

## Improving plunger motion law based on multi-objective optimization algorithm

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### ABSTRACT

At present, there are many ways to improve the pumping efficiency by increasing the pump filling degree to increase the output profit, but many researchers in the study did not pay attention to the pumping rod in the upper and lower stops due to the acceleration generated by the inertia force on the pumping rod pulling, compression and the rod and pipe bias abrasion caused by the shortening of the inspection cycle of the pump and so on, and seldom carry out the optimization of plunger's movement law to achieve the improvement of the pump filling degree under the same stroke, the number of times the study. The study of the optimization of the plunger movement law to improve the pump filling degree under the same stroke and stroke times is rarely carried out. Therefore, this paper establishes the target equation of pump filling degree according to the change rule of fluid volume into the pump and the change rule of pump cavity volume, and uses multi-objective optimization genetic algorithm combined with the constraint equation to optimize the plunger movement law without changing the stroke and stroke times. The research results show that: according to the optimized plunger movement law, the pump filling degree is increased by 5%, and the average acceleration of plunger movement is decreased by 75.36%; this optimization method not only improves the pump filling degree but also reduces the pulling and compression loss of pumping rod due to the inertia force generated by acceleration, and prolongs the checking cycle of the pump. The research method can provide a certain reference for the optimization of the plunger motion law of the pumping machine.

*Keywords:* Plunger; Motion optimization; Objective equation; Inertia force; Genetic algorithm; Pump filling level

### 1. Introduction

Most of the oil beam pumping units in China are used for oil recovery, but most of the pumping units have low pumping efficiency, and the production cost is large while the output profit is small [1]. To improve the pumping efficiency is to improve the pump filling degree [2], the main factors affecting the pump filling degree are the structural parameters of the pumping machine and the plunger movement law [3–8]. Without changing the structural parameters of the pumping unit, the only way to improve the pump filling degree is to optimize the plunger movement.

At present, most of the pumping parameters such as stroke and stroke times are optimized to improve the pump filling degree and pumping efficiency, and the optimization study of the motion law in a cycle is less. And less attention is paid to the optimization before and after the plunger due to the acceleration at the upper and lower stops of the inertial force caused by the pumping rod pulling, compression loss, resulting in rod and pipe bias wear and check the pump cycle is short and other issues. For example, Duan et al. [9] determined the optimal supply and discharge coordination point according to the supply and discharge coordination curve, and then determined a reasonable degree of

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submergence and dynamic liquid level, and optimized the plunger motion law by Fourier transform decomposition and optimization algorithm without changing the stroke and stroke times, and achieved better results, but the acceleration of the plunger at the stop point after optimization is obviously larger, which is prone to rod and pipe misalignment and other problems. Therefore, in this paper, under the consideration of the influence of the acceleration of the plunger movement, the multi-objective optimization algorithm is designed: according to the change law of fluid volume and the change law of the pump cavity volume to establish a mathematical model for calculating the degree of pump filling, with the degree of pump filling as the main optimization objective, the acceleration of the plunger movement as the secondary optimization objective, and the multi-objective optimization model with the use of genetic algorithm to calculate the optimal plunger movement law.

**2. Optimization algorithm**

Genetic algorithms have been widely used in objective optimization problems with constraints since their introduction in the 1970s [10–13]. The main idea of genetic algorithm is derived from Darwin’s theory of evolution, drawing on the natural law of “survival of the fittest”, and modeling the optimization problem as a biological evolution problem. By initializing the population and calculating the fitness of individuals in the population, selecting the individuals with high fitness for replication, generating new population individuals through a series of crossover and mutation operations to further eliminate the individuals with low fitness, thus continuously retaining individuals with high fitness, and after several iterations of screening, an optimal individual will be generated [14–17], which is the optimal result of the value of the objective function. Therefore, this paper mainly adopts genetic algorithm plus constraint function to carry out multi-objective optimization to get the optimal motion pattern of the plunger in the upstroke. The flow chart of the algorithm is shown in Fig. 1.

As can be seen from Fig. 1, the establishment of genetic algorithm before the need to establish the target equation and

constraint equations, so this paper first of all according to the fluid volume change law and the pump chamber volume change law to establish the calculation of the pump filled with the degree of the target equation, and then through the plunger displacement model to get the target equation of the plunger acceleration, and finally through the constraints of the constraints analysis of the constraint equations established.

**3. Optimization model**

*3.1. Fluid volume modeling*

Assume that the following conditions are satisfied when the fixed valve is open:

- (1) The friction loss generated by the fluid entering the pump is neglected;
- (2) The density and pressure of the fluid at each point in the pump are the same;
- (3) In  $t = 0$  moment pump pressure for the opening pressure of the pump valve  $P_0$ .

According to the continuity of the fluid at each point in the pump and the principle of conservation of mass, it can be seen that in the process of upward movement of the plunger, the increase in the mass of the fluid in the pump should be equal to the mass of the fluid that flows into the pump barrel through the gap of the fixed valve.

The increase in fluid  $\Delta M$  in the pump at the moment  $t + \Delta t$  is:

$$\Delta M = M_{t+\Delta t} - M_t \tag{1}$$

Among them:

$$M_{t+\Delta t} = [A(L_0 + L_p + \Delta L_p) - (V_s + \Delta V_s)](\rho + \Delta\rho) \tag{2}$$

$$M_t = [A(L_0 + L_p) - V_s]\rho \tag{3}$$

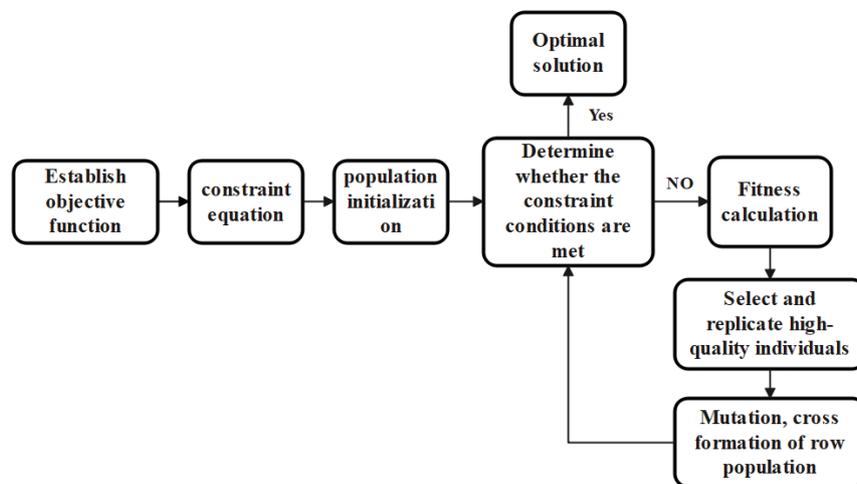


Fig. 1. Algorithm flow chart.

Omitting the second-order minimum term, the increment  $\Delta M$  of the fluid is:

$$\Delta M = [A(L_0 + L_p) - V_s] \Delta \rho - \rho \Delta V_s + A \rho \Delta L_p \quad (4)$$

where  $L_0$  is the anti-punching distance, m;  $L_p$  is the displacement of the plunger at the moment  $t$ , m;  $V_s$  is the volume of the space between the valve ball and the valve seat, m<sup>3</sup>;  $\rho$  is the fluid density, kg/m<sup>3</sup>;  $A$  is the cross-sectional area of the pump, m<sup>2</sup>.

The mass  $\Delta M$  of fluid flowing into the pump through the valve gap at the moment  $\Delta t$  in the upstroke is:

$$\Delta M' = KA_s \sqrt{\frac{2|P_c - P|}{\rho_l}} \rho_l \Delta t \quad (5)$$

where  $K$  is the flow coefficient;  $A_s$  is the valve gap overflow area, m<sup>2</sup>;  $\rho_l$  is the density of the liquid flowing through the valve, kg/m<sup>3</sup>;  $P_c$  is the submerged pressure, MPa;  $P$  is the fluid pressure in the pump, MPa.

Since  $\Delta M = \Delta M'$ , then organizing the two equations yields the following continuity equation:

$$\frac{d\rho}{dt} = \frac{KA_s \sqrt{\frac{2|P_c - P|}{\rho_l}} \rho_l + \rho \frac{dV_s}{dt} - A\rho \frac{dL_p}{dt}}{A(L_0 + L_p) - V_s} \quad (6)$$

Since the fluid density  $\rho$  has a certain correlation  $P = f(\rho)$  with the pressure  $P$  and there is a relationship  $P = f(t)$  between the pressure and time:

$$\frac{dP}{dt} = \frac{d\rho}{dt} \times \frac{dP}{d\rho} \quad (7)$$

Then Eq. (6) can be rewritten as:

$$\frac{d\rho}{dt} = \frac{KA_s \sqrt{\frac{2|P_c - P|}{\rho_l}} \rho_l + \rho \frac{dV_s}{dt} - A\rho \frac{dL_p}{dt}}{A(L_0 + L_p) - V_s} \frac{d\rho}{dP} \quad (8)$$

According to the geometric structure parameters of the fixed valve in the pump can get the space volume  $V_s$  of the valve ball and seat and the valve clearance overflow area  $A_s$ , the specific calculation formula is as follows:

$$V_s = \frac{1}{3} \pi r_{hl}^2 (b - a - 2) + \frac{2}{3} \pi r_h (r_h^2 - a - a^2) - \frac{2}{3} \pi r_h^2 \left[ 1 - \frac{b}{\sqrt{r_{hl}^2 + b^2}} \right] \quad (9)$$

Among them:  $a = b - h$ ,  $b = \sqrt{r_h^2 - r_{hl}^2} + h$

$$A_s = 2\pi r_{hl} \frac{\sqrt{1 - c^2} + 0.5d}{\sqrt{1 + 2d\sqrt{1 - c^2} + d^2}} \quad (10)$$

Among them:  $c = \frac{r_{hl}}{r_h}$ ,  $d = \frac{h}{r_h}$

where:  $r_{hl}$  is the maximum tight radius of the seat port, related to the cone angle angle, ball radius, seat hole width, m;  $r_h$  is the radius of the seat hole, m;  $h$  is the moving height of the valve ball, m.

Consequently:

$$\frac{dV_s}{dt} = \pi r_{hl}^2 \left( \frac{1}{3} + \frac{2}{3} \frac{1}{\sqrt{1 + 2d\sqrt{1 - c^2} + d^2}} \right) \frac{dh}{dt} \quad (11)$$

where  $dh/dt$  indicates the movement speed of the valve ball, which can be obtained by analyzing the force on the valve ball. The movement of the valve ball is mainly affected by gravity and buoyancy, which can be obtained according to the relationship between force and acceleration of the object:

$$m \frac{d^2h}{dt^2} = F - G = A_b (P_s - P) - mg \quad (12)$$

Consequently:

$$\frac{dh}{dt} = \int_0^t A_b (P_s - P) - g dt \quad (13)$$

where  $A_b$  is the working area of the valve, the calculation equation is as follows:

$$A_b = \pi r_{hl}^2 \left( \frac{1}{3} + \frac{2}{3} \frac{1}{\sqrt{1 + 2d\sqrt{1 - c^2} + d^2}} \right) \quad (14)$$

where  $G$  is the gravity of the valve ball, N;  $F$  is the buoyancy force on the valve ball, N;  $m$  is the mass of the valve ball, kg;  $g$  is the acceleration of gravity, 9.81 m/s<sup>2</sup>.

Based on the above continuity analysis within the pump, the following fluid volume calculation model can be established:

$$\left\{ \begin{aligned} \frac{d\rho}{dt} &= \frac{KA_s \sqrt{\frac{2|P_c - P|}{\rho_l}} \rho_l + \rho \frac{dV_s}{dt} - A\rho \frac{dL_p}{dt}}{A(L_0 + L_p) - V_s} \frac{d\rho}{dP} \\ \frac{dV}{dt} &= KA_s \sqrt{\frac{2|P_c - P|}{\rho_l}} \\ \frac{dh}{dt} &= v \\ \frac{dv}{dt} &= A_b (P_s - P) - g \end{aligned} \right. \quad (15)$$

The model is a system of first order ordinary differential equations with the following initial conditions:

$$\begin{cases} P|_{t=0} = P_0 \\ V|_{t=0} = 0 \\ h|_{t=0} = 0 \\ v|_{t=0} = 0 \end{cases} \quad (16)$$

where  $V$  is the fluid volume,  $m^3$ ;  $v$  is the valve ball vertical movement speed,  $m/s$ .

### 3.2. Pump chamber volume model

The volume of the pump chamber is mainly composed of the clearance volume and the volume generated by the displacement of the plunger. Considering that the displacement of the plunger has a certain periodicity and is flexible and changeable, the Fourier series is used here to represent the displacement curve of the plunger:

$$L_p(t) = a_0 + \sum_{k=1}^n \left[ a_k \sin\left(\frac{\pi kt}{0.5T}\right) + b_k \cos\left(\frac{\pi kt}{0.5T}\right) \right] \quad (17)$$

The plunger acceleration is modeled as:

$$A_p(t) = \frac{dL_p^2}{dt^2} \quad (18)$$

The established pump chamber volume model is as follows:

$$V_p(t) = A(L_0 + L_p(t)) - V_s \quad (19)$$

This results in a model for calculating the pump filling level:

$$\beta = \frac{V(t) + AL_0 - V_s}{V_p(t)} \quad (20)$$

### 3.3. Binding equations

The constraints of this optimization model are mainly composed of the following parts:

- (1) Plunger displacement constraints: plunger displacement  $L_p = 0$  at the moment of  $t = 0$  and plunger displacement  $L_p = S$  at the final moment,  $S$  is the pumping stroke and  $N$  is the pumping stroke:

$$L_p(t)|_{t=0} = 0, L_p(t)|_{t=\frac{60}{2N}} = S \quad (21)$$

- (2) Plunger velocity constraints: the model mainly simulates the change of pump filling degree during the upstroke, therefore, the change of plunger displacement  $\Delta L_p > 0$ , that is, it is necessary to ensure that the plunger velocity  $V_{Lp} \geq 0$ :

$$\frac{dL_p(t)}{dt} \geq 0 \quad (22)$$

- (3) Plunger acceleration constraints: according to Newton's second law, the force is related to the acceleration of the object's motion, in order to ensure that the optimized plunger will not produce excessive inertia force during the movement and reduce the service life of the pumping rod, so it is necessary to constrain the acceleration of the plunger motion:

$$a \leq a_{\max} \quad (23)$$

- (4) Pump filling degree constraints: pump filling degree shall not be greater than 1:

$$\beta \leq 1 \quad (24)$$

## 4. Example calculations

Taking a well as a column, the fluid density is  $818 \text{ kg/m}^3$ ; the flow coefficient is 0.6; the diameter of the valve ball is 0.038 m; the diameter of the fixed orifice is 0.03 m; the height of the residual gap is 0.3 m; the mass of the valve ball is 0.2271 kg; the submerged pressure is 0.794 MPa; the density of the steel is  $7,800 \text{ kg/m}^3$ ; the stroke is 3.58 m; and the stroke rate is  $5 \text{ min}^{-1}$ .

Firstly, the above two objective equations and constraint equations are established in simulink, and then the number of optimization parameters is set, and sim as well as assigning functions are used to realize the link between simulink and the main file of the genetic algorithm. In this case, there are 11 optimization parameters; the number of individuals in the population is 50; the crossover probability is set at 0.85; the variation probability is set at 0.09; and the number of iterations is 500.

According to the above basic physical parameters and design method, the calculated equation of the plunger motion law is obtained through optimization calculation as follows:

$$L_p(t) = a_0 + \sum_{k=1}^5 \left[ a_k \sin\left(\frac{\pi kt}{6}\right) + b_k \cos\left(\frac{\pi kt}{6}\right) \right] \quad (25)$$

where the individual coefficient values are:  $a_0 = 3.903$ ;  $a_1 = -3.501$ ;  $a_2 = -1.511$ ;  $a_3 = 1.162$ ;  $a_4 = 0.6687$ ;  $a_5 = -0.05455$ ;  $b_1 = -0.9014$ ;  $b_2 = -2.56$ ;  $b_3 = -1.093$ ;  $b_4 = 0.4584$ ;  $b_5 = 0.2156$ .

The changes of the plunger motion pattern before and after optimization are shown in Fig. 2. From Fig. 2, it can be seen that the stroke and the number of strokes are unchanged before and after optimization, but the changes of the optimized plunger are smoother in the early stage of the motion, that is, around the lower stop, and in the later stage of the motion, that is, around the upper stop, which means that the acceleration of the optimized plunger around the upper and lower stops is smaller. From the figure, it can be seen that the acceleration at the lower stop after optimization is  $0.2144 \text{ m/s}^2$ , which is 56.31% lower than the pre-optimization; the acceleration at the upper stop after optimization is  $0.1206 \text{ m/s}^2$ , which is 74.45% lower than the pre-optimization; and the average acceleration is 75.36% lower than the pre-optimization. The results show that the optimized plunger motion model can reduce the inertia force generated in the

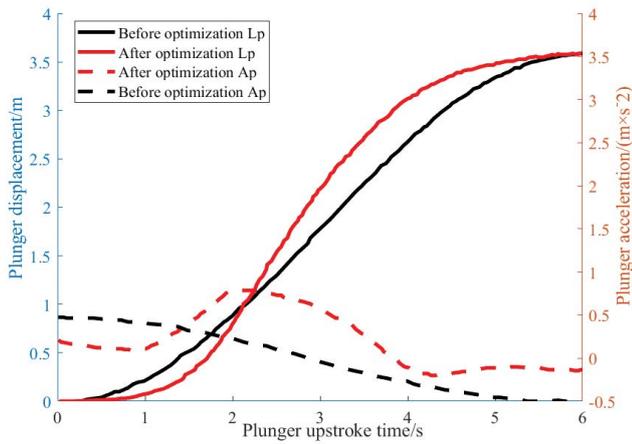


Fig. 2. Plunger upstroke motion law.

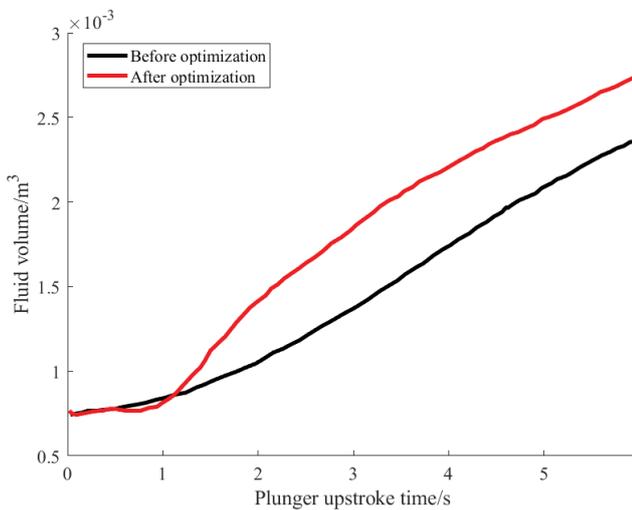


Fig. 3. Plunger upstroke fluid volume change law.

upstroke, reduce the pulling and compression effects on the pumping rod caused by the inertia force, and reduce the rod and pipe misalignment to prolong the pump checking cycle.

Before and after the optimization of the liquid volume and pump filled degree of change rule of law are shown in Fig. 3. Fig. 3 shows that the optimization of the pump fluid volume increased significantly and optimization of the fluid volume before and after the change rule is similar, are the first slow change and then a rapid increase in the last rate of change tends to flatten. Through the differential pressure change, Fig. 4 can be analyzed the reason is that the plunger began to move at a slower speed, the pump pressure change is not big; and after the plunger speed increase, the pump pressure change is obvious; pump inside and outside of the pressure difference is obvious, into the pump fluid volume increases rapidly; late due to the pump fluid pressure increases and the plunger speed reduces the pump inside and outside of the pressure difference reduces the rate of change of the volume of fluid tends to flatten out. Fig. 5 shows that after optimization, the pump filling degree increased from 22% to 27% by 5 percentage points, the optimization effects obvious.

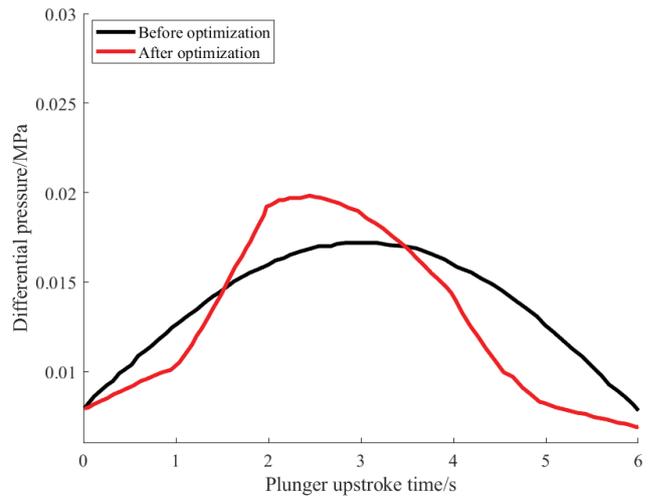


Fig. 4. Variation of differential pressure in the plunger upstroke pump.

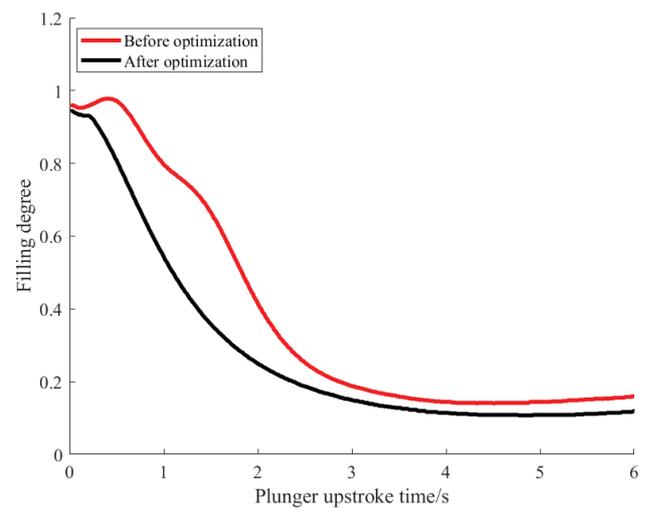


Fig. 5. Variation of filling degree of plunger upstroke pump.

### 5. Conclusion

- The target equation for optimizing the pump filling degree was established by combining the fluid volume model and the pump chamber volume model, and the target equation for optimizing the plunger acceleration was established by the Fourier transform of the plunger motion law. The two target equations combined with the plunger motion constraints to design a set of genetic algorithms to achieve the goal of optimizing the plunger motion in a stroke cycle based on the dynamic change of fluid volume.
- The average acceleration of the plunger is significantly reduced after optimization, and the acceleration at the upper and lower stops is significantly reduced. Since the inertia force is positively correlated with the acceleration, the inertia force of the plunger is significantly reduced compared with that before optimization, which reduces the stretching of the sucker rod due to the inertia force as well as the loss of rod and pipe deflection.

- On the premise of not changing the stroke and stroke times of the pumping unit, the filling degree of the pump is increased by 5% after optimization compared with that before optimization, and the improvement effect is obvious.

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