diffuses widely towards the upper phase, creating high temperature gradients along the vertical walls. For  $\alpha^* = 10$ , that is, the fluid 2 is more diffusive, we notice that the boundary layers are more intense in the lower phase

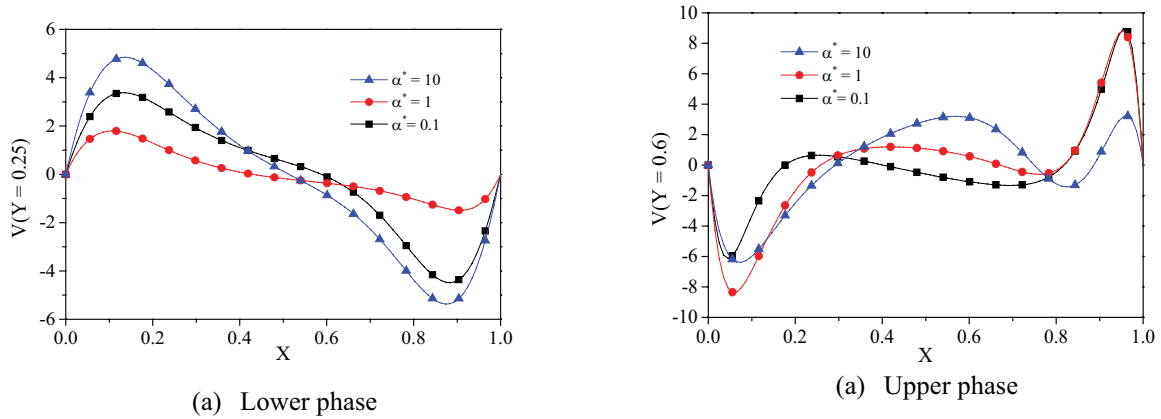


Fig. 6. Vertical velocity component profile at the midlines of each zone, according to the diffusivity ratios.  $Ma = 1,000$  and  $Bi = \rho^* = C_p^* = 1$ .

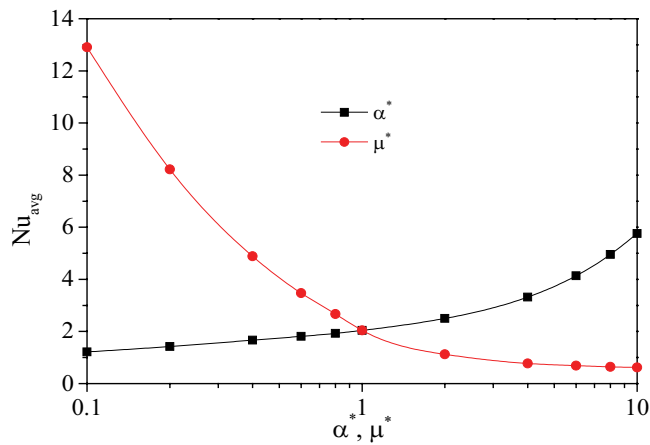


Fig. 7. Average Nusselt number according to dynamic viscosity and thermal diffusivity ratios.  $Ma = 1,000$  and  $Bi = \rho^* = C_p^* = 1$ .

unlike the previous case. This means an excellent heat transfer from the upper phase to the lower phase. On the other hand, when the two fluids have the same capacity to transfer the heat ( $\alpha^* = 1$ ), the distribution of the boundary layer is similar in both phases. In a system composed of two immiscible phases, the heat transfer occurs from the more conductive phase towards the less conductive phase.

The vertical velocity profiles along the median line of each phase for different thermal diffusivity ratios are presented in Fig. 6.

For  $\alpha^* = 10$ , the fluid of the upper phase is more conductive of heat compared to the lower fluid. Therefore, the heat produced in the upper phase is sufficiently transmitted downwards which causes a displacement of the particles at high velocity in this phase. By decreasing the thermal diffusivity ratio of the upper fluid ( $\alpha^* = 0.1$ ), the velocity of the fluid particles decreases in the lower zone because the fluid 2 is less conductive of heat (it has a weak transmission).

#### 4.3. Heat transfer rate within the enclosure

We examine now, the effect of dynamic viscosity and thermal diffusivity ratios on the average Nusselt number through Fig. 7.

It can be seen that by increasing the viscosity of the fluid 2, the heat transfer within the cavity decreases. For a viscosity ratio of 0.1, the lower phase is viscous in comparison with the upper phase. The latter has a direct contact with air. In this case, the fluid particles in this location move and circulate at high velocity, creating a heat transfer caused by the differences in temperatures provoked by a high Marangoni number. By increasing the viscosity ratio, heat transfer weakens within the cavity. The heat formed and stored in the upper part of the cavity (between fluid 2 and air) is not sufficiently transmitted to the lower part.

Regarding the effect of the thermal diffusivity ratio, we notice that the heat transfer is improved by the increase in this ratio. In open cavities, there is a strong coupling between the three phases (liquid 1 – liquid 2 – air) and the heat transfer occurs from the upper part (in contact with air) to the lower part. Heat transfer is reinforced when the upper phase in contact with air is less viscous and more conductive of heat.

### 5. Conclusion

In this paper, thermocapillary convection inside a square enclosure filled with two immiscible liquids has been studied numerically using the finite volume method. The top of the cavity is open to the ambient air. The free surface of the enclosure and the interface which separates the two phases are subjected to surface tension gradients. The effects of two important parameters: the ratios of dynamic viscosity and thermal diffusivity have been interpreted. The main results are expressed as follows:

- A high Marangoni number ( $Ma = 1,000$ ) generates a movement of the fluid from low values of interfacial tension to large values. This is why the fluid particles located below the free surface move from the hot regions to colder regions;
- Particles are more active in the phase showing a low viscosity;
- In a system composed of two immiscible phases, the heat transfer occurs from the more conductive phase towards the less conductive one;
- Heat transfer is reinforced when the upper phase in contact with air is less viscous and more conductive of heat.

**Symbols**

$C_p$	—	Specific heat capacity, J/kg·K
$g$	—	Gravitational acceleration, m/s <sup>2</sup>
$H$	—	Height of cavity, m
$h$	—	Convection heat transfer coefficient
$k$	—	Thermal conductivity, W/m·K
$Nu$	—	Local Nusselt number on the hot wall
$Nu_{avg}$	—	Average Nusselt number along the hot wall
$p$	—	Pressure, Pa
$P$	—	Dimensionless pressure $P = pH^2/\rho\alpha^2$
$Pr$	—	Prandtl number, $\nu/\alpha$
$Ma$	—	Marangoni number $Ma = \gamma\Delta T H/\mu\alpha$
$Bi$	—	Biot number $Bi = Hh_{air}/k$
$T$	—	Temperature, K
$u, v$	—	Velocity components in $x, y$ directions, m/s
$U, V$	—	Dimensionless velocity components, $Hu/\alpha, Hv/\alpha$
$x, y$	—	Cartesian coordinates, m
$X, Y$	—	Dimensionless Cartesian coordinates, $x/H, y/H$

**Greek symbols**

$\alpha$	—	Thermal diffusivity, m <sup>2</sup> /s, $\alpha = k/\rho C_p$
$\beta$	—	Thermal expansion coefficient, K <sup>-1</sup>
$\Delta T$	—	Difference of temperature, K, $\Delta T = T_h - T_c$
$\rho$	—	Density, kg/m <sup>3</sup>
$\sigma$	—	Surface tension
$\theta$	—	Dimensionless temperature $\theta = (T - T_c)/\Delta T$
$\gamma$	—	Surface tension temperature coefficient, $\gamma = -\partial\sigma/\partial T$
$\mu$	—	Dynamic viscosity, kg/m·s
$\nu$	—	Kinematic viscosity, m <sup>2</sup> /s

**Subscripts**

$c$	—	Cold
$h$	—	Hot
$0$	—	Reference state
$1$	—	Lower liquid
$2$	—	Upper liquid
$*$	—	Physical property ratio of liquid 2 to liquid 1

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