



Numerical analysis of the dynamic aspects of flow through a microchannel with sudden expansion

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ABSTRACT

In the present work the results were reported of numerical simulations of the dynamic aspects of a water flow through a circular microchannel with a sudden expansion. The hydraulic diameter was 0.1 mm with inlet velocity ranging from 0.01 to 10.1 m/s corresponding to Reynolds numbers that varied from 1 to 1,000 for three expansion ratios $R_p = 1.51, 2$ and 3. The numerical resolution of the configurations was performed using the calculation code ANSYS CFX 19.2, based on the finite volume method. The simulation results obtained were compared with theoretical ones stated for conventional sized channels and experimental studies with microchannels. The comparison was made in terms of reattachment length and friction factor as well as the singular coefficient due to the expansion. Good agreement was observed with the experimental results in microchannels but a slight deviation was obtained compared to the theory of conventional channels.

Keywords: Numerical analysis; Microchannels; Water flow

1. Introduction

A new discipline known as Microfluidics is a dynamic field of research that deals with technology system conveying fluids with at least one characteristic dimension in the micrometer range. These are flows at the micrometer scale. Mechanical, fluidic, electromechanical, and thermal systems can now be miniaturized to micrometer dimensions thanks to the quick development of microfabrication technology. The theory of fluid mechanics in microchannels was based on models developed for the description of flows at the micrometer scale.

Experimental and numerical studies have been carried out to identify the limits of validity of models in microfluidics. Abdelall et al. [1], for example, experimentally determined the pressure drop across a mini channel with a sudden expansion and contraction with the diameters of the small channel and the large channel being 0.84 and 1.6 mm, respectively. The fluids used were water and air in

single-phase flow and a mixed one with a Reynolds number ranging from 160 to 12,960. The singular pressure drop coefficients for a monophasic flow were close to those predicted by classical theory, except for a liquid flow in the sudden contraction, where a higher pressure drop coefficient was found. A correlation was developed by the authors, concerning the pressure drop in two-phase flow through a mini channel with sudden contraction.

Chalfi and Ghiaasiaan [2] performed an experimental study like that of Abdelall et al. [1] with mini ducts of the same dimensions. The results for air flow through the sudden expansion section were consistent with the theoretical predictions for Reynolds numbers greater than 5,000. However, for water and air flow in the laminar regime, the singular pressure drop coefficient increased significantly with the Reynolds number. In the case of sudden contraction, the singular pressure drop coefficient in the turbulent regime of air, and the laminar regime of water, was close

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to the theoretical value. This was similar with the observations of Abdelall et al. [1]. According to the authors, the theoretical model was insufficient to predict the pressure losses associated with a flow undergoing sudden expansion or contraction. They propose correlations for this type of configuration.

Khodaparast et al. [3] carried out a detailed experimental study of water flow through a circular micro channel provided with a sudden expansion for two inlet diameters 0.508 and 0.257 mm corresponding to expansion ratios R_p (i.e., outlet diameter/inlet diameter) of 1.96 and 1.51, respectively. The Reynolds numbers recorded over all measurement series varied between 1 and 120. The authors noted good agreement with the observations of the numerical results in the case of the pipes of conventional size, of Oliveira et al. [4], Hammad et al. [5] and Dağtekin and M. Ünsal [6]. However, the pressure loss coefficient measured in the range of Reynolds numbers studied did not correspond to theoretical predictions as well as to conventional empirical correlations. Likewise, an experimental study was carried out by Kawahara et al. [7] for a single-phase and two-phase flow through a micro-channel of rectangular section provided with a sudden contraction. Two contraction ratios σ (outlet section/inlet section) were studied, corresponding to the hydraulic inlet diameters 0.32 and 0.38 mm, respectively.

In the present work, the results were reported of numerical simulations of the dynamic aspects of water flow through a circular microchannel with a sudden expansion. The numerical resolution of the configurations was performed using the calculation code ANSYS CFX 19.2, based on the finite volume method. The simulation results obtained were compared with theoretical ones for conventional sized channels and experimental studies with microchannels.

1.1. Physical domain and boundary conditions

The current study consisted of a microchannel with an inlet diameter kept constant at $d = 0.1$ mm while the diameter of the expanded section D was a function of the expansion ratio $R_p = D/d$. The length of the first microchannel L_1 was chosen so that the regime was fully developed before the singularity and L_2 , length of the microchannel of diameter D so that the outlet has no influence on the formation of the vortex zone (Fig. 1). The expansions ratios considered were $R_p = 1.51, 2$ and 3 and the inlet velocities ranged from 0.01 to 10.1 m/s, corresponding to Reynolds numbers varying from 1 to 1,000. Other conditions consisted of:

- Inlet: constant axial velocity, temperature of the fluid (water) = 293.15 K.
- Outlet: pressure = atmospheric pressure.
- No slip condition on the walls is imposed ($u = 0, v = 0$).

2. Mathematical model

To solve the problem, simplifying assumptions were necessary:

- Viscous and incompressible fluid.
- Negligible volume forces.

- Steady flow
- Viscous dissipation and external heat transfer negligible.

The equations for conservation of mass and momentum become:

$$\frac{\partial(u_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial(u_i u_j)}{\partial x_i} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial x_j} + \nu_{nf} \frac{\partial}{\partial x_i} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \quad (2)$$

where ρ is density, P is the pressure, and ν is the fluid's kinematic viscosity.

There was interest in the impacts of changing the expansion ratio, and a Reynolds number change on reattachment length L_r , the linear pressure drop represented by the frictional coefficient λ as well as the coefficient of singular pressure drop ξ due to the suddenly changing diameter of the microchannel.

The Reynolds number was defined as Eq. (3):

$$Re = \frac{\rho U_m D_h}{\mu} \quad (3)$$

where U_m is the average velocity, the working fluid's dynamic viscosity is represented by μ , and D_h is the minichannel's hydraulic diameter.

The theoretical relationship of Borda–Belanger was used to calculate the singular pressure drop coefficient resulting from a sudden expansion [Eq. (4)] [4]:

$$\xi = \left(1 - \frac{1}{R_p^2} \right)^2 \quad (4)$$

The singular pressure drop coefficient is calculated from a momentum balance between the expanded section and the reattachment of the flow at the wall, which leads to the following expression [8]:

$$\xi = \frac{2\Delta P_s}{\rho V_1^2} \quad (5)$$

where ΔP_s : average pressure drop between expansion section S1 and reattachment of the flow at the wall S2 (Fig. 2) and V_1 : average velocity upstream of the expanded section.

3. Numerical method

The resolution of the problem was realized by a commercial code CFX19.2 based on the finite volume method. The SIMPLE algorithm was used to solve the velocity and pressure-coupling problem and second-order central differencing was utilized to discretize the spatial derivative. When the root-mean-square residuals of the governing equations are less than 10^{-5} , convergence is attained.

A tetrahedral mesh was applied, as it was the most suitable for cylindrical geometries. The distribution of nodes

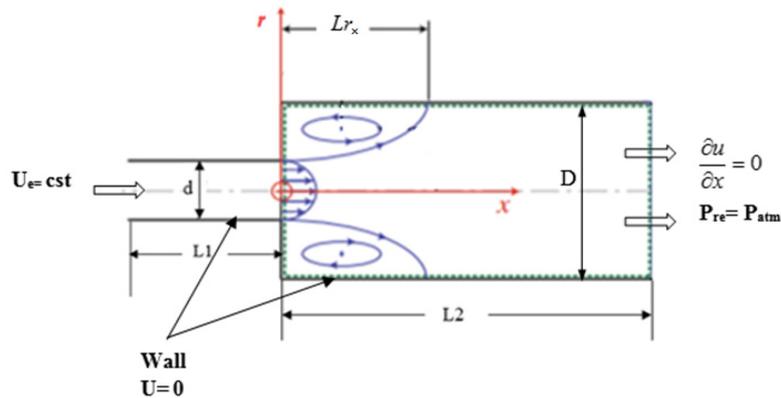


Fig. 1. Geometric flow configuration and boundary conditions.

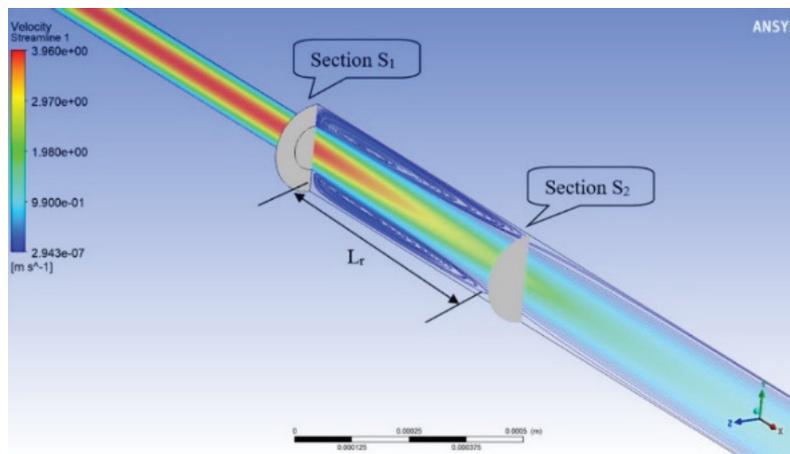


Fig. 2. Streamlines and reattachment length L_r .

was generated while ensuring a good refinement in the areas close to the walls and singularity planes, where the velocity and pressure gradients were very important (Fig. 3).

4. Results

4.1. Reattachment length

The reattachment length L_r was defined as the distance between the expansion section and the point of attachment of the fluid to the wall L_{rx} dimensionless to the inlet diameter d ($L_r = L_{rx}/d$). In other words, the length of the vortex zone (Fig. 4). First, a comparison of the results was obtained by numerical simulation with experimental measurements given by Khodaparast et al. [3] relating to the reattachment length as a function of the inlet Reynolds number varying from 1 to 1,000. This is schematized in Fig. 3, for an expansion ratio $R_p = 1.51$.

It was noted that as the Reynolds number rose, the reattachment length also increased. A first evolution of exponential form was reached until a critical Reynolds number $Re_c = 30$, beyond this value, the increase was linear. These observations were similar to those given by Khodaparast et al. [3]. The results obtained by numerical simulation were close to the experimental measurements [3] for Reynolds

numbers ranging from 2 to 100 with a maximum deviation of 6%. In this case fitting curves of the same shape as the experimental ones could be established.

Fig. 5 represents the evolution of the reattachment length L_r function of the inlet Reynolds number ranging from 1 to 1,000 for the three expansion ratios $R_p = 1.51, 2$ and 3. It was observed that the curve was the same for the three expansion ratios and increased proportionally with an increase of the expansion ratio. Furthermore, it was noticed that for the three ratios a nonlinear evolution for low Reynolds numbers was seen. This was followed by a linear increase while the Reynolds numbers became higher. This change of tendency was characterized by a critical Reynolds number Re_c . This value decreased when the expansion ratio R_p increased. These observations were in agreement with the conclusions of Badekas and Knight in macrochannel [9] and Khodaparast et al. in microchannels [3].

Fig. 6 is a comparison between numerical results of Badekas and Knight [9], Dagetekin and Unsal [6] and experimental results of Hammad et al. [5] for conventional size channels, as well as the experimental data of Khodaparast et al. [3]. in the case of microchannels with sudden expansion. Results obtained by numerical simulation in the current study were also reported in this figure. It was noticed that the results for macro and microchannels

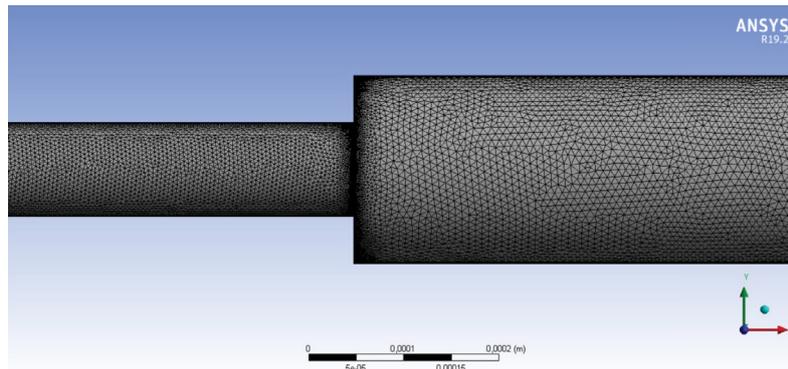


Fig. 3. Mesh generated on the studied physical domain.

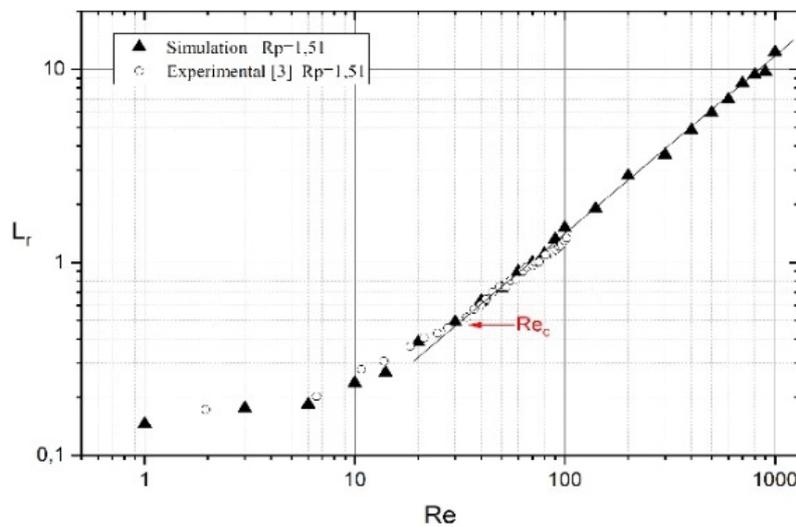


Fig. 4. Comparison reattachment length L_r vs. Re .

followed the same linear evolution for the Reynolds number range between 20 and 200. The current results were close to the numerical predictions as well as the experimental data for the three expansion ratios $R_p = 1.51, 2$ and 3 and all Reynolds number considered.

4.2. Friction coefficient λ

A comparison with experimental data [3] and the classical theory of the friction coefficient λ depending on the outlet Reynolds number for the three ratios of expansion $R_p = 1.51; 2; 3$ is shown in Fig. 7. It was seen that the friction coefficient λ decreased with an increase in Reynolds number, presumably because the inertia forces of the fluid were predominant compared to the viscous forces, It was observed that λ was similar for the three expansion ratios $R_p = 1.51, 2$ and 3 which corresponded to the outlet hydraulic diameters $0.151, 0.2$ and 0.3 mm, respectively. This meant that the scaling effect was negligible in this range of hydraulic diameters. Likewise, good agreement was observed between the results of the numerical simulations and the formula from the theory. In this case a similar correlation can be established of the form $\lambda = \text{constant}/\text{Reynolds}$.

Simulations gave a constant of 57 for the three expansion ratios studied $R_p = 1.51, 2$ and 3 for outlet Reynolds numbers Re_D ranging from 0.33 to 662. This value was close to 64 formulated by the classical theory [Eq. (4)].

4.3. Singular pressure drop coefficient ξ

Fig. 8 illustrates a comparison of experimental data [3] and the singular pressure drop coefficient determined by numerical simulation and theory [Eq. (4)] function of the Reynolds number for an expansion ratio $R_p = 1.51$. The singular pressure drop coefficient ξ decreased significantly with a Reynolds number increase and reached a constant value after a value of Reynolds number of 30. Good agreement was also observed between simulation results and experimental data. However, those results were higher than the theoretical predictions which gave a constant value [Eq. (4)]. It was noticed that the evolution of the coefficient of singular pressure drop ξ obtained from numerical simulation followed the same course as the experimental data [3].

The singular pressure drop coefficient ξ calculated from the numerical results vs. Reynolds number for the three ratios of expansion $R_p = 1.51, 2$ and 3 is shown in Fig. 9. A

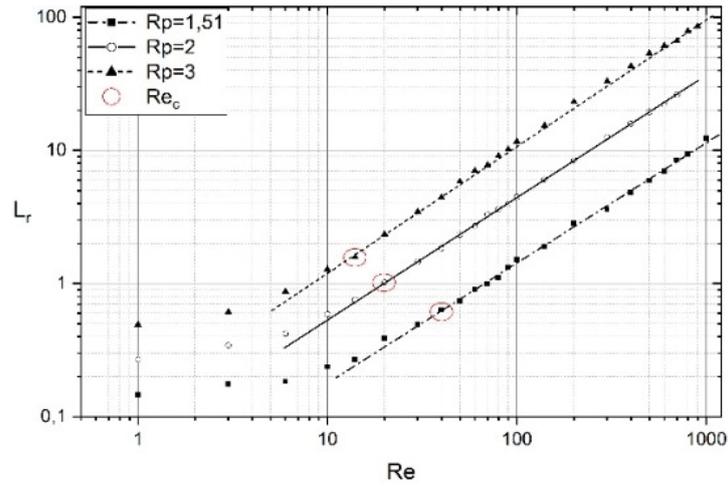


Fig. 5. Evolution of reattachment length L_r vs. Re .

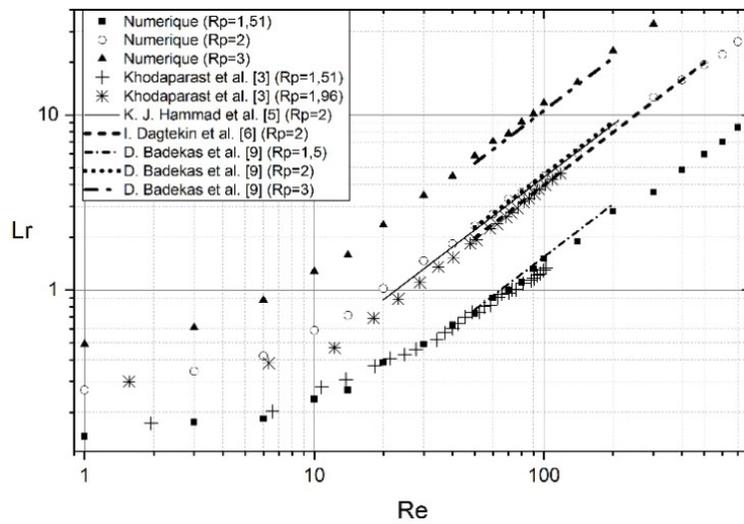


Fig. 6. Reattachment length vs. Reynolds number.

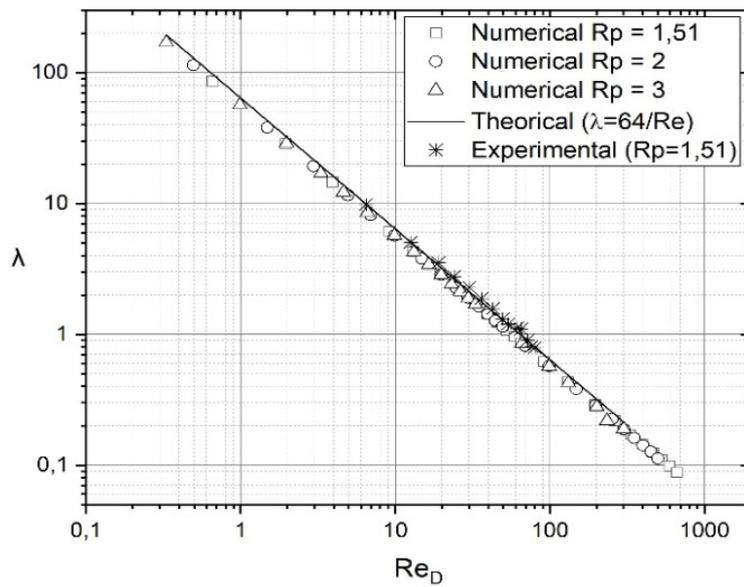


Fig. 7. Friction coefficient vs. Reynolds number.

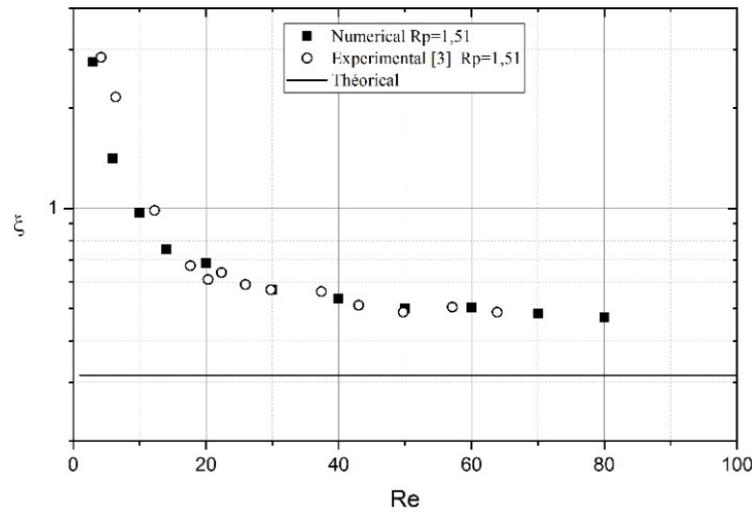


Fig. 8. Comparison coefficient ξ vs. Re.

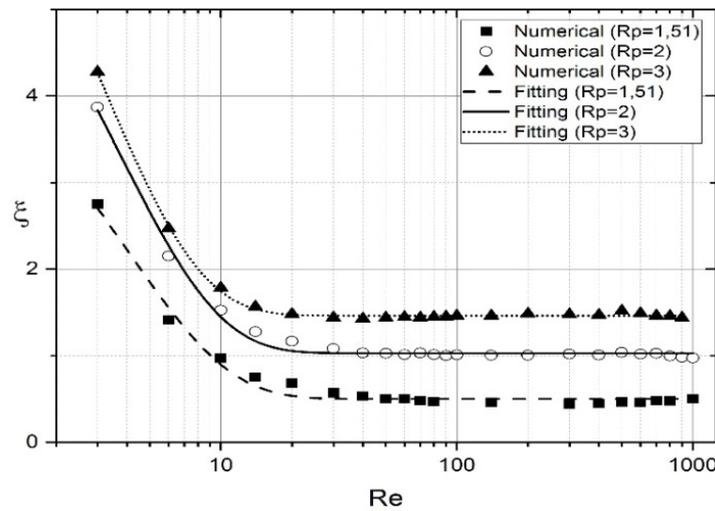


Fig. 9. Coefficient ξ vs. Re.

Table 1

Numerical correlation of the fit curves for the reattachment length and the singular pressure drop coefficient ($R_p = 1.51, 2, 3$)

R_p	Re_c	$L_r (Re > Re_c) (R^2 = 0.99)$	$L_r (Re < Re_c) (R^2 = 0.998)$	$\xi (R^2 = 0.993)$
1.51 Exp [3]	30	$L_r = 0.165e^{0.04Re}$	$L_r = 0.154 + 0.011Re$	$\xi = 0.509 + 5.143e^{-0.185Re}$
1.51	30	$L_r = 0.14129e^{0.04939Re}$	$L_r = 0.117Re + 0.19426$	$\xi = 0.50254 + 4.59631e^{-0.24552Re}$
2	14	$L_r = 0.25758e^{0.08263Re}$	$L_r = 0.03704Re + 0.60681$	$\xi = 1.02809 + 6.32774e^{-0.27097Re}$
3	10	$L_r = 0.45311e^{0.10419Re}$	$L_r = 0.10423Re + 0.76197$	$\xi = 1.46355 + 7.54081e^{-0.32984Re}$

significant decrease was noticed for low Reynolds numbers up to a critical value $Re_c = 30$. After this, the singular pressure drop coefficient tended to a constant value which was predicted by the classical theory. Table 1 represents the fit curves calculated from numerical results for the reattachment length L_r and the coefficient of singular pressure drop for the three expansion ratios $R_p = 1.51, 2$ and 3 for a Reynolds number range from 1 to 1,000.

5. Conclusions

A comparison of the results obtained with conventional theory revealed a difference in the singular pressure drop coefficients. The current results were in good agreement with experimental results established in mini and micro-channels, reporting on the length of reattachment, and the fiction coefficient. Many researchers working in the field

of mini and microfluidics have found similar results in the range of Reynolds numbers studied. Based on the present numerical simulation data, fit correlations were established for Reynolds numbers ranging from 1 to 1,000 for expansion ratios of 1.51, 2 and 3. In summary, the applicability of the classical laws of hydrodynamics can be extended to micrometer scales, at least up to the values studied in the case of a laminar regime.

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